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Title: The philosophical and mathematical commentaries of Proclus on the first book of Euclid's elements (Vol. 2 of 2)
To which are added, A history of the restoration of Platonic theology, by the latter Platonists: And a translation from the Greek of Proclus's Theological elements

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*** START OF THE PROJECT GUTENBERG EBOOK THE PHILOSOPHICAL AND MATHEMATICAL COMMENTARIES OF PROCLUS ON THE FIRST BOOK OF EUCLID'S ELEMENTS (VOL. 2 OF 2) ***

THE
PHILOSOPHICAL and MATHEMATICAL
COMMENTARIES OF PROCLUS,
ON
THE FIRST BOOK OF EUCLID'S ELEMENTS.
TO WHICH ARE ADDED,
A History of the Restoration of PLATONIC THEOLOGY,
BY THE LATTER PLATONISTS:
And a Translation from the Greek of
PROCLUS'S THEOLOGICAL ELEMENTS.
IN TWO VOLUMES.



VOL II.

LONDON, PRINTED FOR THE AUTHOR:

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[Price Two Guineas in Boards.]

P R E F A C E .

Now present the reader with the remaining part of the Commentaries of Proclus on Euclid: with the addition of **T**his Theological Elements, and a History of the Restoration of the Platonic Theology, by the latter disciples of Plato. Should my design be enquired into, in combining works which the superficial observer will consider as opposite and heterogeneous, I answer that it is no less novel than certain, no less important than apposite and connected. Its novelty will be evident by assuring the reader, that a vindication of philosophical Polytheism, as embraced by the wisdom, and supported by the general voice of antiquity, is the ultimate tendency of its execution. Its connection too with geometry will be manifest to every Tyro in Platonism, and has been so copiously proved in the former volume, that it would be superfluous to repeat the demonstration in the present. I am well aware that nothing has been so much the subject of ridicule and declamation, of ignorant aspersion and impotent contempt, as the theology of the ancients. It has supplied the harangues of the pulpit with an endless variety of popular argument, and an exhaustless source of *priestly* elocution. It has been equally derided by the man of learning and the merchant, by the noble and the vulgar, by the peasant and the priest. But it still lives in the works of the ancients, it is still capable of being supported by sound reasoning, and sublime philosophy; and its intrinsic excellence and truth will extend its existence beyond the wreck of modern systems, and the desolation of ages. Like a strong and capacious ship it sails with majestic security through the ocean of time; and sustains with careless dignity the storms of opposition that roar round its well-compacted sides. The blasts of calumny may indeed impede its progress, but are unable to shatter its indissoluble fabric; and the prosperous gales of philosophy will always succeed the tempests of folly, and waft it with rapidity to the *enlightened* regions of mankind. The time perhaps is not far distant, when this fortunate change may commence. Above twelve hundred years have elapsed since the vessel of ancient wisdom visited the civilized parts of the world, and the nations were blest with its invaluable contents: and during this dreadful interval, ignorance and delusion, *jargon* and *reverie*, have held an undisturbed and universal reign. The depravity of the times is the subject of general complaint: genius no longer soars; learning has evaporated into words; and philosophy is but a name. Yet, though the restoration of ancient theology is the object of my most ardent desires, I much fear that a period still more barbarous, with respect to philosophy; that an age still darker and more debased must precede its establishment on the earth. Prodigies and destruction attended, as we shall observe in the ensuing history, its departure from mankind; and desolation will doubtless be the harbinger of its future appearance. The orb of vicissitude produces renovation and decay in regular succession; and marks, as it revolves, the dormant events of future periods with *the ruinous* characters of the past. Let us, therefore, patiently wait for, and joyfully expect the happy moment when the breezes of philosophy shall arise with abundance and vigour; and impel the vessel of theology laden with the riches of wisdom, on our natal coast. The revolution is certain, however remote: and the prospect is of itself sufficient to increase the vigour of exertion, and animate the expectations of hope; to enable us to brave the storms of ecclesiastical persecution, and vanquish the resistance of folly.

C O M M E N T A R I E S
OF
P R O C L U S .

B O O K III.

Concerning Petitions and Axioms.

[1] Since the principles of geometry are triply divided into Hypotheses, Petitions, and Axioms, the difference between these we have explained in the preceding books. But we now intend to discourse more accurately of petition and axiom, as especially necessary to our present design. For hypotheses, which are also called definitions, we have already explained. It is common, therefore, as well to axioms as to petitions, to require no demonstration, and no geometrical faith: but to be received as manifest, and to become the principles of the rest. But they differ mutually from each other, in the same manner in which we have distinguished theorems from problems. For as in theorems we propose to perceive and know that which follows a subject; but in problems we are ordered to compare and do something: in the same manner also in axioms, we must receive whatever is manifest of itself, and easily apprehended by our untaught conceptions; but in petitions we must receive whatever is easy to be done and compared, (since in admitting these, thought is not fatigued) and whatever requires no variety, and no kind of construction. Hence evident and indemonstrable cognition, and unconstructed assumption, distinguish petitions from axioms. Just as demonstrative cognition, and an assumption of things sought, together with construction, separates theorems from problems. For it is every where requisite, that principles in simplicity, indemonstrability, and self-evidence, should excel things posterior to principles. For universally (says Speusippus) of the things which cogitation pursues, some of its energies it produces without a various progression, prepares them for future enquiry, and has a more evident apprehension of these than of visible objects: but others which it is not able immediately to follow, by a transition proceeding from their nature, these it endeavours by *consequence* to pursue. Thus for example, *to draw a right line from one point to another*, it receives as evident, and easy to be done. For since in this case the line is composed from the indeclinable flux of a point, and at the same time advances in an orderly progression, because it no where more or less declines, it necessarily falls in another point. Again, if one extremity of a right line abiding, the other is moved about it, it will describe a circle without any labour. But if any one wishes to describe a helix of one revolution, it requires a more various operation. For it is generated by various motions. Likewise if any one wishes to construct an equilateral triangle, he will require a certain method for its construction. For the geometrical intellect says, when I understand a right line, which abides according to one of its extremities, but is moved about it according to the other, and at the same time conceive a point, which is moved in the line from the abiding extreme, I have described a helix of one revolution. For when at the same time both the extremity of the right line, which describes the circle, and the point which is moved in the right line, arrive at the same point, and coincide, they produce for me such a helix. And again, when I describe equal circles, and draw right lines from the common section to the centre of the circles, and a right line from one centre to the other, I shall have an equilateral triangle. The production of these, therefore, is very remote from a simple apprehension, and primary notion. For we are content to pursue the progressions of their origin. Hence it happens that these are compared with greater ease or difficulty, and are exhibited with many or fewer mediums, according to the habit of those who enter on this undertaking: but that they require demonstration and construction, on account of the property of the things sought, which wants the evidence of petitions and axioms.

Petition, therefore, and axiom, are simple and easy to be apprehended. But petition, indeed, commands us to fabricate, and provide a certain matter, in order to the assignation of the *symptom*, which possesses an easy and

simple apprehension: but axiom pronounces a certain essential accident, of itself known to the hearers. As that *fire is hot*, or any other of those manifest truths, he who doubts of which, we consider as either wanting sense or punishment. Hence, petition and axiom are of the same genus; but they differ in the above-mentioned manner. For each is an indemonstrable principle, but this after one mode, and that after another, as we have already observed. But some think that all these should be called petitions, in the same manner as all problems, *things sought*. For Archimedes beginning his book of *Equiponderants*, *we desire it may be granted* (says he) *that things equally heavy, from equal lengths, will equally ponderate*; though some would rather chuse to call this an axiom. But others call all these axioms, in the same manner as they denominate every thing a theorem, which requires demonstration. For, according to the same proportion, as it seems they pass from proper names to such as are common. Nevertheless, as a problem differs from a theorem, so petition from axiom: though both these last are indemonstrable, and the former require demonstration. And the one, indeed, is assumed as easy to be done, but the other is granted as easy to be known by the common consent of all men. After this manner, therefore, Geminus distinguishes petitions from axioms.

But others will perhaps say, that petitions are indeed proper to the geometrical matter: but that axioms are common to the universal theory, which is conversant about the *how-much*, and the *how-many*. For the geometrician knows *that* which requires *that all right angles are equal*, and *that every finite right line may be produced straight forwards*: but *that* which says, *things equal to one and the same are equal to each other*, is a common conception, which not only the arithmetician employs, but every one endued with science, accommodating that which is common to his own particular matter. But Aristotle (as we have before observed^[2]) says, that petition, since it is demonstrable, is not granted by the hearer, yet is received as a principle: but that axiom is of itself indemonstrable, and that this is confessed by all, according to habit, though some, for the sake of disputation, have doubted its evidence. Since then, there are these three differences, according to the first, which by operating, and knowledge only distinguishes petition from axiom, it is manifest that which says *all right angles are mutually equal*, is not a petition. Nor the fifth, which says, *if a right line falling on two right lines makes the internal angles towards the same parts less than two right, those right lines infinitely produced, shall coincide towards the parts in which the angles less than two right subsist*. For these are neither assumed in construction, nor do they command any thing to be done: but they exhibit a certain *symptom*, inherent in right angles, and in right lines, departing from angles less than two right. But, according to the second difference, that will not be an axiom which says, that *two right lines cannot comprehend space*, which some at present consider as an axiom. For this is proper to the geometric matter, as likewise that which affirms *that all right angles are equal*. But according to the third difference, which is Aristotelic, all those which produce their own credibility by a certain demonstration, are petitions; but whatever are indemonstrable, are axioms. Apollonius, therefore, in vain endeavours to deliver the demonstrations of axioms: for Geminus very properly observes, that some have attempted demonstrations of indemonstrables, and have endeavoured from more unknown mediums, to prove things manifest to all, into which error Apollonius has fallen, who wishes to prove the axiom true, which says, *that things equal to one, and the same, are equal to each other*: but that others assume in the place of indemonstrables, things requiring demonstration. As is the case with Euclid himself, in the fourth and fifth petition. For some say, that this last, as ambiguous, requires demonstration. Indeed, is it not ridiculous, that theorems should be assigned as indemonstrable, the converse of which are demonstrable? For that the internal angles of coincident right lines are less than two right, Euclid himself shews in the theorem, which says, *that two angles of every triangle, however taken, are less than two right*: besides, it may be perspicuously shewn, that not every thing equal to a right angle is a right angle. Hence, says Geminus, the converse of these are not to be granted indemonstrable. It seems therefore, according to the ordination of this man, that there are, indeed, three petitions: but that the other two, and the converse of these, require demonstrating science: and that in the axioms, the one which says, that two right lines cannot comprehend space, is superfluously added, since its credibility must be derived from demonstration. And thus much concerning the difference of petitions and axioms. Again, of axioms, some are proper to arithmetic, but others to geometry; and others are common to both: for that which says, *every number is measured by unity*, is an arithmetical axiom. But that which says *equal right lines agree amongst themselves*, as also this which affirms *that every magnitude is divisible in infinitum*, are geometrical axioms: but the one which says *that things equal to the same, are mutually equal*, and all of this kind are common to both. However, it must be observed, that each science uses such as the last, according to its proper subject; as geometry in magnitudes, but arithmetic in numbers. In like manner of petitions, some are peculiar to particular sciences, but others are common to all. For you must call the petition which requires to be granted, *that a number may be divided into the least parts*, peculiar to arithmetic: but this, *that every finite straight line may be produced*

straight forwards, peculiar to geometry; and the one which desires us to grant, *that quantity may be infinitely increased*, common to both; for this passion is equally found to reside in number and magnitude.

PETITIONS or POSTULATES.

I.

Let it be granted that a straight line may be drawn from any one point to any other point.

II.

That a terminated straight line may be produced to any length in a straight line.

III.

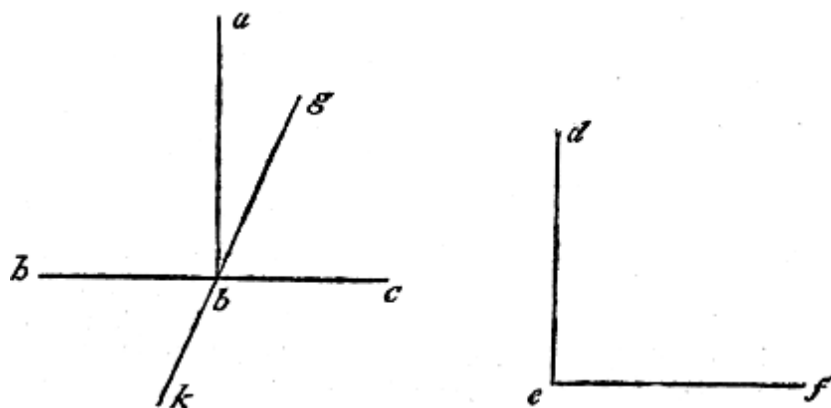
And that a circle may be described from any centre, at any distance from that centre.

According to the opinion of Geminus, these three are necessarily placed among petitions, as well on account of their facility, as because they command us to do something. For this, *to draw a right line from every point, to every point*, follows the definition, which says, *that a line is the flux of a point*, and a right line *an indeclinable and inflexible flow*. If then we conceive a point to be moved with an uninclined, and the shortest motion, we shall fall upon another point, and the first petition will be produced, and we shall understand nothing various or difficult. But if when the right line itself is terminated by a point, we conceive its extremity moved with the shortest indeclinable motion, the second petition will arise from an easy and simple apprehension. But if we again imagine that the terminated right line abides according to its other extreme, but that it moves about that which abides according to the rest, the third petition will be produced; for the centre is the point which abides, but the interval the right line. Since the distance of the centre, from all parts of the circumference, is always equal to the quantity of this interval. But if any one should doubt how we apply motion in geometrical concerns, which have an immoveable existence; and how we can move impartibles, (since this is impossible) we request him to call to mind what we have demonstrated in the beginning of these Commentaries. I mean that the reasons of things subsisting in the phantasy, describe there all the images of cogitation, of which cogitation itself possesses the reason: for an intellect of this kind is an unwritten, ultimate, and passive tablet. Hence, it receives forms from another, accompanied with motion; but we must not understand a corporeal but imaginative motion, and must by no means admit that impartibles are moved with corporeal motions, but that they suffer imaginative progressions. For intellect, though impartible, is moved, yet not according to place, and the phantasy has a proper motion according to the impartible which it contains: but we only regarding corporeal motions, neglect those which are made in things destitute of interval. Impartibles, therefore, are pure from corporeal place, and external motions: but another species of motion, and another place congenial to such motions, is considered in their progressions. For, indeed, we should say, that a point also has position in the phantasy, and should not enquire how an impartible can abide, which is at the same time moved elsewhere, and comprehended by place. Since the place of things, with dimension, possesses itself dimension; but the place of impartibles is destitute of all dimension. The proper species therefore of geometrical concerns, are different from the things they produce; and the motion of bodies is different from that of the forms in the phantasy; and the place of impartible is different from that of impartible natures; and it is requisite, by distinguishing these, neither to confound nor disturb the essences of things. But it appears that the first of these three petitions declares to us in images, how *the things which are*, are contained in their own impartible causes, and are terminated by their immaterial bound; and that previous to their constitution, they are on all sides comprehended in their indivisible embrace: for the points existing, a right line is drawn from the one to the other, is terminated by, and received between them. But the second indicates how *the things which are* by possessing proper causes proceed to all things, preserving in them a continuation not derived from the natures into which they proceed; but that through a cause of infinite power, they endeavour to permeate every where, with a never-failing progression. And the third petition shadows forth the manner in which these progressions return again to their proper principles: for the convolution of a point producing a circle, by moving about an abiding point, imitates a circular regression. But it is requisite to know, that every line cannot be infinitely produced, for the circle and cissoid, and all such as describe figure, are incapable of this property; as likewise some which produce no figure. For the helix of one revolution cannot be infinitely produced, since it is constituted between two points; nor any other lines similarly formed. But neither is it possible to extend every line from every point, to every point; for every line cannot subsist between all points: and thus much for the three first petitions; let us now proceed to the rest.

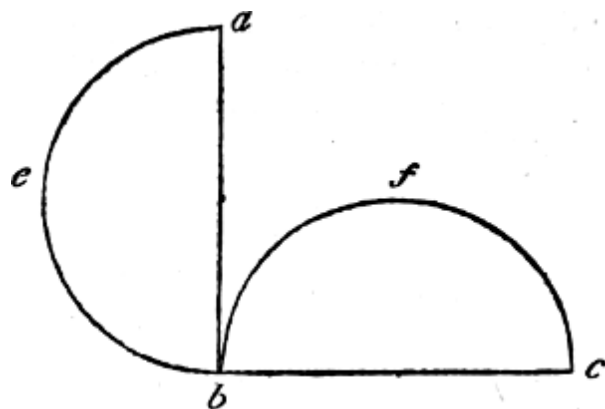
IV.

All right angles are equal to each other.

If the present petition is considered by us as manifest, and as requiring no demonstration, it is not a petition according to the opinion of Geminus, but an axiom; for it affirms a certain essential accident of right angles, not commanding us to perform any thing according to a simple conception. But neither is it a petition according to the division of Aristotle: for petition, according to his opinion, requires some demonstration. But if we should say it is demonstrable, and enquire after its demonstration, yet according to the opinion of Geminus, it ought not to be placed among petitions. The equality, therefore, of right angles, appears from our common conceptions; for since a right angle has the relation of unity or bound to the infinite increase and decrease of the angles on each side, it is equal with respect to every right angle, since we constitute the first right angle after this manner, by a right line making angles on each side of the right line on which it stands equal to each other; but if it be requisite to produce a linear demonstration of this, let there be two right angles, one $a b c$, the other $d e f$.

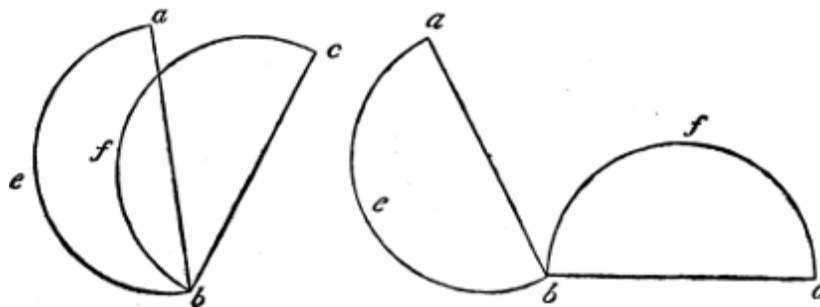


I say that they are equal; for if they are not equal, one of them must be greater, suppose the angle at b . If then the line de be adapted to the line ab , the line ef shall fall within. Let it fall as bg , and let the line bc be produced to h ; because, then abc is a right angle, abh also shall be a right angle, and they shall be mutually equal to each other, from the tenth Definition: the angle abh therefore, is greater than the angle abg . Let again the line gb be produced to k , because, therefore abg is a right angle, the successive angle abk shall be a right one, and consequently equal to abg . Hence, the angle abh , shall be less than the angle abg ; but it was also greater, which is impossible: but this has been shewn by other expositors, and requires no great consideration. But Pappus very properly admonishes us, that the converse of this Petition is not true; I mean, that every thing equal to a right angle, is a right angle; though if it be rectilinear, it is without doubt a right angle. But a curvilinear angle may also be exhibited equal to one that is right: for let there be conceived two equal right lines, ab , and bc , making the angle at the point b , right;



and on them let the semicircles aeb , bfc , with a proper centre and interval be described; because, therefore, the semicircles are equal, they shall have a mutual congruence, and the angle $e b a$, is equal to the angle $f b c$, and $a b f$ is common: the whole right angle, therefore, is equal to the lunular, i.e. to $e b f$, and yet the lunular is not a right angle. In the same manner, if the angle abc should be obtuse or acute, a lunular angle may be shewn equal to it

(for this is that genus of curvilinear angles which agrees with such as are rectilinear), only this is to be observed, that in a right and obtuse angle, it is requisite to add the middle angle, which is contained by the line $a b$, and the circumference $b f$; but in an acute angle to take this away: for the right line $c b$, in these cases, cuts the circumference $b e$. The truth of which, will be evident from the following figures:



And hence, it appears, that all right angles are mutually equal to each other, and that not every thing equal to a right angle, is consequently a right angle: for if it be not rectilinear, how can it be called right. But it is also manifest from this Petition, that angular rectitude is allied to equality, in the same manner as acuteness and obtuseness are related to inequality. For rectitude and equality, as also similitude, are of the same co-ordination, (for each exists under bound): but acuteness and obtuseness, as also dissimilitude, are of the same series with inequality. For they are all produced from *bound* and *infinite*. Hence some, regarding the quantity of angles, say, that a right angle is equal to a right: but others, considering their quality, affirm that one is similar to another. For similitude in qualities is the same as equality in quantities.

V.

If a right line falling upon two right lines, makes the internal angles towards the same part less than two right, those right lines, if infinitely produced, shall coincide in that part, in which the angles less than two right, are placed.

This ought to be entirely blotted out from the number of Petitions, for it is a theorem including many doubts, which Ptolemy in one of his books proposes to solve; but it requires in its demonstration both many definitions and theorems; and Euclid also exhibits its converse as a theorem. But perhaps some, from an erroneous conception, may think that this should be placed among the petitions, as that which produces credibility of itself, respecting the inclination of right lines, on account of the diminution of two right angles. To such as these, Geminus rightly answers, that from the authors of this science, we learn not entirely to give credit to imaginative probabilities, for the purpose of accomplishing geometrical reasons: for it is similar (says Aristotle) to require demonstrations from a rhetorician, and patiently listen to a geometrician, disputing from probability. And Simmeas in the Phædo of Plato, says, "I know that those who demonstrate from appearances, are vain." Hence, in the present instance, it is true and necessary that right lines should incline, while right angles are diminished: but this, that the inclining lines, while they are more produced, should at length coincide, is probable, but not necessary, unless some reason demonstrates that this is true in right lines: for there are certain lines infinitely inclining, and never coinciding, and though this appears incredible and admirable, yet it is true, and has been observed in other forms of a line. Is it therefore possible that this can be accomplished in right lines which takes place in others? For before we procure conviction of this, from demonstration, the properties exhibited in other lines molest the phantasy by the contrary images they produce. But if the reasons doubting against the coincidence of lines are very strong, ought we not much more to expel this improbable and irrational supposition from our doctrine? And thus it appears that a demonstration is to be sought for of the present theorem, and that it is foreign from the property of Petitions: but how it is to be demonstrated, and by what reasons the objections urged against it are to be removed, we shall shew in our comment on the proposition, where it is used by the institutor of the elements as manifest. For then it will be necessary to exhibit its evidence, since it does not present itself to our view with indemonstrable clearness, but becomes manifest through the medium of demonstration alone.

AXIOMS.

I.

Things which are equal to the same, are equal to one another.

II.

If equals be added to equals, the wholes are equal.

III.

If equals be taken from equals, the remainders are equal.

IV.

If equals are added to unequals, the wholes are unequal.

V.

If equals be taken from unequals, the remainders are unequal.

VI.

Things which are double of the same, are equal to one another.

[13]

VII.

Things which are halves of the same, are equal to one another.

VIII.

Things which coincide with each other, are mutually equal.

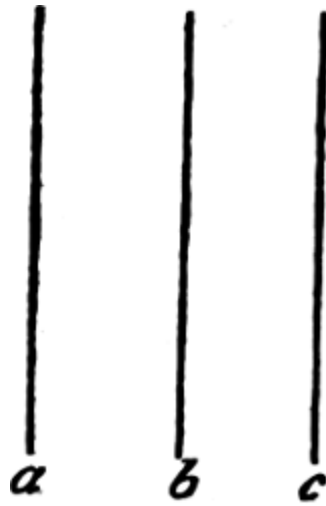
IX.

The whole is greater than its part.

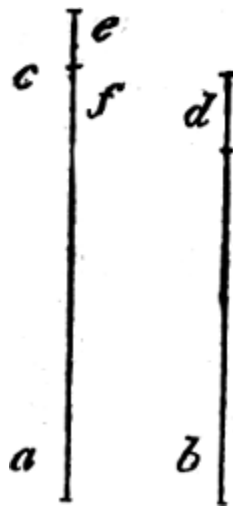
X.

Two right lines cannot comprehend space.

These are the things which, according to the opinion of all men, are called indemonstrable axioms, so far as their certainty is admitted by all, and no one disputes their evidence. For propositions also are often simply called axioms, of whatever kind they may be, whether they are immediately proper, or require some declaration; and the Stoics, indeed, are accustomed to call every simple enunciative speech an axiom: and when they write on dialectic arts, they say that they discourse on axioms. But some, distinguishing more accurately axioms from other propositions, give this appellation to a proposition immediate, and producing credibility of itself, on account of its evidence: as also Aristotle and geometers themselves affirm. For, according to the opinion of these, an axiom is the same as a common conception. By no means, therefore, must we praise Apollonius the geometrician, who writ (as it appears) demonstrations of axioms, because he performs the very opposite to Euclid: for he, indeed, enumerates that which is demonstrable among Petitions; but Apollonius endeavours to find out demonstrations of indemonstrables. But these naturally differ from each other, and the genus of the sciences is different: I mean of the things which take place about immediate propositions, which are entirely subject to our knowledge, on account of their evidence; and of things which use demonstrations, which receive principles from them; and which, when received, they orderly employ in their proper conclusions. But that the demonstration of the first axiom, which Apollonius persuades himself he has invented, possesses a medium, not more known, but more dubious than the conclusion may be known by any one from a slight inspection. For let (says he) a be equal to b , and b to c , I say that a also is equal to c .



For since a is equal to b , it occupies the same place as b . And because b is equal to c , it occupies the same place as c ; and so a occupies the same place as c , they are therefore equal. Now in this demonstration it is requisite that two things must be previously assumed; one, that things occupying the same place, are mutually equal; but the other, that things occupying the same place, with the same thing, mutually occupy the same place: but these are evidently more obscure than the present axiom. For it is proper to enquire how are things, which fill the same place equal, according to the whole, or according to a part; or according to a figure of speech: hence we must by no means admit a transition to place,^[3] which is more unknown than the natures it contains; for the invention of its essence is difficult and ambiguous. That we may avoid prolixity, therefore, all axioms are to be delivered as things immediate and self-manifest, since they are of themselves known and credible; for he who brings demonstration to things the most manifest, does not confirm their truth, but diminishes the evidence we possess in the untaught and innate conceptions of the soul: but this is to be received concerning axioms, as a judgment of their peculiarity; and that all of them are of the common kind of the mathematical sciences; and that each of them is said to be verified, not only in magnitudes, but also in numbers, and motions, and times: and this indeed is necessary. For equal and unequal, the whole and part, and the more and the less, are common to discreet and continued quantities. The contemplation, therefore, which is conversant with times and motions, numbers and magnitudes, requires all these, as things evident by their own intrinsic light; and in all of them both that is true, which says, *things equal to the same, are equal to one another*; as likewise each of the axioms we have assumed: but as they exist in common, each science uses them according to its proper matter, and one indeed, as in magnitudes; but another, as in numbers; and another, as in times; and after this manner in each science, the conclusions become peculiar and apposite, though the axioms are common. Besides, it is likewise requisite not to contract the number of these to the least, as is done by Heron, who only establishes three axioms; for this also is an axiom, *the whole is greater than its part*, and the geometrician every where assumes this in his demonstrations; as also, that *things which mutually coincide, are equal*; for this is employed with advantage in the solution of the fourth Proposition. Nor is it proper to join some with others, of which some are proper to the geometric matter, as *that two right lines cannot comprehend space*, (since axioms are, as we have said, of a common kind); but others are consequent to things established, as that which says, *things double of the same, are equal*. For this is consequent to the axiom, affirming, that *if to equals you add equals, the wholes are equal*, since things equal to the half, because they assume the half, become double to the same, and mutually equal, on account of an equal addition: and according to this reason, not only the doubles, but also the triples, and all multiples of the same quantity will appear equal. But with these axioms, Pappus says, that certain others are to be classed, as *if unequals are added to equals, the excess of the whole, will be equal to the excess of the adjuncts*. And on the contrary, *if equals are added to unequals, the excess of the wholes is equal to the excess or difference of the unequals themselves*. And these also are manifest from themselves, yet they may be made manifest as follows.



Let a be equal to b , and add to each the unequals c d , but let c be greater than d by e , and the remainder be f ; because, therefore, a is equal to b , and also f to d ; a f will be equal to b d . For if equals are added to equals, the wholes are equals: a c , therefore, exceeds b d , by e only, by which alone c exceeds d . Again, c and d are unequals, to which, let the equals a and b be added, and let e be the excess of c , above d , and the remainder be f ; because, therefore, a is equal to b , and f to d a f will be equal to b d ; the whole, therefore, a c , will exceed b d , by e only, by which c also exceeds d . These, therefore, are consequent to the aforesaid axioms, and are, not undeservedly, in many copies, omitted. But whatever others he adds to these, have been previously assumed by definitions, to which they are consequent. As for example: *that all the parts of a plane and a right line mutually agree*; for things placed in their extremities, possess a nature of this kind; and *that a point divides a line, but a line a superficies, and a superficies a solid*. For all things are divided by the natures by which they are proximately bounded; and *that infinite subsists in magnitudes, by addition and diminution, but according to capacity only, in both these respects*: for every thing continuous may be infinitely divided and increased. But, as we have summarily spoken concerning these, it remains that we consider things consequent to principles; for thus far principles extend themselves. But of those who oppose geometry, some very much doubt concerning principles, endeavouring to shew that the terms have no subsistence, whose arguments, indeed, are known in common, who endeavour to take away all science, and, like hostile foes from a foreign region, demolish the fruits and fecundity of philosophy, as is the case with the Pyrrhonian philosophers; but others only propose to themselves the subversion of geometrical principles, as the Epicureans. Others, again, admitting the principles, affirm, that things consequent to the principles cannot be demonstrated, unless something else is granted, which was not previously assumed in the principles. Zeno exercised this mode of contradiction, who was a Sidonian by birth, but of the Epicurean sect, against whom Possidonius wrote an entire book, exhibiting the whole of his imbecile opinion; and thus much may suffice for the difference of opinions concerning principles. We shall shortly consider the troublesome objection of Zeno: but now, after we have briefly resumed the consideration of theorems and problems, their difference, and the divisions they receive, we shall proceed, to an exposition of the things exhibited by the institutor of the elements, gathering the more beautiful observations upon the propositions found in the writings of the antients, and contracting the infinite prolixity of their discourses; but delivering such things as are more artificial, and full of methods producing science, dwelling more on an accurate treatise of things than on the variety of cases and assumptions, to which young men, for the most part, eagerly incline.

PROPOSITION I. PROBLEM.

Upon a given terminated right line to describe an equilateral triangle.

Since all science is two-fold, and one is conversant about immediate propositions, but another about things, which are exhibited and provided from the propositions, and universally about the consequents to principles; this, again, divides itself in geometrical discourses, into the solution of problems, and the invention of theorems. And problems, indeed, geometry denominates things in which it proposes to procure, manifest, and fabricate that, which, in a certain respect, has no existence; but it calls theorems, things in which it appoints to perceive, know, and demonstrate that which either exists, or does not exist. For problems command us to undertake the origin, positions, applications, descriptions, inscriptions, circumscriptions, coaptations, and contacts of figures, and

every thing of this kind: but theorems endeavour to procure our assent to symptoms, and things essentially inherent in the subjects of geometry, and to convince by demonstrations. For geometry discourses concerning every object of enquiry, which is possible to be effected, referring some things to problems, but others to theorems; since it enquires concerning the *what*, in a two-fold respect: for it either seeks for the reason and intelligence of the thing; or for intelligence, and the essence of the subject. I say, for example, as when it requires what a line of similar parts may be: for in an enquiry of this kind, it either desires to find the definition of such a line, as, *that a line of similar parts is that which has all its parts agreeing with all*; or to receive the species of lines of similar parts, as that it is either *right*, or *circular*, or a *cylindric helix*. Besides, prior to this, it enquires, by itself, concerning the *if*, and this especially in its determinations, agitating, whether the object of its enquiry is possible or impossible, what place it possesses, and in how many ways. It likewise seeks concerning the *what kind*; for when it considers the essential accidents of a triangle, circle, and parallels, it is manifest, that in such cases it seeks after the *what kind*; but many have thought that geometry very little contemplated the *cause*, and the *why*. And of this opinion is Amphinomus, led by the decisions of Aristotle: but (says Geminus) an enquiry into these may be found in geometry. For does it not belong to geometry to enquire for *what cause* infinite equilateral multangles may be inscribed in circles, but to describe solid equilateral and equiangular multangles, and constructed from similar planes, in spheres, is impossible? To whom does an investigation of this kind belong, except to a geometrician? When, therefore, to geometricians the syllogism is by an impossibility, they alone desire to find the symptom; but when by a principal demonstration, then again if the demonstrations are in that which is particular or partial, the cause is not yet manifest; but if in that which is universal, and in all similars, the *why* becomes immediately manifest: and thus much concerning objects of enquiry.

But every problem and theorem which receives its completion from its own perfect parts, ought to possess in itself all the following parts: *proposition*, *exposition*, *determination*, *construction*, *demonstration*, and *conclusion*. But of these, *proposition* informs us what the object of enquiry is from a given datum; for a perfect proposition is composed from both; but *exposition* receiving the datum essentially, prepares for the question. Again, *determination* separately explains the thing sought for according to the *what*; but *construction* adds to the datum what is wanting to the investigation of the thing sought; and *demonstration* skilfully collects the proposition from the concessions. But the *epilogue*, or conclusion, is again converted to the proposition, by confirming that which is exhibited. And so many, indeed, are all the parts of problems and theorems; but *proposition*, *demonstration*, and *conclusion*, are especially necessary, and exist in all; for it is requisite that the thing sought for should be previously known; and that this should be shewn by proper mediums, and that what is exhibited should be concluded; and it is not possible that any one of these three can be wanting; but the rest are, indeed, received in many places; but in many, because they produce no utility, are omitted. For *determination* and *exposition* are not found in the problem, which says, *to construct an isosceles triangle, which will have each of the angles at the base double of the other*; but *construction* has frequently no subsistence in many theorems, the demonstration being sufficient to exhibit the thing proposed from the data, without any addition. When, therefore, shall we say that *exposition* fails, when no datum is given in a proposition? Because, though *proposition*, for the most part, is divided into *datum*, and *the thing sought for*, yet this is not always the case; but sometimes *the thing sought for*, alone affirms that which it is requisite to know or effect, as in the aforesaid problem; for it does not previously say from what datum it is requisite to construct an isosceles triangle, which shall have each of the angles at the base, double of the remaining one; but that it is required to effect this. And here, indeed, the admission of the proposition takes place from things previously known; for we must know the meaning of the terms *isosceles*, *equal* and *double* (since this, as Aristotle observes, is the property of all ratiocinative discipline^[4]), yet nothing is subjected to us as in other problems, as in that which says, *to bisect a given terminated right line*. For here the right line is given, but we are ordered to divide it into two parts; and the datum is separately determined from the object of enquiry. When, therefore, a proposition has both of these, then also *determination* and *exposition* are found; but when the datum is deficient, these also fail, since *exposition* and *determination* belong to the datum: for this will be the same with the proposition. Indeed, what else do we say, when determining in the aforesaid problem, unless that it is requisite to find an isosceles of this kind? But such was the proposition: if then the proposition has neither this *datum*, nor *thing sought*, *exposition* will, indeed, be silent, because there is no datum; but *determination* will be neglected, lest it should become the same with the *proposition*: but you may find many other problems of this kind, especially in arithmetic, and in the tenth book of these Elements, as, *to find a medium comprehending two right lines commensurable in power, and every thing of this kind*.

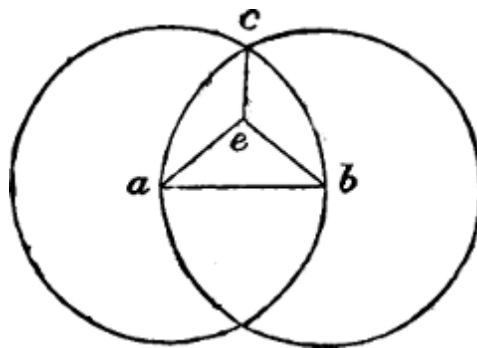
But every datum may be given in these four modes, either in *position*, or *proportion*, in *magnitude* or *form*; for a point, indeed, is given in *position* only, but a line and the rest in all the four. Thus, when we say, *to bisect a given rectilinear angle*, we declare the species of the angle given, as that it is right lined, lest we should also seek to bisect a curvilinear angle by the same methods. But when we say, *from the greater of two unequal right lines, to cut off a part equal to the less*, the lines are given in magnitude; for the less and the more, finite and infinite, are the proper predications of magnitude. But when we say, that *if four magnitudes are proportional, they shall be also alternately proportional*, the same proportion is given in the four magnitudes: but when it is requisite, *from a given point to place a right line equal to a given right line*, then the point is given in position. From whence, since position may be various, construction also receives variety; for the point is given either without the right line, or in the right line, and in the extremity, or without the extremity of the right line. Since, therefore, a datum has a four-fold acceptation, it is manifest, that exposition also is four-fold; but sometimes it connects two or three modes. Again, we find that demonstration sometimes possesses things proper to demonstration, exhibiting the thing sought for from mediate definitions; for this is the perfection of demonstration, but that sometimes it argues from certain signs. And it ought not to be concealed, that geometrical discourses have every where that which is necessary, on account of the subject matter, but are not every where perfected by demonstrative methods. For when, because *the external angle of a triangle is equal to the two internal and opposite ones*, it is shewn, that *the three internal angles of the triangle are equal to two right*, how is this demonstration from the cause? And is not a sign the medium in this case? For the external angle not yet existing, since the internal angles exist, they are equal to two right, since it is a triangle, though the side is not produced; but when, by a description of circles, the triangle, which is constituted, is shewn to be equilateral, the apprehension takes place from the cause. For we say, that the similitude and equality of the circles is the cause of the triangle's equality with respect to its sides.

But geometrical discourses are likewise accustomed to make the conclusion, in a certain respect, ~~twofold~~ ^{fourfold}. And this, when they exhibit things agreeable to the data, and reason universally, recurring from a particular conclusion to that which is universal; for when they do not use the property of the subjects, but placing the data before our eyes, describe an angle or right line, they think that which is concluded in this, is to be concluded in every thing similar: they pass on therefore to *universal*, lest we should think that the conclusion is particular. But their transition is effected in the best manner, since they employ, in demonstration, *the things placed*, not considered as such, but considered as similar to others: for it is not because such a particular angle is proposed that they effect a bipartite section, but because it is rectilinear only. But quantity, is indeed, proper to the proposed angle; but rectilinear is common to all right lines: let then the given angle be a right one. If therefore, we receive rectitude in the demonstration, we cannot pass to every species of right lines; but if we do not subjoin its rectitude, or being right angled, but alone consider its being rectilinear, the discourse may be adapted to all right lined angles; and all that we have previously observed we may contemplate in this first problem. For that it is a problem, is evident, since it commands us to construct an equilateral triangle: but *proposition* in this, consists from a *datum* and *thing sought*. For a terminated right line is given, but it is *enquired* how an equilateral triangle may be constructed upon it, and the datum indeed precedes, but the thing sought follows; so that we may say, by conjoining the two, *if there be a terminated right line, it is possible to construct upon it an equilateral triangle*; for a triangle cannot be constructed without the existence of a right line, since it is comprehended by right lines; nor upon an unlimited line, for an angle cannot be constructed unless it is made on one point, but in an infinite line there can be no extremity or bounding point. But after proposition, *exposition* follows, as, *let there be given a terminated right line*. And here we may see that *exposition* alone pronounces the *datum*, but by no means subjoins *the thing sought*; but after this we shall find *determination*: *it is required upon the given terminated right line to construct an equilateral triangle*; and here we may observe that *determination* is in a certain respect, the cause of attention, for it makes us more attentive to the demonstration, by pronouncing the thing sought, as *exposition* causes us to be more docile, by placing the datum before our eyes. Again, after determination, *construction* follows, *from one extremity of the right line, as a centre, but with the remainder as an interval, let a circle be described*. And again, *with the other extremity, as a centre, and with the same interval, let a circle be described; and from the common point of the sections of the circles, to the extremities of the right line, let right lines be continued*. And here we may observe, that Petitions are used in the construction, this for one, *from every point to every point, to draw a right line*; and also this, *with every centre and interval to describe a circle*; for universally Petitions are the sources of utility to constructions, but Axioms to demonstrations; demonstration therefore follows, *because, then each extremity of the given right line is the centre of the circle surrounding it, the right line which reaches to the common section is equal to the given right line; hence, because the other extremity of the right line is the centre of its containing circle, the right line reaching to the common section of the circles, is also*

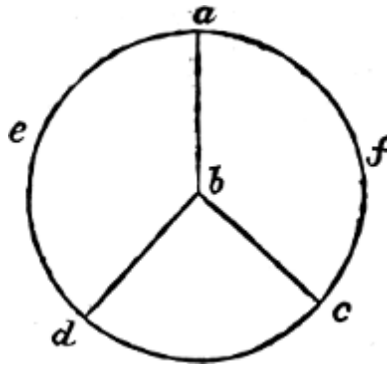
equal to the given line. And the admonition of these, is derived from the definition of the circle, which says, that *all lines from the centre to the circumference are equal.* Each of these lines, therefore, is equal to the same; but *things equal to the same, are equal among themselves,* by the first axiom. The three right lines, therefore, are mutually equal; hence, *upon this given right line an equilateral triangle is constructed;* and this, indeed, is the first *conclusion* which follows the exposition. But after this, that universal one; *upon a given right line, therefore an equilateral triangle is constructed:* for whether you make the line double of the one now proposed, or triple; or receive any one greater or less, the same constructions and demonstrations will accord. But to these he adds the particle *which was required to be done,* shewing from hence, that the conclusion is problematical; for in theorems, he adds the particle *which was required to be shewn;* the former announcing the production of something, but this the ostension and invention of a thing required. He therefore subjoins this to the conclusions, for the purpose of shewing that every part of the proposition is accomplished by this means, uniting the end with the beginning, and imitating intellect convolved, and again returning to its principle. But he does not always add the same, but sometimes the particle *which was required to be done,* and sometimes the particle *which was required to be shewn,* on account of the difference between problems and theorems: and thus, in this one problem, we have exercised and made perspicuous all this variety of considerations. But the reader ought to make a similar enquiry in the rest; investigating what propositions receive these leading properties, and in what they are omitted. Likewise in how many ways a *datum* is given, and from what principles we receive either constructions or demonstrations; for a perspicacious contemplation of these affords no small exercise and meditation of geometrical discourses.

But here it is necessary that we should briefly determine the nature of *assumption*, *case*, *corollary*, *instance*, (*ενχασις*) and *induction*. They say therefore that *assumption* is often predicated of every proposition assumed in the construction of another proposition, affirming at the same time that the demonstration of such a proposition is composed from so many *assumptions*. But *assumption*, properly considered by those who are conversant in geometry, is a proposition indigent of credibility; for when either in construction or demonstration we assume any thing which has not been exhibited, but requires a reason for its admission, then that which is assumed, as of itself ambiguous, being considered as worthy of enquiry, we call an *assumption*; and this differs from Petition and Axiom, because it is demonstrable, but they are assumed without demonstration, for the purpose of giving credibility to others. But the best aid in the invention of *assumptions*, is an aptitude of cogitation; for we may see many naturally acute in solutions, and discovering them without any method, as was the case with our Cratistus, who was adapted to the investigation of a thing sought from the first and shortest methods possible; and had a natural promptitude for invention; but there are nevertheless certain most excellent methods delivered, one which reduces the thing sought, by resolution to its explored principle, which, as they say, Plato delivered to Leodamas, and from which he is reported to have been the inventor of many things in geometry: but the second is that which has a power of division; because it distributes the proposed genus into articles, but affords an occasion of demonstration, by an ablation of other things from the proposed construction. And this likewise is praised by Plato, as that which affords assistance to all sciences; but the third is that which by a deduction to an impossibility, does not of itself shew the thing sought, but confutes its opposite, and discovers the truth by accident; and thus far is the contemplation of *assumption* extended. But *case* enunciates different modes of construction, and the mutation of position, points, or lines, superficies, or solids being transposed; and in fine, all its variety is beheld about description: hence, it is also called *case*, because it is the transposition of construction. Again, *Corollary* is affirmed, indeed, of certain problems, as the Corollaries which are ascribed to Euclid; but *Corollary* is properly predicated, when, from the things demonstrated, a certain unexpected theorem appears, which on this account they have denominated *Corollary*, as a certain gain, exceeding the intention of demonstrative science; but *instance* impedes the whole passage of the discourse, either opposing the construction or the demonstration: and here it is not necessary, that as he who proposes a case, ought to shew the proposition true; so he who proposes an *instance*: but it is requisite to destroy the *instance*, and convict its employer of falsehood. Lastly, *induction* is a transition from one problem or theorem to another, which being known or compared, the thing proposed is also perspicuous. For example: when the duplication of the cube is investigated, geometricians transfer the question into another to which this is consequent, i.e. the invention of two mean proportionals, and afterwards they enquire how between two given right lines two means may be found. But Hippocrates Chius is reported to have been the first inventor of geometrical induction; who also made a quadrangle equal to a lunula, and invented many other things in geometry, and excelled all in his ingenuity respecting appellations; and thus much for these.

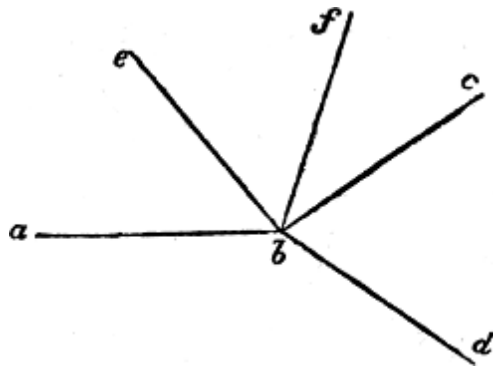
But let us return to the proposed problem: that an equilateral triangle, therefore, is the best among triangles, and is particularly allied to a circle, having all lines from the centre to the circumference equal, and one simple line for its external bound, is manifest to every one; but the partial comprehension of two circles in this problem, seems to exhibit in images how things which depart from principles, receive from them perfection, identity, and equality. For after this manner, things moving in a right line, roll round in a circle, on account of continual generation; and souls themselves, since they are indued with transitive intellections, resemble by restitutions and circumvolutions, the stable energy of intellect. The zoogonic or vivific fountain of souls too, is said to be contained by two intellects. If, therefore, a circle is an image of the essence of intellect, but a triangle of the first soul, on account of the equality and similitude of angles and sides; this is very properly exhibited by circles, since an equilateral triangle is included in their comprehension. But if also every soul proceeds from intellect, and to this finally returns and participates intellect in a two-fold respect; on this account also it will be proper that a triangle, since it is the symbol of the triple essence of souls, should receive its origin comprehended by two circles. But speculations of this kind, as from bright images in the mirror of phantasy, recall into our memory the nature of things. And here, because some object to the constitution of an equilateral triangle, thinking by this means to overthrow the whole of geometry, let us briefly answer and confute them. Zeno then, whom we have mentioned before, says, that if any one admits the principles of geometry, yet he will not obtain from common consent, things consequent to the principles, while this is not admitted, that there are not the same segments of two right lines: for unless this is given an equilateral triangle cannot be constructed. For let there be (says he) a right line $a b$, upon which an equilateral triangle is to be constructed.



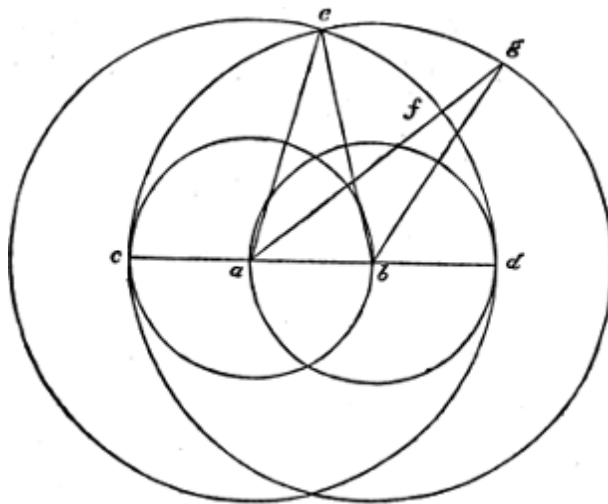
But let circles be described, and from their common section let the right lines $c e a$, $c e b$, be extended, having the common segment $c e$. It will therefore happen, that the lines extended from the common section, will be equal to the given line $a b$, and yet the sides of the triangle will not be also equal, but two will be less than the remainder, that is, than $a b$. And so this not being constituted, neither can the rest be constructed. Can then (says Zeno) the rest follow, though the principles are given, unless this also is previously received, that there are no common segments either of circles or of right lines? Against this objection then, we must affirm in the first place, that it was in a certain respect previously understood, that two right lines have no common segment. For the definition of a right line comprehends this property, since *that is a right line which is equally situated between its bounding points*; and the equality of the interval between the points to the right line, causes that which joins the points to be one, and the shortest line; so that if any one adapts it to another line, according to one of its parts, it must also agree with the line according to its remaining part; for since it is constituted in its extremities, because it is the shortest line, it is necessary that the whole should fall on the whole. But again, this was manifestly received in the Petitions: for the Petition which says, *that a terminated right line may be produced straight forwards*, perspicuously shews that the produced line ought to be one, and produced by one motion;



but if any one is desirous to receive a demonstration of this assumption, let, if possible, $a b$ be the common segment of $a c$ and $a d$, and with the centre b , and interval $b d$, let the circle $a c d$ be described; because therefore the right line $a b c$, is drawn through the centre, $a f c$ is a semicircle; and because the right line $a b d$ likewise is drawn through the centre, $a e d$ is a semicircle. The semicircles, therefore, $a f c$, $a e d$, are equal to each other, which is impossible. But against this demonstration Zeno will perhaps say, that it is likewise requisite to demonstrate that the diameter bisects the circle, because we previously assume that there is not a common segment of two circumferences. Thus too we take for granted, that one circumference coincides with another, or if it does not coincide, that it either falls externally or internally. But nothing hinders (he will say) that the whole may not coincide with the whole, but according to some part. But to this Possidonius rightly answers, who laughs at the acute Epicurean, as if conscious that though the circumferences do not coincide according to a part, yet the demonstration will succeed; for according to that part in which they do not coincide, the one will fall within, and the other without, and the same absurdities will follow when right lines are extended from the centre to the external circumference; for those from the centre will be equal, as well the greater which is drawn to the external, as the less which is extended to the internal circle: either therefore the whole will coincide with the whole, and they will be equal; or coinciding according to a part, it will alternately vary according to the remainder, or no part will coincide with no part; and in this case it either falls within or without: but of this, enough. But Zeno also condemns the following demonstration of this particular:



Let $a b$ be the common segment of two right lines $a c$, $a d$, and let $b e$ be erected at right angles to $a c$, the angle $e b c$, therefore, is a right one. Hence, if the angle $e b d$ is also right, they shall be equal, which is impossible; but if not, let $b f$ be erected at right angles to $a d$. The angle $f b a$, therefore, is right; but the angle $e b a$ was also right; and they are therefore mutually equal, which is impossible. This is the demonstration which Zeno opposes, as assuming that which is to be exhibited afterwards; I mean from a given point to raise a right line, at right angles, to a given right line. However, Possidonius observes, that indeed, a demonstration of this kind is never to be introduced into elementary institutions; but that Zeno calumniates Geometricians using their own as a flagitious demonstration; though there is some reason in their conduct. For there are right lines existing at right angles; since any two right lines are capable of forming a right angle; and this is previously assumed in our definition of a right angle. For we alone constitute a right angle from such an inclination; and it may perhaps be this which we have erected. Indeed, Epicurus himself, and all other philosophers admit, that not only many things possible may be supposed, but likewise many of an impossible matter, for the purpose of contemplating something consequent; and thus much concerning an equilateral triangle.



But it is requisite to construct other triangles, and in the first place an isosceles. Let $a b$, therefore, be a right line, upon which it is required to construct an isosceles triangle. Describe circles as in the construction of an equilateral triangle, and produce the line $a b$ on each side to the points $c d$; the line $c b$, therefore, is equal to $a d$. Again, with the centre b , and interval $c b$, let the circle $c e$ be described; and with the centre a , and the interval $d a$, the circle $d e$; and from the point e , in which the circles intersect each other, to the points a and b , let the lines $e a$, $e b$, be extended. Because therefore, $e a$ is equal to $a d$; but $e b$ to $b c$, and $a d$ is equal to $b c$, $e a$ will also be equal to $e b$; but they are also greater than $a b$. The triangle $a b e$, therefore, is isosceles, which it was required to constitute. But let it be ordered to construct a scalene triangle upon the given right line $a b$. Describe circles with centres and intervals, as before, and let there be taken in the circumference of the circle, whose centre is a , the point f , and let the right line $a f$ be extended and produced to the point g ; and likewise let the right line $g b$ be extended. Because, therefore, a is a centre, $a f$ is equal to $a d$; and hence, $a g$ is greater than $a d$, that is, than $g b$. But b also is a centre, $g b$, therefore, is equal to $c b$; and hence, $g b$ is greater than $b a$: but $g a$ is greater than $g b$; the three lines therefore $g b$, $b a$, $a g$, are unequal; and hence, the triangle $a b g$ is scalene. Hence too, three triangles are constructed; but these things are commonly known: however, this is beautiful in these triangles, that the equilateral existing on all sides equal, is constructed by one mode alone; but the isosceles, endued with equality in two sides only, has a two-fold construction: for the given right line is either less than both the equal ones (according to our present construction), or it is greater than both; but the scalene being unequal in all its sides, receives a triple construction; for the given right line is either the greatest of the three, or the least; or greater than the one, and less than the other; and indeed, it is proper to be exercised in each supposition, either by enlarging or contracting; but to us, what is already delivered, is sufficient. Let us now contemplate problems universally, some of which are produced simply, but others manifoldly, and others according to infinite modes. But (as Amphinomus observes) those which are simply constructed are *ordinate*: but those which receive a manifold composition, and are constructed according to number, are *middle*; and those which are varied in infinite ways, are *inordinate*. The manner, therefore, in which problems are constructed, simply or manifoldly, becomes manifest in the preceding triangles; for the equilateral is constituted simply; but of the other two, the one receives a two-fold, and the other a triple construction. But problems of the following kind, may take place in infinite modes; I mean *to divide a given right line in three proportional parts*; for if it be divided in a duple ratio, and the deficient quadrangular form, resulting from the less, be applied to the greater, it will be divided into three equal parts; but if the greater segment be more than double of the less, as for instance, triple, and a deficient quadrangular form, equal to that which results from the less, be applied to the greater, the line will be divided into three unequal parts. Because, therefore it may be divided into two parts, in infinite ways, the greater of which is either double or triple, (for multiplex proportion proceeds in infinitum), hence, it may be divided into three parts, according to infinite variations.

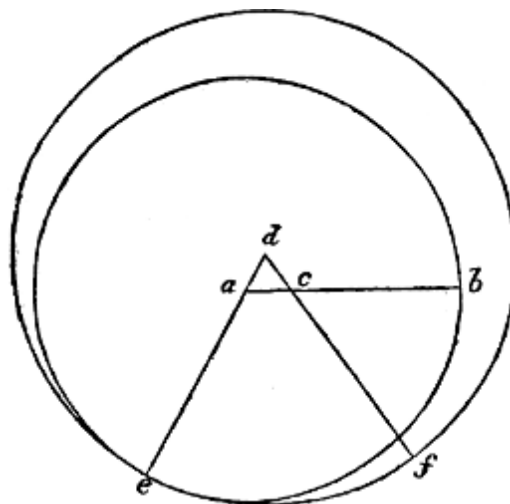
But it is requisite to know, that problem also is manifoldly predicated; for whatever is proposed may be called a problem, whether it is proposed for the sake of learning or operating. But in mathematical disciplines, that is properly called a problem, which is proposed for the purpose of contemplative energy. Since that which is performed in these, has contemplation for its end; and often, indeed, certain things, impossible to be executed, are called problems: but more properly that which is possible to be done, and neither exceeds, nor is deficient, is

allotted an appellation of this kind; and the problem *exceeds*, which says, *to construct an equilateral triangle, having its vertical angle two thirds of one right*; for this is superfluous, and is added in vain: since it is a property inherent in every equilateral triangle. But of those which exceed, whatever are redundant with incongruous and non-existent symptoms, are called *impossibles*; but whatever are redundant with accidents, are called *greater problems*. But a *defective* problem (which is also called a *less problem*) is that which requires some addition, that it may be reduced from inordination into order and scientific bound, as if any one should say, *to constitute an isosceles triangle*: for this is mutilated and indeterminate, and requires some one who may subjoin, what kind of an isosceles triangle, whether that which has its base greater than either of the equal sides; or that which has it less. Likewise, whether that which has the vertical angle double of each at the base, as a semiquadrangle; or that which has each of the angles at the base double of the vertical angle; or that which possesses these angles according to some other proportion, as triple or quadruple: for it is possible that it may be varied in infinite modes. From hence, therefore, it is manifest, that such things as are properly denominated problems, ought to avoid indetermination, and not to be of the number of things capable of infinite variation; though such as these are also called problems, through an equivocation of the word problem. The first problem, therefore, of these elements, excels the rest in the manner we have explained; for it neither *exceeds*, nor is *deficient*; it is neither constructed in a variety, nor according to infinite modes; and such ought to be the conditions of that which is to be the element of the rest.

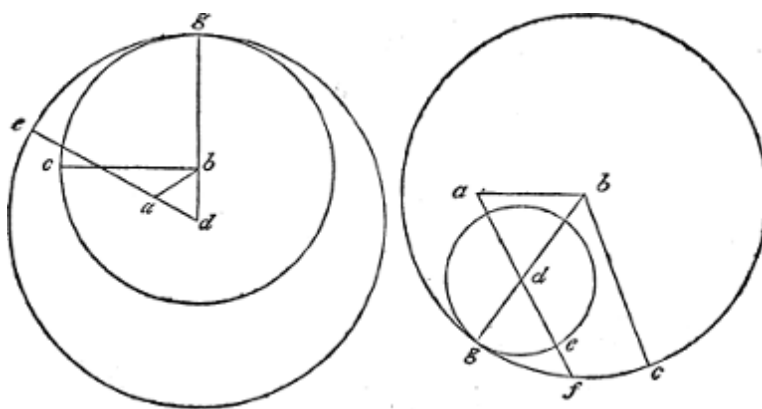
PROPOSITION II. PROBLEM II.

To a given point to place a right line equal to a given right line.

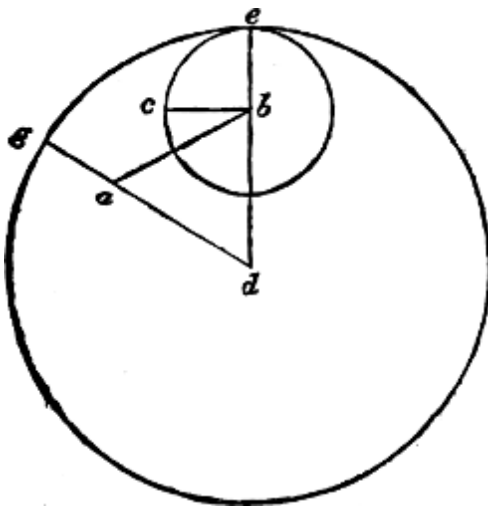
Of problems, as well as of theorems, some are without case, but others possess a multitude of cases. Whatever, therefore, have the same power acceding to many descriptions, and when their positions are changed, preserve the same mode of demonstration, these are said to have *case*; but such as proceed according to one position only, and one construction, are without *case*; for simply, *case*, appears about the construction both of theorems and problems. The second problem, therefore, has many cases; but a point is given in it *in position*, since it can only be given in this manner; but a right line, both in form and position, (for it is not simply *line*, but of such a kind.) For it is here enquired, *how to a given point to place a right line equal to a given right line*. But it is manifest that the point is entirely in the subject plane, in which the right line exists, and not in one more elevated. For in all problems and theorems respecting planes, we must conceive that one plane is subjected. But if any one should doubt how a line is to be placed equal to a given right line, for what if the given line be infinite? Since the present datum pertains both to finite and infinite: for every datum signifies that which is proposed and supposed by us for the sake of investigation. But this Euclid himself declares sometimes, saying, *upon a given terminated right line to construct an equilateral triangle*; but at other times, *upon a given infinite right line to let fall a perpendicular*. In answer then to this doubt, we must say, that when he orders us to place the line equal to a given right line, at a given point, he sufficiently evinces that the given line is finite; for every thing placed at a point, is terminated according to that point. Hence, the line equal to that which is given, must have a much prior termination. At the same time, therefore, in which he says *to a given point*, he terminates both the given right line, and its equal which is investigated.



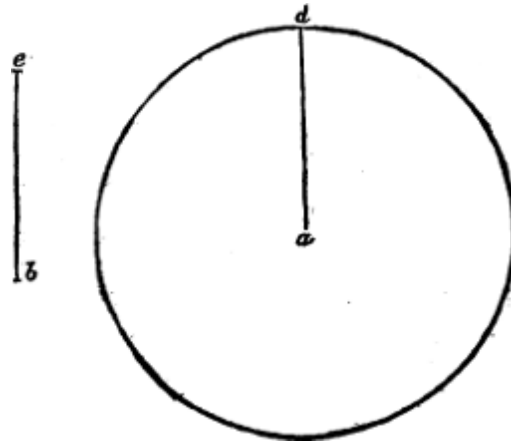
But that the cases of the present problem are formed from the various position of a point, is manifest. For the given point is either placed external to, or in the given right line; and if in it, it will either be one of its extremities, or it will be situated within the extremes; and if external, it will either have a lateral position, so that a line drawn from it to the extremity of the given line will form an angle, or a direct position; so that if the line were produced, it would coincide with the external point. But the geometrician, indeed, considers the point as external, and receives it according to a lateral position; however, for the sake of exercise, all the positions are to be assumed, the more difficult of which we shall exhibit. For let there be given a right line ab , and a given point c , which lies between its extremes, and let there be constituted according to the doctrine of the elements, an equilateral triangle upon the right line ac , and let dc , da , be produced; then, with the centre a , and the interval ab , let the circle be be described. And again, with the centre d , but with the interval de , let the circle df be designed. Because, therefore, a is the centre, ba is equal to ae ; and hence, de is equal to df , the parts of which, da , dc , are equal: for the triangle dac was established as equilateral. The remainder, therefore, ae , is equal to cf ; but ae , as it was shewn, is equal to ab , and hence, cf is equal to ab . To a given point, therefore, c , a right line cf is placed equal to ab . With respect to the position of the point then, so many cases arise. But there are many more with respect to the constitution of the equilateral triangle, the extension of its sides, and the description of circles.



For let there be assumed, as in this element, a point a , and a right line bc , but let ba be extended. The equilateral triangle, therefore, will not be constituted on ba , with its vertex above (because there is no place for it), but beneath; let it, therefore, be adb ; ad , therefore, is either equal to bc , or greater or less. If then it be equal that which was required is performed. But if less with the centre b , and the interval bc , let a circle be described, and let ad , db be produced to the points e and g ; and with the centre d , but the interval dg , let a circle ga be designed. Because, therefore, dg is equal to de , for they are drawn from the centre; and likewise because ad is equal to db , for the triangle is equilateral, the remainder ae is equal to the remainder bg . But bg is also equal to bc , for they proceed from the centre; and hence, ae is equal to bc , which was required to be done.



But if $a d$ is greater than $b c$ (for this is the last case), then with the centre b , and the interval $b c$, let a circle $e c$ be described. The line $d b$, therefore, shall cut the circle $e c$. Again, with the centre d , and interval $d e$, let the circle $e g$ be described. Because therefore, d is the centre of the circle $e g$, $g d$ is equal to $d e$. But $d a$ was also equal to $d b$; the remainder, therefore, $a g$ is equal to the remainder $b e$. But $b e$ is equal to $b c$, for both proceed from the centre. Hence, $a g$ is equal to $b c$; and it is placed at the point a , as was required to be done. And though there are many other cases, the description of the above is sufficient for our present purpose. For from these it is possible for the more curious to exercise themselves in the rest. But formerly some destroying the construction and variety of this problem, reasoned thus.

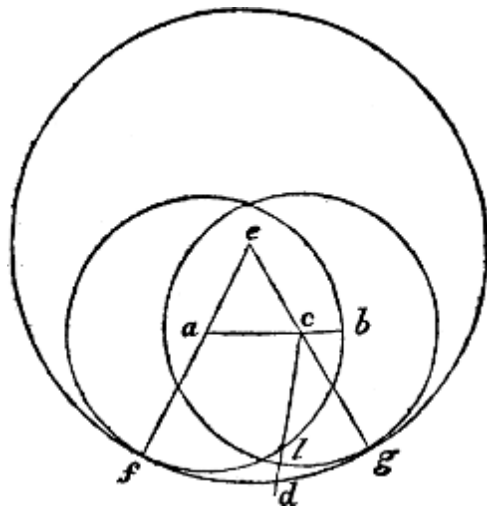


Let a be a given point, but $b e$ a given right line, and with the centre a , but with an interval equal to $b e$, let a circle $d e$ be described. Then let a certain right line $a d$ be extended from the point a to the circumference; and this shall be equal to $b e$: for the magnitude of the line from the centre, was equal to that of $b e$: and so that is done which was required. But he who thus reasons, *begs*, in the very beginning. For when he says with the centre a , but interval $b e$ describe a circle $e d$, he receives, in a certain manner, a line equal to $b e$, placed at the extremity a ; and preserving the Petition, he makes one extremity of the interval a centre, but with the other describes a circle: however, in this case, the centre is in one place, but the interval in another. We by no means, therefore, approve this method of demonstration.

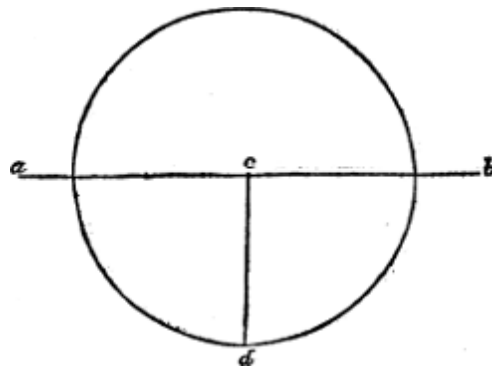
PROPOSITION III. PROBLEM III.

Two unequal right lines being given, from the greater to cut off a part equal to the less.

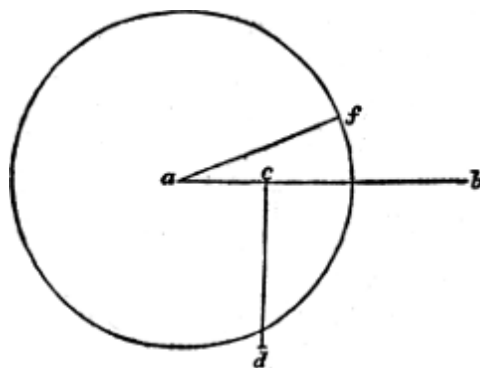
This third problem, likewise, has a variety of cases. For the given unequal right lines are either mutually distant from each other, as with the institutor of the elements, or they are united according to one extreme; or the one cuts the other according to one of its extremities, and this in a two-fold manner. For either the greater cuts the less, or the less the greater. But if they are united according to one extreme, the demonstration is manifest. For employing the common extremity as a centre, and the lesser of the lines for an interval, you will describe a circle, and cut off from the greater, a part equal to the less; since as much as the circle intercepts within itself, will be equal to the less. But if the one cuts the other according to its extreme, either the greater will cut the greater, or the contrary. And if they mutually cut each other, they will either be mutually cut into equal parts, or into unequal; or the one will be cut into equal, and the other into unequal parts, and this in a two-fold respect. For all these present us with an admirable variety of exercise, some of which, out of a many, we shall exhibit.



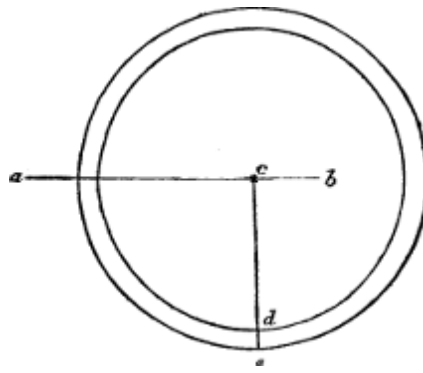
Let there be given the unequal right lines ab , cd , the greater of which is cd , and let it cut ab in one of its extremities c ; then with the centre a , but interval ab , let a circle bf be described, and let an equilateral triangle aec be constructed upon ac , and produce ea , ec . Again, with the centre e , but interval ef , let the circle gf be described; and with the centre c , and interval cg , the circle gl . Because therefore, ef is equal to eg (for the centre is e) of which ea is equal to ec , the remainder af , shall be equal to the remainder cg . But af is likewise equal to ab ; for the centre is a . Hence, cg will be equal to ab , and this is equal to cl , for the centre is the point c : ab , therefore, is equal to cl , which was required to be done.



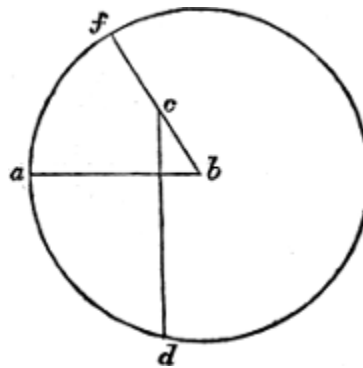
But let cd be less than ab , and let it cut ab according to its extremity c ; either, therefore, it will cut it in the middle, or not in the middle. Let it in the first place cut it in the middle; cd , therefore, is either the half of ab , and ac is equal to cd , or it is less than half. And in this case with the centre c , and interval cd , describe a circle, and you will cut off from ab a part equal to cd : Or it is greater than half;



and then at the point a , placing af , equal to cd , and describing a circle with the centre a , and interval af , you will cut off from ab a part equal to af , that is to cd . But if cd does not cut ab in the middle, cd shall either be its half, or greater than the half, or less.



If therefore cd is the half, or less than the half of ab , employing c as a centre, and cd as an interval, you will cut off from ab , a part equal to cd , as was required to be done. But if cd is greater than the half, again at the point a ^[5] placing af equal to cd , you will accomplish the same.



For with the centre a , but interval af , you will describe a circle, cutting off from ab a line equal to af , that is, to cd . But if they mutually intersect, as cd , ab , then with the centre b , but interval ba , describe the circle af , and let bc be extended to the point f . Because therefore, bf , cd , are the two unequal right lines, and cd cuts bf , according to its extremity, it is possible from cd to make a line equal to bf ; for this has been shewn in the first case of this problem. It is therefore possible, that a line equal to ab may be cut off from cd ; for ab and bf are mutually equal. Having, therefore, received these cases from division, we have endeavoured to exhibit their variety. But the demonstration of the elementary institutor is admirable, since it accords with all the preceding constructions. And it is possible, in every position, at the extremity of the greater, to place a line equal to the less, and using the same extreme as a centre, and placing the interval to describe a circle, which shall cut off from the greater, a line equal to the less, whether they mutually intersect, or one cuts the other, or they are constituted in a still different position.

PROPOSITION IV. THEOREM I.

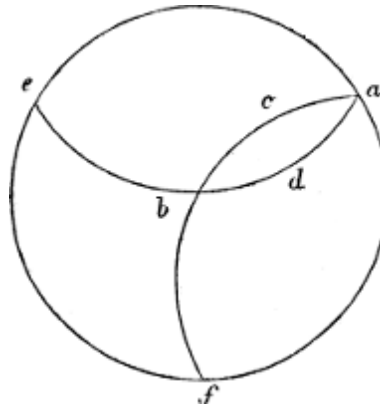
If two triangles have two sides equal each to each; and have likewise the angles equal;
which are comprehended by the equal sides; then they shall have their bases equal;
and the two triangles shall be equal; and the remaining angles opposite to the equal
sides shall be equal.

This is the first theorem in the institution of the elements, for all those which preceded were problems. The first, indeed, treating concerning the origin of triangles: but the second and third proposing to procure one right line equal to another. And of these the one produced an equal from an unequal line, but the other discovered an equal line by an ablation from one unequal. Since, therefore, equality, which is the first symptom in quantity, is to be constructed by us in a triangle and right line, it is delivered in the following theorem. For how can he who has not previously constructed triangles, and procured their origin, be learned in their essential accidents, and in the equality of angles and sides which they contain? How can he receive sides equal to sides, and right lines to other right lines, who has neither problematically investigated these, nor fabricated the invention of equal right lines? For if he should say it may happen before they are fabricated, that if two triangles have *this* for a *symptom*, they

shall likewise have *this particular symptom*; would it not, in this case, be easy to object to him, that we by no means know whether a triangle can be constructed? And should it be afterwards inferred, that if there are two triangles, they may have two sides equal to two sides, may we not also doubt this, whether it is possible that right lines may be mutually equal? And this particularly in geometrical forms, in which inequality not entirely existing, equality is likewise inherent. For we must learn that the cornicular is always unequal to an acute angle, and the same is true of the semicircular angle, and the transition from the greater to the less does not entirely take place through that which is equal. The institutor of the elements, therefore, first of all removing these objections, delivers also the construction of triangles (for it is common to three forms) and the origin of equal right lines, in a two-fold order. For he produces the one, not yet existing: but he acquires the other by an ablation from an unequal line. But after these he very properly subjoins the theorem, by which it is shewn how triangles having two sides equal to two, each to each, and the angles comprehended by the equal sides equal, have also the base equal to the base, the area equal to the area, and the remaining angles to the remaining angles. For there are three particulars exhibited in these triangles: but two data. Hence, the equality of the two sides is given, or two equal sides (and it is manifestly given in proportion) and the equality of the angle contained by the equal sides: but three particulars are investigated, the equality of *base to base*, of *triangle to triangle*, and of the *remaining angles*. But because it is possible that triangles may have two sides equal to two, and yet the theorem not be true, because the one is not equal to the other, but both together, on this account he adds in the data, that the sides are equal not simply, but one to the other. For if one of the triangles should have one of its sides of three units, but the other of four; and again, if the sides of the other triangle are respectively two, and five units, the angle comprehended by these being right, the two sides of the one triangle, will, indeed, taken together, be equal to the two sides of the other, or to seven units, yet the two triangles will not be equal. For the area of the one is six units^[6], but of the other five. And the reason of this is, because the sides are not equal each to each. Hence, many, not observing this in the division of land, when they have received a greater, have thought just the same as if they had received an equal field; and this because both the sides containing one field, have been together equal to both the sides containing the other field. It is requisite, therefore, to receive the one equal to the other, and to mark wherever the institutor of the elements subjoins this, because he does not add it without occasion. For discoursing on the equality of equal angles, he adds the particle *comprehended by equal sides*, lest by speaking indeterminately we should assume some one of the angles at the bases. Besides, when in triangles no side is previously named, we must conceive the base to be the side opposite to our sight; but when two are previously received, the remaining side is necessarily the base. Hence, here too, the institutor of the elements having previously assumed two sides equal to two, calls the remainder the bases of the triangles. But a triangle is then said to be equal to a triangle, when their areas are equal. For it is possible, that though the ambits are equal, yet the areas may be unequal, on account of the inequality of angles. But I call the area, the space intercepted by the sides of the triangle: as also I denominate the ambit, the line composed from the three triangular sides. Each, therefore, is different, and it is requisite, indeed, that besides the equality of the ambits, according to each side, the angles should also be equal, if also area ought to be equal to area. But it happens in certain triangles, that though the areas are equal, yet the ambits are unequal; and that the ambits being equal, the areas are unequal. For if there be two isosceles triangles, each of whose equal sides contains five units, but the base of the one is eight, and of the other six units; he who is ignorant of geometry, will say that the greater triangle is that whose base contains eight units. For the whole ambit will be eighteen. But the geometrician will say, that the area of each triangle contains twelve units, and this he will demonstrate, by drawing in each triangle a perpendicular from the vertex, and multiplying this with either part of the segments of the base^[7]. But it happens (as I have said) that though the ambits are equal, the spaces are unequal. Hence, certain persons formerly fraudulently deceived their partners in the division of fields, on account of the equality according to ambit, receiving a larger field. But one base is said to be equal to another, and one right line to another, when their extremes conjoined make the whole coincide with the whole. For every right line, indeed, agrees with every right line; but equal right lines mutually coincide according to their extremes. Again, one right-lined angle is said to be equal to another, when one of the comprehending sides of one angle being placed upon one of the other, the remaining side also coincides with the remainder: but when one of the remaining sides falls external to the other, the greater angle is that whose side falls externally; and the less whose side falls within. For there, indeed, the one contains, but in this case it is contained.

But we must assume the equality of angles according to the convenience of sides in right lines, and in all of the same species, as in lunulars and systroides^[8], and figures on both sides convex; because, it is possible that they may be equal, and yet the sides not mutually coincide. For a right angle is equal to a certain lunular angle,

and yet it is not possible that right lines can coincide with circumferences. Besides, this also must be previously understood, that the angles are said to subtend the opposite sides. For every triangular angle is contained by two sides of the triangle, but is subtended by the remaining side. Hence, the geometrician, when he says that the angles are equal, adds, *which are opposite to the equal sides*, lest we should conceive it of no consequence whatever angle is received, and should think that he denominated any other two angles of the triangles equal, but we must call those equal which subtend equal sides. For equal sides mutually subtend equal angles. And such are the considerations necessary to the declaration of the present theorem.



But against the objection of our adversary^[9], this must be previously assumed, that two right lines^[46] cannot comprehend space. For this the geometrician receives as evident. For if (says he) the extremes of the bases mutually coincide, the bases also shall coincide: but if not two right lines, will comprehend space. From whence, therefore, is the impossibility of this derived? Let there then be two right lines comprehending space acb , adb , and let them be infinitely produced. Then with the centre b , and interval ab , let a circle aef be described. Because, therefore, the line $acbf$ is a diameter, acf is the half of the circumference. Again, because the line adb is a diameter, ade is one half of the circumference. Hence, ace and acf are equal to the circumference, which is impossible. Two right lines therefore, cannot comprehend space; which the institutor of the elements knowing said, in the first Petition, *from every point, to every point, to draw a right line*, because one right line is always capable of uniting two points, but this is impossible for two right lines to effect. Many circumferences, indeed, may conjoin two points, both in the same, and in contrary parts: for by this means the extremities of a diameter conjoin two circumferences, but only one right line. But it is possible that both within and without semicircles, infinite circumferences conjoining given points may be described. And the reason of this is, because a right line is the least of lines, having the same extremes. But there is every where one *minimum*, and this always becomes the measure of the infinity of others. As therefore a right line; since it is one, becomes the measure of the infinity of right-lined angles (for by this we discover their quantity) so likewise a right line procures us the greatest utility in the mensuration of such as are non-rectilineal. And thus much may suffice concerning these.

But that the whole demonstration of the present theorem depends on common conceptions, rising as^[47] were spontaneously, and emerging from the evidence of hypotheses, is manifest to every one. For since two sides are equal to two sides, each to each, they will mutually coincide. But since the angles contained by the equal sides are equal, they also shall mutually coincide. And when angle is placed on angle, and sides on sides, so as to touch, in every part, the extremities of the sides beneath shall also coincide. But if these, then *base*, shall agree with *base*. And if three with three, the whole triangle shall accord with the whole triangle, and all shall be equal to all. Hence, therefore, equality considered in things of the same species, appears to be the cause of the whole demonstration. For here are two axioms endued with a power of containing the whole method of the proposed theorem. One, indeed, affirming, that *things which mutually coincide, are equal*; and this is simply true, requiring no limitation, and is employed by the institutor of the elements both in the base, and in the space, and in the other angles. For these, says he, are equal, because they mutually coincide. But the other affirming that *things which are equal mutually coincide*. This, however, is not true in all, but in those of a similar species. But I call things similar in species, such as a right line when compared with a right line, one circumference with another of the same circle, and the angles comprehended by similar lines endued with a similar position. But of these, I say, that such as are equal, mutually coincide: so that in short, the whole demonstration is of this kind. These equals, therefore, are given, viz. two sides equal to two sides, and the angles which they comprehend, and these accord

among themselves. But if these mutually coincide, the base also shall agree with the base, and all coincide with all. And if these accord, they are also equal. If then these are equal, it may at the same time be shewn that all are equal to all. And this appears to be the first mode of knowing triangles on all sides equal. And thus much concerning the whole demonstration.

But Carpus, the mechanist, who, in an astrological treatise, discourses of problems and theorems, says⁴⁹ “that they must not be passed over in silence, since they opportunely present themselves for investigation;” and lastly, entering on their distinction, he observes, “that the problematical genus precedes theorems in order. For in problems (says he) the invention of subjects is investigated prior to symptoms. Likewise, a problematical proposition is simple, and requires no artificial intelligence. For this commands us to accomplish something evident, as *to construct an equilateral triangle, or from two given unequal right lines, to cut off from the greater a part equal to the less*. For what is there in these difficult and obscure. But he affirms that the proposition of a theorem is difficult, and requires the most accurate power, and a judgment productive of science, that it may appear neither to exceed, nor to be deficient from truth; such, indeed, as the present, which is the first of theorems. Add too, that in problems, there is one common way invented by resolution, by proceeding according to which, we can happily accomplish our purpose. For after this manner the more easy kind of problems are investigated. But the treatise of theorems is so very difficult, that even to our time (says he) no one has been able to deliver any common method of their invention. Hence, on account of facility also, the problematical genus is more simple. But these being distinguished, it is on this account (says he) that in the elementary institution problems precede theorems, and from these the institution of the elements begins; and the first theorem is in order the fourth, not because the fourth is exhibited from the preceding, but because it is necessary they should precede as being problems, and this a theorem, though it should require none of the antecedent propositions for its demonstration. For the present theorem entirely employs common conceptions; and in a certain respect receives the same triangle in a different position. Since coincidence, and its consequent equality possess a sensible and manifest apprehension. But such being the demonstration of the first theorem, problems with great propriety precede, because they are universally allotted the primary place.” And perhaps, indeed, problems antecede theorems in order; and particularly among those who ascend to contemplation from the arts, which are conversant with sensible particulars: but theorems excel problems in dignity of nature. And it appears, that all geometry, so far as it conjoins itself with a variety of arts, energizes problematically: but so far as it coheres to the first science, it proceeds theorematially from problems to theorems, from things secondary to such as are first, and from things which more regard the arts, to such as are endued with a greater power of producing science. It is, therefore, vain to accuse Geminus, for affirming that theorems are prior to problems. For Carpus assigns a precedency to problems, according to order: but Geminus to theorems, according to a more perfect dignity. But of this fourth theorem, we have already observed, that in a certain respect it is indigent of the preceding problems, in which we learn the origin of triangles, and the invention of equality. But we now add, that since it is the most simple and principle of theorems (for it is naturally, as I may say, exhibited from primary conceptions alone), but demonstrates a certain symptom appearing about triangles, having two sides equal to two, each to each, and the two angles equal contained by the equal sides, it is with great propriety placed the first after problems, in which things subject to this symptom, and the data themselves are constructed.

PROPOSITION V. THEOREM II.

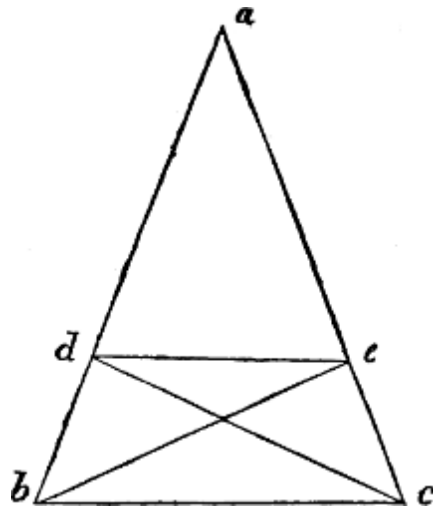
The angles at the base of an isosceles triangle are mutually equal; and the equal right lines being produced, the angles under the base shall be mutually equal.

Of theorems some are *simple*, but others *composite*. I call those *simple*, which, both according to hypotheses and conclusions, are indivisible, possessing one *datum*, and one object of investigation. Thus for example, if the institutor of the elements had said, *every isosceles triangle has the angles at the base equal*, it would have been a simple theorem. But theorems are composite, which are composed from many particulars, either having composite hypotheses, or conclusions from a simple hypothesis, or both. And of these, some are *complex*, but others *incomplex*. The *incomplex* are such composites as cannot be divided into simple theorems, as the fourth proposition. For in this, both the *datum* is a composite, and its consequent, yet it is impossible that the *datum* can be divided into things simple, and become theorems. For if a triangle has its sides alone equal, or the angle at the vertex, the same consequences will not ensue. But the *complex* are such as may be divided into things simple, as the theorem which says, *triangles and parallelograms of the same altitude, have the same proportion as their bases*. For it is possible to say by division, that *triangles of the same altitude, have the same proportion as their*

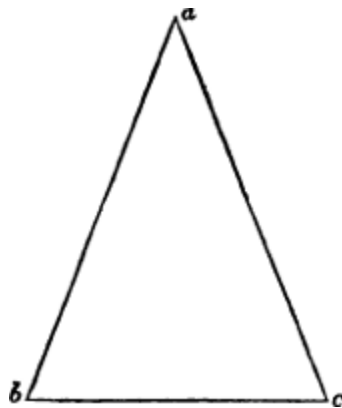
bases, and in parallelograms after a similar manner. But of all composites, some are composed according to the conclusion, being excited from the same hypothesis: but others have their conclusion according to hypotheses, and infer the same conclusion in all: and others, lastly, are composed both according to the conclusion, and according to hypotheses. *Composition*, therefore, in the present case, is according to the *conclusion*, for there are three particulars concluded in this theorem, *that the bases are equal, that the triangles are equal, and that the remaining angles, under the base, are equal to the remaining angles*. But composition, according to *hypothesis*, is found in the common theorem *of triangles and parallelograms of the same altitude*. And according to *both*, in the theorem that *the diameters both of circles and ellipses, bisect as well the spaces as the lines containing the spaces*. But of *complex* theorems, some are universal: but others conclude that which is universal from particulars. For if we should say that a diameter divides a circle, ellipsis, and parallelograms, we receive, indeed, every part of the complex, not universally, but we make that universal which is composed from all. But if we should say, that *in a circle, all lines passing through the centre, mutually bisect each other, and make equal angles of all the segments*, we should affirm a universal. For in an ellipsis all the angles of the segments are not equal, but those only which are formed by the diameter. But these compositions are entirely fabricated, for the sake of geometrical brevity and resolutions. For many things incomposite are not resolved, but composites alone afford convenience to a resolution tending to principles.

In consequence of these previous considerations then, we must call the fifth theorem a *composite*⁵² and a composite, both with respect to the *datum*, and the *object of investigation*; and this the institutor of the elements exhibiting, divides this theorem, being one, and gives a separate position to the *data*, and the *things to be investigated*, for he says that *the angles at the base of an isosceles triangle are equal*; and again, that *the equal sides being produced, the angles under the base are equal*. For we must not think that there are two theorems, but one; and that this is a composite, both according to the *data*, and *thing sought*: and that each of these composites is perfect and true. Hence, conversion also is true in each. For if the angles at the base are equal, the triangle is isosceles: but if those under the base are equal, the equal right lines are produced, and the triangle is isosceles. But the institutor of the elements *converts* the equality of the angles at the base; but not the equality of those under the base, though this is likewise true; the reason of which we shall shortly explain. But we shall now, in the first place, enquire on what account he demonstrates that the angles under the base are equal. For he never employs this in the construction or demonstration of other problems or theorems. It may be doubted, therefore, why, since it is useless, it was requisite to insert it in the present theorem? To this we must reply, that though it is never employed in the elements, yet it is most useful for the destruction of objections, and the solution of oppositions to theorems^[10]. But it is artificial, and belongs to science to prepare solutions of things resisting its propositions, and to provide subsidies of answers; that not only true demonstrations may be fabricated from things previously demonstrated, but that from hence confutations of error may be produced. And from this geometrical order, you will likewise receive a rhetorical emolument. For he who can effect this in the discourses of rhetoric, who can foresee the oppositions to his following heads, and previous to their delivery, can first of all prepare solutions of them to others, he, indeed, will fabricate in a wonderful manner, a most excellent mode of disputation. The institutor of the elements, therefore, teaching us this in reality, previous to the theorems by which we solve opposing objections, employing such as are now exhibited, at the same time demonstrates, that the angles under the base of an isosceles triangle, are equal, and thus prepares a confutation of the falsehood such objections contain. But that from the present theorem we may solve the objections urged in the seventh and ninth propositions, will be perspicuous as we proceed. Hence, it appears, why Euclid does not convert the latter part of this theorem in the sixth, because it does not produce a principal utility, but confers to our advantage, accidentally, with respect to the whole of science.

But if any one should desire us without producing the equal right lines, to prove the angles at the base⁵³ of an isosceles triangle equal, (for it is not requisite to demonstrate the equality of these, by those under the base) by transposing, in a manner, the construction, and fabricating those constructions within, which are made without the isosceles triangle, we may exhibit the thing proposed.



Thus let abc be an isosceles triangle, and in the side ab , take any point d , and from ac , take ae , equal to ad , and draw the lines be , dc , de . Because, therefore, ab is equal to ac , and ad to ae , and the angle a is common, be also shall be equal to cd , and the remaining angles to the remaining angles. Hence, the angle abe , is equal to the angle acd . Again, because db is equal to ec , and be to cd , and the angle dbe to ecd ; hence, the base, since it is common to both, is equal to itself, and all are equal to all. The angle, edb , therefore, is equal to the angle dec : and the angle deb , is equal to the angle edc . Hence, since the angle edb , is equal to the angle dec , from which the equal angles deb , edc , are taken, the remaining angles bdc , ceb are equal. But the sides also bd , dc , are equal to the sides ce , eb , each to each, and the base bc is common. All, therefore, are equal to all. Hence, the remaining angles also, subtending equal sides, are equal. The angle, therefore, dbc , is equal to the angle ecb . For the angle dbc , subtends the line dc : but the angle ecb , the line eb . The angles, therefore, at the base of an isosceles triangle, are equal, the equal right lines not being produced.



But Pappus demonstrates this yet shorter, without any addition in the following manner. Let abc be an isosceles triangle, having ab , equal to ac . We must conceive, therefore, this one triangle as if it was two, and reason thus. Because ab is equal to ac , and ac to ab , the two sides ab , ac , are equal to the two ac , ab , and the angle bac , is equal to the angle cab , (for it is the same.) All, therefore, are equal to all. The base bc , to the base cb . But the triangle abc , to the triangle acb ; and the angle abc , to the angle acb , and the angle acb , to the angle abc . For they subtend equal sides, i.e. ab , ac . The angles, therefore, at the base of an isosceles triangle, are equal. And it seems that Pappus invented this mode of demonstration, when he considered that the institutor of the elements also, in the fourth theorem, when he had united two triangles, and had made them mutually coincide, thus forming one of two, by this means observed their equality throughout. In like manner it is possible, that we also, by an assumption contemplating two triangles in one, may demonstrate the equality of the angles at the base. Thanks, therefore are to be given to the ancient Thales for the invention of this theorem, as well as a multitude of others. For he, first, is said to have perceived and affirmed, that the angles at the base of every isosceles triangle are equal: and after the manner of the ancients, to have called them similar. But still more deserving of praise are those moderns, who have yet more universally demonstrated (among which number is Geminus) that equal right lines falling from one point, on a line of similar parts, form equal angles. For Geminus

using this theorem, shews, that there are only three lines, and not more of similar parts, the *right*, the *circular*, and the *cylindric helix*; and this is properly universal, to which this symptom first agrees, just as the possession of two sides greater than the third, is shewn to be essentially inherent in every triangle. It is not, therefore, the property universally of every isosceles, though it belongs to every one, to possess angles at the base equal: but of equal right lines falling on a line of similar parts. For to subtend equal angles, is in these primarily inherent.

PROPOSITION VI. THEOREM III.

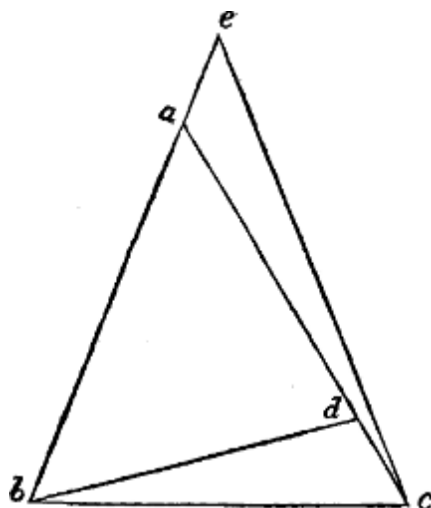
If two angles of a triangle be equal to each other, the sides also which subtend the equal angles, shall be equal to one another.

The present theorem exhibits these two properties of theorems, *conversion*, and a *deduction*^{56b} to an impossibility. For it is converted, indeed, in the preceding theorem, but its certainty is evinced by a deduction to an impossibility. It is requisite, therefore, to speak of each, whatever belongs to the present treatise. One kind of *conversion* then, among geometricians, is denominated principally and properly, when the conclusions and hypotheses alternately receive theorems; so that the conclusion of the former becomes hypothesis in the latter; and hypothesis is inferred as the conclusion. As *that the angles at the base of an isosceles triangle are equal*. For here the isosceles triangle is the *hypothesis*: but the *conclusion*, the equality of the angles at the base. And *that where the angles at the base are equal, the triangles are isosceles*, which the present 6th theorem affirms. For here the equality of the angles at the base is the *hypothesis*; but the *conclusion*, the equality of the sides subtending the equal angles. But another kind of conversion, is alone according to a certain mutation of composites. For if the theorem be composite, beginning from many hypotheses, and ending in one conclusion, by receiving the conclusion, and one or more of the hypotheses, we infer some one of the other hypotheses as a conclusion. And after this manner the eighth theorem is the converse of the fourth. For the one says, that *equal bases subtend equal sides and angles*: but the other, that *equal sides being placed on equal bases, contain equal angles*. Of which the predication concerning *equal bases* in the latter proposition, is the *conclusion* of the former: but the predication concerning the position of *equal sides*, is one of the previously assumed hypotheses in the former theorem; and the *comprehension of equal angles* is another hypothesis which this fourth proposition contains. In consequence therefore of these two *conversions*, the one which is called the principle, is uniform and determinate: but the other is various, advancing into a great number of theorems, and not converting in one, but in many, on account of the multitude of hypotheses, in composite theorems. But oftentimes in that which begins from two hypotheses, there is one which is converted, when the hypotheses are not all determinate, but some of them indeterminate.

It is here, however, requisite to observe, that many false and improper conversions take place. As *that every sexangular is a triangular number*^[11]. For the converse is not also true, that *every triangular number is sexangular*. But the reason of this is, because the one is more common, but the other more particular. And one is alone predicated *totally*^[12] of the other. But things in which, *that which is primary*, is inherent, and according to which it is received, in these, conversion also follows. And these observations, indeed, were not unknown to those mathematicians, the familiars of Menæchmus, and Amphinomus. But of theorems receiving conversion, some are usually called *precedents*, but others *converse*. For when supposing a certain genus, they demonstrate some symptom of its nature, they call this a *precedent* theorem. But when on the contrary, they make the hypothesis a symptom, and the conclusion a genus, they denominate the theorem to which this happens *converse*. As for instance, the theorem which says, *every isosceles triangle has the angles at the base equal*, is a *precedent*. For that is subjoined which precedes by nature. I mean the genus itself, or the isosceles triangle. But that which says, *every triangle possessing two equal angles, has likewise the sides subtending those equal angles equal, and is isosceles*, is a *converse* theorem. For it changes the subject, and its passion, supposing the latter, and from this exhibiting the former. And thus much concerning geometrical conversions.

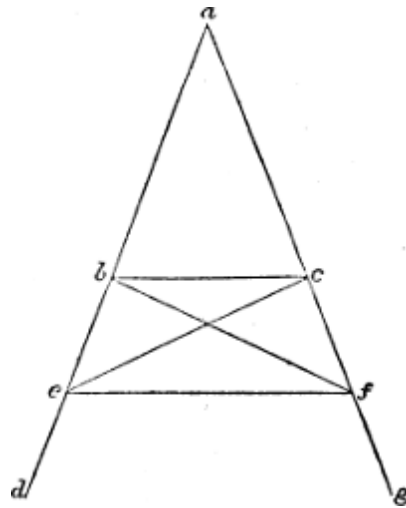
But deductions to an impossibility, entirely end in an evident impossible, the contrary of which is confessed by all. It happens, however, that some of them end in such things as are opposed to Axioms, or Petitions, or Hypotheses; but others in things contradicting prior demonstrations. For the present sixth theorem shews that which happens to be impossible, because it destroys the common conception, affirming that *the whole is greater than its part*. But the eighth theorem falls, indeed, on an impossible, yet not on that endued with a power of destroying a common conception, but that exhibited by the seventh theorem. For what the seventh denies, this affirming exhibits to such as do not admit the object of investigation. But every deduction to an impossibility,

which being received, opposes *the thing sought*, and on this hypothesis advances, until it falls upon the explored absurdity, and by this means destroys the hypothesis, corroborates that which was investigated from the first. But it is requisite to know, that all mathematical proofs are either *from principles*, or *to principles*, as Porphyry in a certain place affirms. And the proofs *from principles*, are two-fold. For they either emanate from common conceptions, and things self-evident: or from things previously exhibited. But proofs *to principles* are endued with a power of either *establishing* or *destroying principles*. And those, endued with a power of *establishing principles*, are called *resolutions*; and to these compositions are opposed. For it is possible that we may proceed in an orderly method from those principles to the object of investigation; and this is nothing else than composition. But those possessing a power of *destroying principles*, are called *deductions to an impossibility*. For it is the business of this mode to destroy some of the concessions, and objects of investigation. And in this, also, there is a certain ratiocination, though not the same as in resolution. For in deductions to an impossibility, *complexion* is according to the second mode of hypothetical reasonings. As if *in triangles possessing equal angles, the sides subtending the equal angles are unequal; and the whole is equal to its part*: but this is impossible. *In triangles, therefore, possessing two equal angles, the sides subtending the equal angles are equal*. And thus much concerning what is called by geometricians, deduction to an impossibility.



But the institutor of the elements uses *conversion* in the present proposition, for he receives the conclusion of the fifth as a datum, and adds its hypothesis as an object of enquiry: but he employs *deduction to an impossibility*, in the construction and demonstration. But if any should rise up, and assert that it is not necessary by taking a part from *a c* equal to *a b*, to make the ablation at the point *c*, but at the point *a*, upon this hypothesis, we shall fall into the same impossibility. For let *a b* be equal to *a d*, and having produced *b a*, let *a e* be placed equal to *d c*. The whole *b e*, therefore, is equal to the whole *a c*.

Let *e c* be connected. Because, therefore, *a c* is equal to *b e*, but *b c* is common, the two are equal to the two, and the angle at the point *b*, is equal to the angle *a c b*. For so it was established in the hypothesis. All, therefore, are equal to all, by the fourth theorem. Hence, the triangle *e b c*, is equal to the triangle *a b c*, the whole to the part, which is impossible. But because this also is manifest, it remains that we exhibit the rest of the conversion. For the institutor of the Elements converts the whole sixth theorem from a part of the fifth. But it is requisite to adjoin the remaining conversion. This, then, he receives as an hypothesis, *that the angles at the base of a certain triangle are equal*: but he shews that the triangle is *isosceles*.



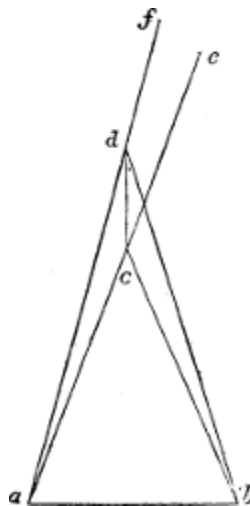
Let acb , therefore, be a triangle, and let ab , ac , be produced to the points d g , and let the angles under the base be equal. I say that the triangle abc , is isosceles. For let there be assumed in the line ad , the point e , and let be be taken equal to cf ; and connect the lines ec , bf , ef . Because, therefore be is equal to cf , but bc is common, the two will be equal to the two. And the angle ebc , is equal to the angle fcg ; for they are under the base. All, therefore, are equal to all, by the fourth theorem. Hence the base ec , is equal to the base fb , and the angle bec , to the angle cfb ; and the angle cbe , to the angle bce : for they subtend equal sides. But the whole angle ebc , was equal to the whole fcg , of which the angle fbc , is equal to the angle ecb . The remainder, therefore, ebf , is equal to the remainder fce . But be is equal to cf , and bf to ce , and they contain equal angles. All, therefore, are equal to all. Hence, also, the angle bef , is equal to the angle cef . Wherefore, the side ae , is equal to the side af (for it is shewn by the sixth) of which be , is equal to cf . The remainder, therefore, ab , is equal to the remainder ac . And hence, the triangle abc , is isosceles. It is, therefore, as well isosceles, if it possesses angles at the base equal: as if the sides being produced it has the angles under the base equal. Why then did not the institutor of the Elements convert the remaining part? Shall we say it was because the equality of the angles under the base in the fifth theorem, was exhibited for the sake of solving other doubts. But that proving the triangle to be isosceles, from the equality of the angles under the base, neither confers to a principal demonstration, nor to the solution of things investigated, the truth of which is confirmed in the following theorems, and that from the equality of the angles under the base, he is enabled to demonstrate that the triangle is isosceles? For if every right line, standing upon a right line, and forming two angles, makes them equal to two right; when the angles under the base are equal, those upon the base will be equal. And these being equal, the sides subtending them shall be equal. Euclid, therefore, having used this in the whole elementary institution, was enabled to conclude, that when the angles under the base are equal, the triangle is isosceles. Indeed he requires this also, for the demonstration of certain theorems: For shortly a theorem will appear, evincing, that if a right line standing on a right line, forms angles, it will either make two right, or angles equal to two right. And the theorems, indeed, preceding this, require no such conversion; but those which follow, are indigent of this, and establish their credibility from the present theorem.

PROPOSITION VII. THEOREM IV.

[13] Upon the same right line, two right lines cannot be constituted equal to two other right lines each to each, *drawn* to different points, to the same parts, and having the same extremes with the two right lines first drawn.

The present theorem possesses a rare property, which is not frequently found in propositions producing science. For to be formed by negation, and not by affirmation, is not their sufficiently distinguishing property. Indeed, the propositions, as well of geometrical as of arithmetical theorems, are for the most part affirmations. But the reason of this is, (as Aristotle says) because, an affirmative universal, especially agrees with sciences, as more proper, and not indigent of negation: but a universal negative requires affirmation, in order to produce evidence; for from negatives alone, there is neither demonstration nor reasoning. Hence, demonstrative sciences exhibit a multitude of affirmations, but rarely employ negative conclusions. However, the proposition of this theorem is full of admirable diligence, and is bound with every addition, by which it is rendered so certain and

indubitable, that it cannot be confuted and overturned by the efforts of opposing calumniators. For in the first place, the particle *upon the same right line*, is assumed, lest we should exhibit upon *another*, two right lines equal each to each, and employ the proposition for the purpose of circumvention. In the second place, he does not say upon what right line, to constitute two right lines simply equal to two (for this is possible) but *each to each*. For what wonderful thing is it, that he should take both equal to both, who extends one of the constituted lines, and contracts the other? But each to each, (says he) is impossible. In the third place, he adds the particle, *to different points*. For what, if some one, when he has formed two lines equal to the first two, each to each, should connect these with those in the same point, which joins the subject right lines in the vertex; and should constitute these? For the extremes of equal right lines perfectly coincide. In the fourth place, he adds the particle *to the same parts*^[14]. For what if one subject right line being given, we should place two of the right lines on one side, and the other two on the opposite side, so that this common right line should be the basis of the two triangles with opposite vertexes? Lest, therefore, we should form an erroneous figure, and charge our deception on the institutor of the Elements, he adds the particle *to the same parts*. In the fifth place, he subjoins, *having the same extremes with the two right lines first drawn*. For it is possible to constitute *upon the same right line*, two right lines equal to two, *each to each*, drawn to different points, and to the same parts, by employing the whole right line, and constructing upon it, these two right lines; but then the lines last drawn, will not have the same extremes with those constituted at first. For if we conceive in a quadrangle two diagonals drawn on one of its sides, two lines shall be equal to two; a side and diameter to its parallel side, and the other diameter. But in this case the equal right lines will not have the *same* extremes. For neither the parallel sides, nor the diameters, will mutually possess the same extremes; and yet they will be equal. These distinctions, therefore, being preserved, the truth of the proposition, and the certainty of the reasoning, is evinced.



But perhaps, some, notwithstanding all these terms producing science, will dare to object, that these hypotheses being admitted, it is possible to effect what the geometrician affirms to be impossible. For let there be a right line ab , and upon this two lines ad, db , equal to two ac, cb , and let the former be external to the latter, being drawn to different points d, c , and terminated in the same extremes a and b . Let ac too, be equal to ad : but bc to bd . This objection, then, we shall confute, by connecting the line dc , and producing the lines ac , and ad , to the points e, f . For these being constructed, it is manifest that the triangle acd is isosceles, ad , being equal to ac , from hypothesis; and the angles under the base ecd, fdc are equal. The angle fdc , therefore, is greater than the angle bdc . Much more then is the angle bcd greater than the angle bdc . But again, because the line db , is equal to the line bc , the angles also at the base are equal, i.e. the angle bcd , to the angle bdc . The same angle, therefore, is both greater and equal, which is impossible. And this is what we said in our exposition of the fifth theorem, that though the equality of the angles under the base, was not useful to the demonstrations of the following theorems, yet it procured the greatest utility in the solution of objections. For in the present instance we have confuted the objection, by inferring that, because ac , and ad , are equal, the angles ecd , and fdc , are also equal. In a similar manner in other theorems, it will appear to be peculiarly useful for the solution of doubts^[15].

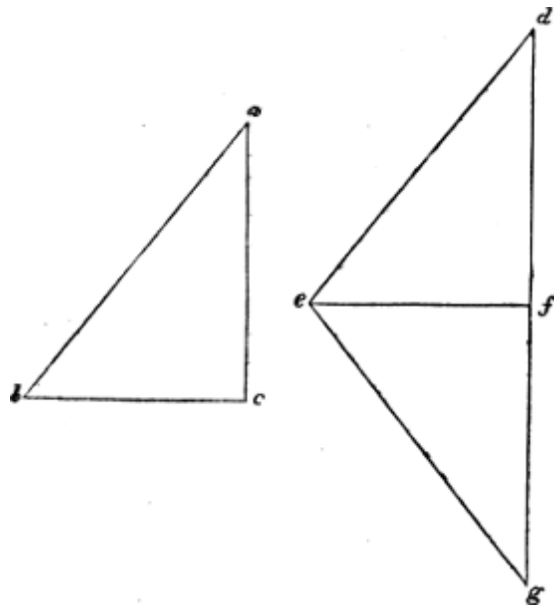
But if any one should say that there may be constituted upon the right line ab , right lines bd, bc , equal to the right lines ac, ad , of which bc may be equal to ac , but bd to ad ; and that in this case they will be drawn to different points a and b , to the same parts, and will have the same extremes with ac , and ad , viz. c , and d , what

shall we reply to this assertion? Shall we say that it is requisite to constitute the first lines, upon the right line $a b$, and their equals upon the same right line? For this is what the institutor of the Elements affirms in the proposition. But here, $a c$, and $a d$, are not constituted upon the right line $a b$, but only on one of its points. Hence, the lines $a c$, $c b$, and $a d$, $d b$, which stand on the right line $a b$, are different from the right lines, which were placed in the beginning, and to which they ought to be constituted equal. Though at the same time it is necessary that the right lines constituted upon $a b$, should be equal to those constituted upon $a b$. And thus much may suffice for objections against the present question, But that the present theorem is exhibited by the institutor of the elements, by a deduction to an impossibility, and that this impossible opposes the common conception, affirming that *the whole is greater than its part*; and that *the same thing cannot be both greater and equal*, is sufficiently manifest. But this theorem seems to have been assumed for the sake of the eighth theorem. For it confers to its demonstration, and is neither simply an element, nor elementary: since it does not extend its utility to a multitude. And hence, we find it very rarely employed by the geometrician.

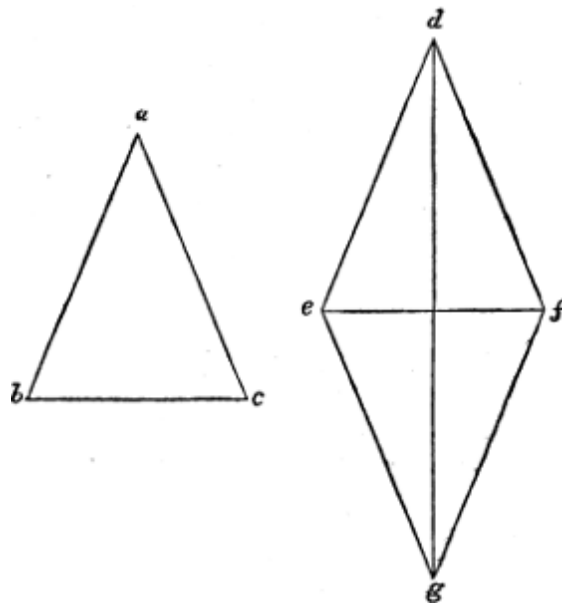
PROPOSITION VIII. THEOREM V.

If two triangles have two sides equal to two, each to each, and have the base equal to the base: then the angles contained by the equal right lines, shall be equal to each other.

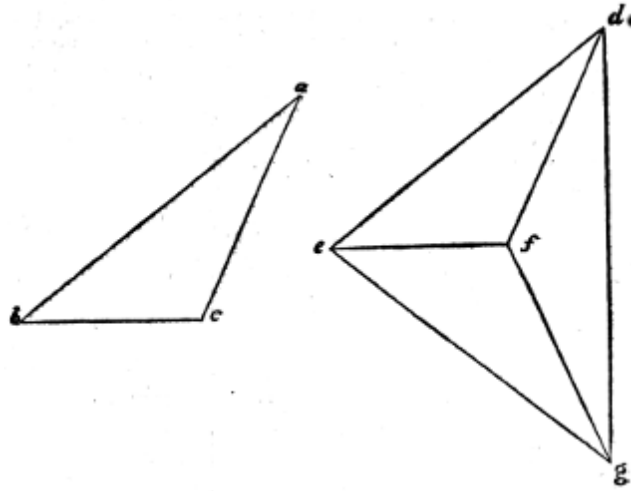
This eighth theorem is the converse of the fourth: but it is not assumed according to a principal conversion. For it does not make the whole of its hypothesis a conclusion; and the whole conclusion an hypothesis. But connecting together some part of the hypothesis of the fourth theorem, and some part of the objects of enquiry, it exhibits one of the data which it contains. For the equality of two sides to two, is in each an hypothesis; but the equality of base to base, is, in the fourth, an object of investigation, but in the present a datum; and the equality of angle to angle, is, in the former, a datum, but in the latter, an object of enquiry. Hence, a change alone of data, and objects of investigation, produces conversion. But if any one desires to learn the cause why this theorem is placed in the order of the eighth proposition, and not immediately after the fourth, as its converse, in the same manner as the sixth after the fifth, of which it is the converse, since many converted propositions follow their precedents, and are exhibited after them without any intervening medium, to this we must reply, that the eighth, indeed, is indigent of the seventh proposition. For its truth is evinced by a deduction to an impossibility, but the nature of an impossible becomes known from the seventh. And, this again, in its demonstration, is indigent of the fifth. Hence, the seventh and fifth theorems were necessarily assumed, previous to the present. But because the *converse* to the fifth obtained a demonstration easy, and from *things first*, it was very properly placed after the fifth, on account of its alliance with that theorem; and because, since it is shewn by a deduction to an impossibility, it confutes that which is impossible from common conceptions, and not as the eighth from another theorem. For things opposing common conceptions, are more evident for the purpose of confutation than such as contradict theorems: since these are assumed by demonstration, but the knowledge of axioms is better than demonstration. But the institutor of the elements exhibits what is now proposed from the previously demonstrated seventh theorem.



But the familiars of Philo assert, that they can demonstrate this theorem, without being indigent of any other. For let there be conceived (say they) two triangles, abc , def , having two sides equal to two, and the base bc equal to the base ef . Likewise let the bases coincide with each other; and let the two triangles abc , def , be so placed in the same plane, that their vertices may be opposite, and so that efg may be the equal substitute of abc . And let eg be equal to de , but fg to df . Hence, fg will either be placed in a right line with df , or not in a right line. And if not in a right line, it will either make with it an angle according to the internal part, or according to the external. Let it first be placed in a right line. Because, therefore, de is equal to eg , and dfg is one line, the triangle deg , is isosceles, and the angle at the point d , is equal to the angle at the point g . But if it does not lie in a right line, it will make an angle inward; and in this case let $d g$ be connected.



cause, therefore ed , eg , are equal, and the base is dg , the angle edg also, is equal to the angle egd . Again, because df is equal to fg , and the base is dg , the angle, also, fdg , is equal to the angle fgd . But the angle edg was also equal to the angle egd . Hence, the whole edf , is equal to the whole fgd , which was required to be demonstrated. But in the third place, let fg make an angle with df , externally, and let the right line dg be connected.



Because, therefore $d e$, $e g$, are equal, and the base is $d g$, the angles $e d g$, $d g e$, are equal. Again, because $d f$, $f g$, are equal, and the base is $d g$, the angle $f d g$, is equal to the angle $f g d$. But the whole angles $e d g$, $d g e$, were mutually equal. Hence, the remaining angles $e d f$, $f g e$, will be equal to each other. And thus the thing proposed is invented according to any position of the right line $f g$, and we may demonstrate the theorem, without employing the seventh proposition.

Is, then (say they), the seventh proposition introduced in vain by the institutor of the elements? For if we only assume it on account of the eighth, but the eighth may be exhibited without it, does not the seventh appear entirely useless? To these enquiries we must reply in the words of our predecessors, that the seventh theorem, being demonstrated, is of the greatest utility to such as are skilled in astronomical concerns, when they discourse concerning the eclipses of the sun and moon. For, employing this theorem, they shew that three consequent eclipses, distant from each other by an equal space, cannot subsist. I say, in such a manner, that the second may be distant from the first by as great a space of time as the third from the second. For example, if the second is produced after the first, when six months and twenty days are elapsed; the third, will by no means be produced after the second, by the same, but by either a greater or less interval of time. But that this is the case may be demonstrated by the seventh theorem. And the institutor of the elements has not only exhibited the present as conferring to astronomy, but a multitude of other theorems and problems. For to what other end shall we say that the last problem of the fourth book was proposed, by which we are taught how to inscribe the side of a figure of fifteen angles in a circle, than for its relation to astronomy? For those who describe in a circle a quindecangle passing through the poles, will, by this means, obtain the distance of the poles of the equator from the poles of the zodiac. Since they are distant from each other by the side of a quindecangle. The institutor of the elements, therefore, appears by regarding astronomy, to have previously exhibited many things preparative to our advancement in that science. But when, at the same time, he saw that this seventh theorem is exhibited from the fifth, and proves the eighth without any variety, he assigned it the present place. The addition of Philo is, indeed, beautiful, but is not sufficiently adapted by its variety of cases to an elementary institution. And thus much in reply to the present question.

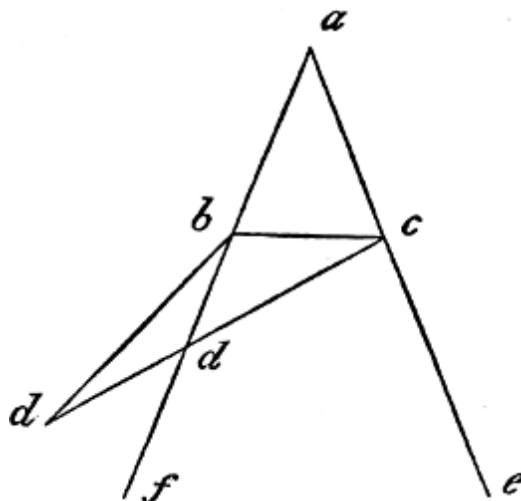
But if any one should doubt why he does not add so much in the eighth as in the fourth theorem, I mean, *that the triangles and the remaining angles are equal*; we must say, that because the equality of the vertical angle is demonstrated, it follows, that all are equal to all, by the fourth theorem. It was therefore alone necessary to demonstrate this by itself, but to assume all the rest as consequents. But it seems that the equality of the vertical angles causes the equality of the bases, and of the sides comprehending those angles. For when the bases are unequal, the same angles will not remain, though the containing equal sides are supposed, but while the base becomes less, the angle is at the same time diminished, and while that increases, the angle also receives a correspondent increase. Nor while the same bases remain, but the sides become unequal, will the angle remain; but while they are diminished, it will be increased; and while they are increased, it will be diminished: for angles, and their containing sides, suffer a contrary passion. Thus, if upon the same base, you conceive the sides descending to the lower part, you will diminish the sides, but increase the angle which they comprehend, and enlarge their distance from each other. But if you conceive the sides to be elevated, and to receive an addition as they rise, you will diminish the angle which they contain: for they will coincide the longer, when their vertex is

more remote from the base. We may therefore certainly affirm that the identity of the basis and equality of the sides, in a triangle, determine the equality of its angle.

PROPOSITION IX. PROBLEM IV.

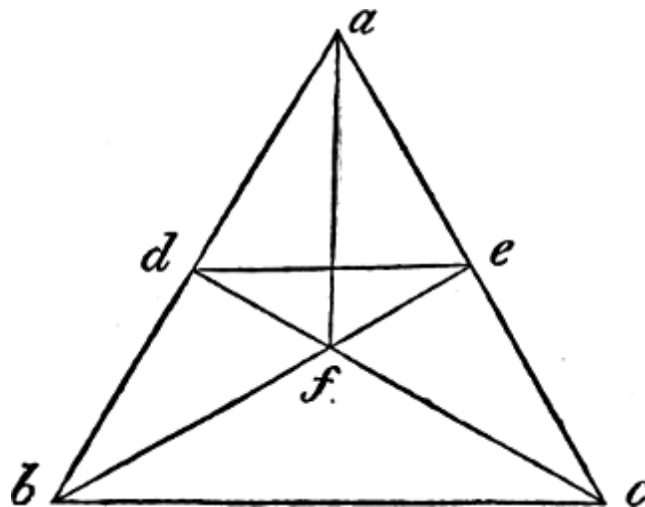
To bisect a given rectilinear angle.

Our author mingles theorems with problems, and connects problems with theorems, and through both completes the whole of his elementary institution, comparing as well subjects as the *symptoms* subsisting about subjects themselves. Since, therefore, he had shewn in the preceding propositions, both in one triangle, from the equality of the sides, the consequent equality of the angles, and the contrary: and in a similar manner in two triangles, with this exception, that the mode of conversion in one and two triangles is different, he now passes to problems, and orders us to bisect a rectilinear angle. And it is manifest, that the angle here is given according to form: for it is called right-lined, and not of any kind whatever. Indeed, we cannot bisect every angle by the elementary institution; since it is doubtful whether every triangle can be bisected. For, perhaps, you may doubt whether it is possible to bisect a cornicular angle. But the ratio of the section is also distinguished in this problem, and this again not in vain. For to divide an angle in any given ratio, transcends the present construction: as, for example, into three, four, or five equal parts. Indeed, to trisect a right angle is possible, by employing a few of the propositions which are afterwards delivered^[16]: but this cannot be effected in an acute angle, without passing on to other lines of a mixt species.^[17] And this is manifested by the geometricians who propose to trisect a given rectilinear angle. For *Nichomedes*, indeed, from conchoidal lines, the origin, order, and symptoms of which, he delivers, as he was the inventor of their properties, trisects every right-lined angle. But others effect this from the quadrantal lines of *Hippias* and *Nichomedes*, by employing mixt quadrantal lines. Others, again, being incited from the Helices of *Archimedes*, divide a given rectilinear angle, in a given ratio. But the consideration of these, because difficult to learners, we shall for the present omit; as it will, perhaps, be more convenient to examine this in the third book^[18], where the institutor of the *Elements* bisects a given circumference. For there the same mode of enquiry presents itself with respect not only to bisection, but also trisection; and the ancients endeavoured, by employing the same lines, to divide every circumference into three equal parts. With great propriety, therefore, he who only mentions a right line and a circumference, alone bisects a right angle and a circumference. But conceiving that the species composed from these, through mixture, are difficult to explain and enumerate, without a curious examination, he omits all such enquiries as involve mixt lines in their consideration, and proposes to investigate in first and simple forms alone, such things as can either be produced or considered from these. And such, indeed, is the proposition of the present problem, *to bisect a given right lined angle*. For in the construction of this he uses one petition, and the first and third problem: but in the demonstration he employs the eighth theorem alone. Since problems entirely require demonstration (as we have already observed^[19]) and through this they obtain a power of producing science. But perhaps, some may oppose the geometrician, by asserting that an equilateral triangle may be constituted by him, not having its vertex within the two right lines, but either upon, or external to each; and that this may be manifested by the elements. For let there be an angle $b a c$, which it is required to bisect.

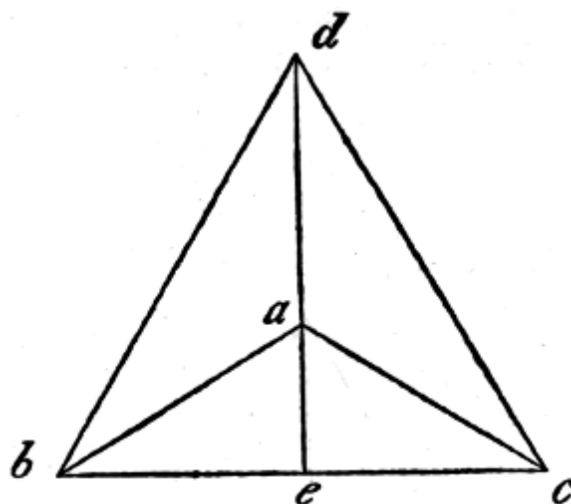


Then let ba be taken equal to ac , and let bc be connected, and upon it, let an equilateral triangle bcd be constructed. This point d , therefore, is either within the right lines ab , ac , or upon ab , or ac , or external to both. Now the institutor of the Elements assumes them within; and hence, those who oppose the demonstration, will say the point is either placed on one of the right lines, or external to both. Let the point d then be placed on the line ab , so that the triangle bcd may be equilateral: db , therefore, is equal to dc , and the angles at the base $c b d$, $b c d$, are equal. Hence, the whole, $b c e$, is greater than the angle $c b d$. Again, because ab , ac , are equal, the triangle abc , is isosceles, and the angles under the base bc , will be equal. The angle, therefore, $b c e$, is equal to the angle $c b d$. But it was also greater, which is impossible. Hence, the vertex of the equilateral triangle cannot be in the right line abd . In like manner we may shew that it cannot be in the right line ace . Let it therefore, if possible be placed externally. Because, then bd is equal to cd , the angles at the base are equal, viz. $b c d$, and $c b d$. Hence, the angle $b c d$, is greater than the angle $c b f$. Much more, therefore, is the angle $b c e$, greater than $c b f$: but it is also equal, because these angles are under the base bc , of an isosceles triangle abc , and this is impossible. Hence, the point cannot fall in these parts external to the two right lines; and it may be similarly shewn that this is impossible in other parts. Here too you may again observe, that we destroy objections by using the second part of the fifth proposition, *that the angles under the base of an isosceles triangle are equal*. And this is what we have previously observed, that many things opposing science, are shewn to be debile, and easy of confutation, by the assistance of this theorem; and that such is the utility it affords to geometry.

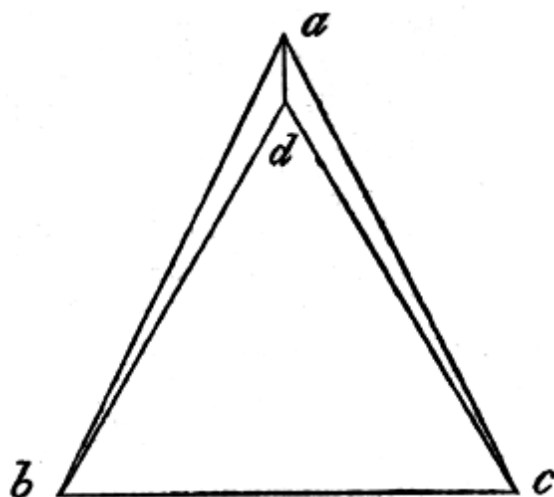
But if any one should say that there is no place under the base, and yet that it is requisite to constitute the equilateral triangle at the same parts, in which the lines ba , ac , are situated; it will be necessary that the lines which are constituted should either coincide with ba , ac , if they also are equal to the base bc : or that they should fall external to them, if they are less than the base bc : or within, if ba , ac , are greater than bc .



Let them, in the first place, coincide, and let bac be an equilateral triangle, and let there be taken in the side ab , the point d , and make ae in the side ac , equal to ad , and connect the lines de , be , cd , af . Because, therefore, ab is equal to ac , and ad to ae , the two ba , ae , are equal to the two ca , ad , and they comprehend the same angle. Hence, they are all equal to all, and the angle dbe , is equal to the angle ecd . But db is also equal to ec , and be to cd . All, therefore, are equal to all. Hence, the angle deb , is equal to the angle edc : for they subtend equal sides. And df is equal to ef , (by the sixth.) Because, therefore, ae is equal to ad , and af is common, and the base df , is equal to the base ef , the angle dae is bisected, which was required to be done.



But if the sides of the equilateral triangle fall external to the right lines ba , ac , let them be bd , dc , and having connected da , let it be produced to the point e . Because, therefore bd , dc , are equal, but da is common, and the bases ba , ac , are equal, the angle, also, bda , (by the eighth) is equal to the angle cda . Again, bd , dc , are equal, and de is common, and they contain equal angles as we have shewn, the base also be , is equal (by the fourth) to the base ec . Because, therefore, ab is equal to ac , and ae is common, the angle, also bae , is equal to the angle cae , which was to be shewn.



But if the sides of the equilateral triangle fall within the right lines ab , ac , as bd , dc , let again ad be connected. Because, therefore, ba , is equal to ac , and ad is common, but the base bd , is equal to the base cd , hence, the angle bad (by the eighth) is equal to cad . The angle, therefore, at the point a , is bisected, in whatever manner the equilateral triangle may be constituted. And having thus summarily spoken concerning these, we shall now proceed to the following theorems, only adding, that the given angle may be given in a four-fold respect. *In position*, as when we say *to this right line, and to this point to place an angle*: for after this manner it is given. But *in form*, as when we call the angle right, or acute, obtuse, right-lined, or mixed. And *in proportion*, as when we call it double, or triple, greater, or less. And lastly, *in magnitude*, as when we call it the third part of a right angle. But the present angle is only given in form.

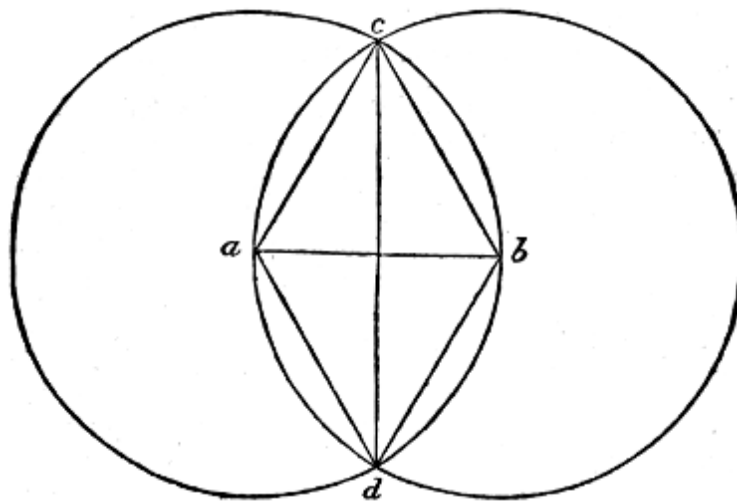
PROPOSITION X. PROBLEM V.

To bisect a given finite right line.

This, also, is a problem which supposes a finite right line, since we cannot terminate a line on both sides infinite. But the section of a line infinite on one side only, wherever the point is assumed, is made in unequal parts. For that part of the section which takes place on the infinite side, is necessarily greater than the remainder, because finite. Hence, the line required to be bisected, must be necessarily both ways finite. But perhaps, some

excited by this problem, may think, that the doctrine of a line, not being composed from impartibles, is only previously received by geometricians as an hypothesis. For if it consists from impartibles, it either becomes finite, and receives its completion from *odd*, or from *even* parts. But if from such as are *odd*, it will appear that an impartible also may be cut, while a right line is bisected. And if from such as are even, the section will be unequal, because, one part, as composed from more impartibles, will be greater than the remainder. It is therefore impossible to bisect a given right line, if magnitude consists from impartibles. But if it be not composed from impartibles, it may be divided in infinitum. It appears, therefore, (say they) to be received by common consent, and to be a geometrical principle, that magnitude is among the number of things infinitely divisible. Against these we reply in the words of Geminus, that geometricians previously receive according to a common conception, that continued quantity is divisible. For we call that continuous, which is composed from conjoined parts, and this it is in every respect possible to divide. But that continued quantity may be infinitely divided, they do not previously assume, but demonstrate from proper principles. For when they shew that incommensurability is found in magnitudes, and that all are not commensurable with each other, what else can we say they evince by this means, except this, that every magnitude may be divided into parts always divisible, and that we can never arrive at an impartible, by the most unwearied analysis, since this minimum would be the common measure of all magnitudes? This then is demonstrable, but that which says, *every thing continuous is divisible*, is an axiom. Hence, since a finite line also is continuous, it is divisible. And from this conception the institutor of the Elements cuts a finite right line into equal parts, but not as pre-assuming, that it is divisible in infinitum. For to be merely divisible, and to be infinitely divisible is not the same.

But the discourse of Xenocrates inferring indivisible lines, is confuted by this problem. For if it be a ~~line~~ ^{line}, it is either right, and may be bisected; or circular, and it is greater than a certain right line; (since every circular has a certain right line less than itself); or it is mixt, and on this account is the more divisible, since composed from simple divisible lines. But this must be deferred to some posterior speculation. However, the geometrician bisects a finite right line, employing in the construction the first and ninth propositions; but using in the demonstration the fourth alone; for by the angles he shews the equality of the bases. But Apollonius Pergæus bisects a given finite right line after the following manner.

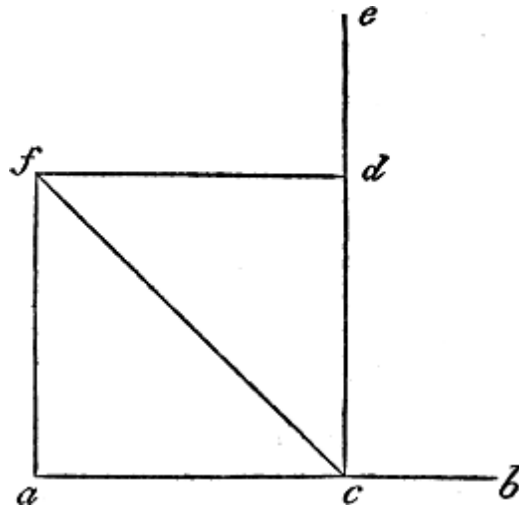


Let there be (says he) a finite right line ab , which we are required to bisect, and with the centre a , but interval ab , let a circle be described. And again, with the centre b , but interval ba , let another circle be described, and let the right line cd , connect the common sections of the circles; this shall bisect the right line ab . For let the equal lines da , db , ca , cb , be connected; these being equal, because each is equal to ab . But cd is common, and da is equal to db on the same account. Hence the angle acd , is equal to the angle bcd ; and so (by the fourth) ab is bisected. Such then, according to Apollonius, is the demonstration of this problem, assumed, also, from an equilateral triangle; but instead of exhibiting the bisection of the line, from the bisection of the angle at the point c , it shews this from the equality of the bases. The demonstration, therefore, of the institutor of the Elements, is much better, since it is both more simple, and emanates from principles.

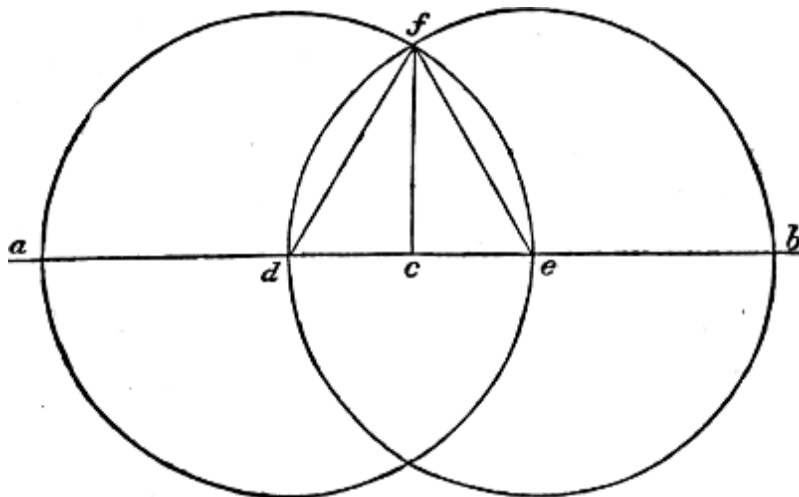
PROPOSITION XI. PROBLEM VI.

To raise a right line at right angles, to a given right line, from a given point in that line.

Whether we receive a right line on both sides finite, or on both sides infinite, or on one side infinite but on the other finite, and a point in it, the construction of the present problem will conveniently succeed to the geometrician. For though the given point should be on the extremity of the right line, by producing it we can accomplish our purpose. But it is manifest that the point in the present problem is given in *position*, since it can only be placed in position in a right line. But the right line is given according to *form*; since its magnitude is not distinguished either by proportion or position. Hence, the institutor of the Elements, employing the first and third problem, together with the eighth proposition, and the tenth definition, exhibits the thing proposed. But if any placing the point on the extremity of the right line, should ask us without producing the line, to erect upon this a right line at right angles, we can likewise shew that this is possible to be effected.



For let there be a right line ab , and a given point in it a , and let there be assumed in the line ab , any point c , and from this (as the present element teaches us) let a right line ce be erected at right angles to ab . Then from c , let cd be taken equal to ac , and let the angle at the point c be bisected by the line cf , and at the point d let a right line be erected at right angles, coinciding with cf in f ; and lastly from the point f , to the point a , let fa be connected. I say that the angle at the point a is right. For since dc is equal to ca , but cf is common, and contains equal angles, (for the angle at the point c was bisected) hence, df is equal to fa , and all in like manner (by the fourth) are equal to all. The angle, therefore, at the point a , is equal to the angle at d . But the angle at the point d is right; and so consequently is the angle at a . And thus the thing required is effected. But the institutor of the Elements was not indigent of any such artifice: for he commands us to raise a line at right angles, but not at one right. It is requisite, therefore, not to receive the point in the extremity of the right line, because the perpendicular line forms angles with its subject right line, but not one angle alone.



But Apollonius raises a perpendicular as follows. Let the given right line, says he, be ab , and a given point in it c , but let there be assumed in ac any point d , and from c to b , take away ce , equal to cd . Then with the centre d , but interval de , let a circle be described; and again with the centre e , but interval ed , let another circle be described, and let a right line be drawn from f to c . I say that fc is a perpendicular. For if fd , fe , are connected, they shall be equal. But dc , ce , are equal, and fc is common. Hence, also, the angles at the point c (by the eighth) are equal. They are therefore right. And here, is it not again obvious, that this demonstration is more various than that of Euclid, and requires the description of circles, that by this means an equilateral triangle may be described upon de , and the problem exhibited? For all the rest are common to the demonstrations. But the demonstration by a semicircle is not worthy to be remembered, since it supposes many things which are afterwards exhibited, and entirely falls from the order of an elementary institution.

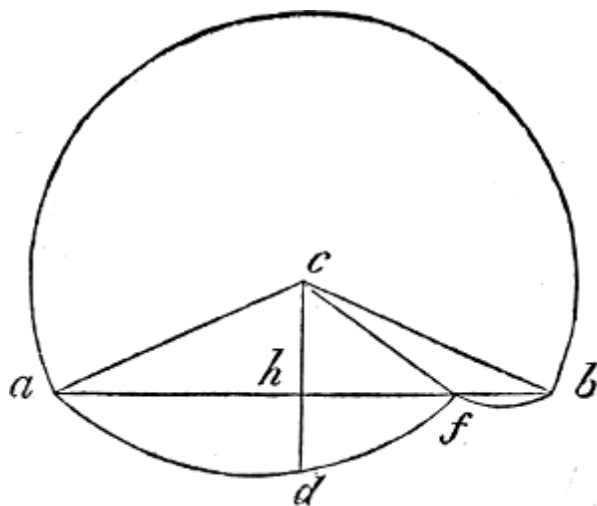
PROPOSITION XII. PROBLEM VII.

Upon a given infinite^[20] right line, and from a given point which is not in that line, to let fall a perpendicular.

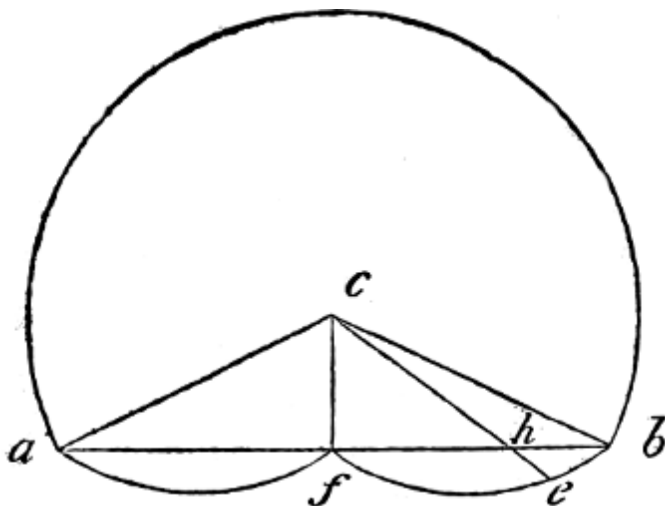
Oenopides first investigated this problem, believing it useful for astrological purposes. But he^[24] calls a perpendicular, after the manner of the ancients, a gnomon, because a gnomon, also, is at right angles to the horizon, but the same line is at right angles with a perpendicular, from which it differs only in habitude, since, as he observes a gnomon has the same subject with a perpendicular. But again, a perpendicular is two-fold, that is, it is either plane or solid. Hence, when the point from which the perpendicular right line is drawn, is in the same plane, the perpendicular is called plane; but when the point is on high, and external to the subject plane, it is called solid. And the plane perpendicular, indeed, is drawn to a right line: but the solid to a plane. Hence, it is necessary, that this last should not only form right angles, with one right line, but with all right lines in the same plane. For the perpendicular is let fall on a plane. In the present problem, therefore, the institutor of the Elements proposes to let fall a plane perpendicular. For the deduction is proposed to a right line, and the discourse proceeds, so far as all are supposed to be in the same plane. Hence, in the line at right angles we do not require infinity, because the point is supposed to be in that right line. But in the present problem, respecting a perpendicular, he supposes the given right line infinite, because the point from which the perpendicular is to be drawn is placed external to the right line. For if it was not infinite, the point might be received externally, and yet in a direct position, so that the protracted right line would fall upon it, and the problem not succeed. Hence, he places the right line infinite, so that the point may be received at either of its parts; and that no place may be left, in which it can be in the same direction with the given right line, unless it is in the line, and has not an external position. And on this account the right line to which the perpendicular is to be drawn is considered as infinite.

But in what manner infinite can subsist, is a matter well worthy our contemplation. For it is manifest^[25] that a right line existing infinite, a plane also will be infinite, and this in energy, if the thing proposed by Euclid be true. That among sensible particulars, therefore, there can be no magnitude infinite, according to any distance, both the dæmoniacal Aristotle, and those who received their philosophy from him, have abundantly shewn. For neither that which is moved circularly, nor any other simple body can be infinite; since the place of each is limited. But neither in separate and impartible reasons is an infinite of this kind possible. For if they neither contain dimension, nor magnitude, much less can they contain infinite magnitude. It remains, therefore, that infinite can alone subsist in the phantasy, which at the same time the phantasy does not comprehend. For as soon as it understands, it induces form and bound to that which is understood, stops the transit of the phantasm by its intellection, pursues its progress, and infolds it in its shadowy embrace. The phantasy, therefore, is not infinite by intellection, but rather by advancing infinitely about that which is understood; and calling whatever it leaves innumerable, and incomprehensible by intelligence, infinite. For as the sight by not seeing understands darkness; so the phantasy by not understanding perceives infinite. Hence it pursues the progress of the infinite, because it is endued with an impartible power, capable of perpetually advancing: but it understands as if stopping in its progression, because infinite surpasses its comprehension. For it calls that infinite, which it leaves as unable to pass over in its pursuit. On this account when we place a given infinite line in the phantasy, in the same manner as we establish all other geometrical species, viz. triangles, circles, angles, lines, and all of this kind, we must not wonder how a line is infinite in energy, and how advancing infinitely, it applies itself to finite intellections. But cogitation, in which reasons and demonstrations reside, does not use infinite for the purpose of science, since infinite is by no means perceptible by science, but receiving it from hypothesis, it employs finite alone in its

demonstrations, and assumes infinite not for the sake of infinite, but of that which is bounded and finite. For if we should grant to cogitation, that the given point, neither lies in a right line with the given finite right line, nor yet is so distant from it, that no part of the right line is subjected to the point, we shall no longer require an infinite line. That cogitation, therefore, when employing a right line, may use it without controversy and reproof, she supposes it to be infinite; and employs the infinity of the phantasy, as the foundation of infinite generation. And thus much may suffice for the present concerning the nature of infinite.

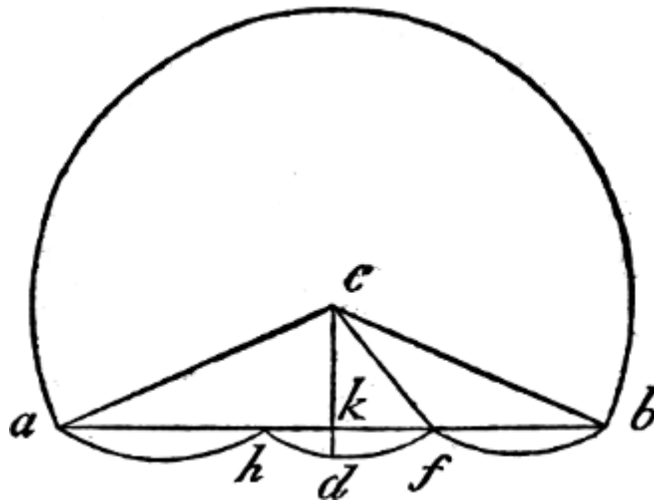


But it is now requisite that we should consider the objections which are urged against the construction of this problem. Let there be received, say they, an infinite right line ab , and let the given point be c , from which it is required to let fall a perpendicular, and let d be a point on the other side, according to the geometrician. But the circle which cuts the right line ab , in the points a and b , will cut it also in f , and will have a situation according to the figure. In answer to this, we must say, that it affirms an impossible case. For let the right line ab be bisected in h , and let ch be connected, and produced to the circumference, to the point d , and let ca , cb , cf , be connected. Because, therefore, these lines are from the centre, and ah is equal to hb , but ch is common, all are equal to all. Hence ch forms right angles at the point h . Again, because ca , cb , are equal, they form equal angles at the points a and b . But ca also, is equal to cf , on which account the angle caf , is equal to the angle cfa . In like manner the angle cbf is equal to the angle cfb . Because, therefore, the angles at the points a and b , are equal, the angle, also, cfa , is equal to the angle cfb , and they are successive, and consequently right. But each of the angles at the point h is right. Hence, ch is equal to cf . But cf is also equal to cd , since they are from the centre. Therefore ch is equal to cd , which is impossible. Hence, the circle does not cut the right line in any other points than a and b .



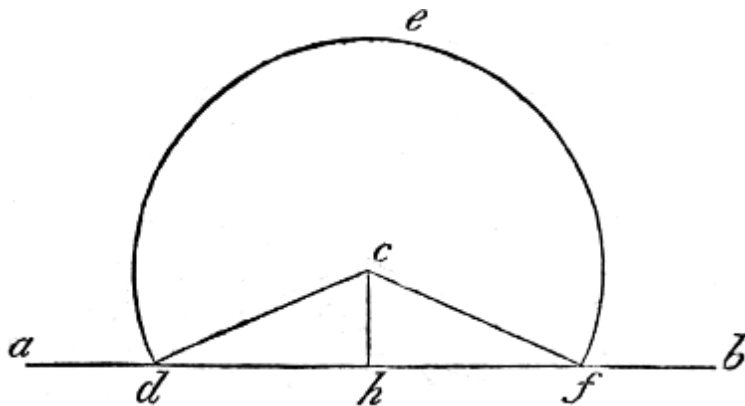
But if any one should say, that he who describes a circle will bisect ab in f , we can again shew that this is impossible. For let all be described as before, and let the right line fb , be bisected in the point h . Because, therefore, af , fb , are equal, but cf common, and the base ca , is equal to the base cb , all are equal to all. Hence,

the angles at the point f are right. Again, because fh is equal to hb , and ch being connected, is common, and the base cf is equal to the base cb , for they are from the centre, the angles at the point h , are right; for they are equal and successive. Because, therefore, each of the angles cfh , chf , is right, cf is equal to ch . But cf is equal to ce , for they are from the centre, and hence ch is not unequal to ce , which is impossible.

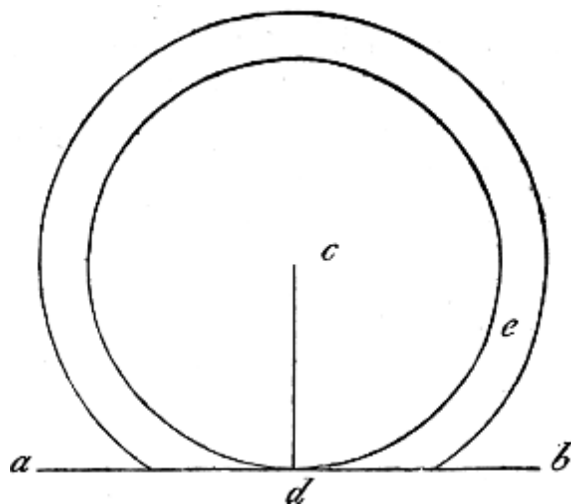


It now remains that we run over the third objection. For the circle which is described (say they) will cut the right line in the points a , b , and in the points f , h . We therefore bisecting the right line ab in the point k , and connecting the lines ca , cf , ck , cb , can shew that this is impossible. For since ka , kb , are equal, and ck is common, and the bases ca , cb , are equal, hence the angles at the points a and b are equal, and those at the point k right. But each of the lines is equal to cf ; and hence, the angles at the point f , are right; for they are equal, because successive. Therefore, cf is equal to ck : for they subtend right angles. But cf is equal to cd , since they are from the centre; and hence, cd is equal to ck , which is impossible. Hence then, it is impossible that the circle which is described should cut the line ab in one, two, or in more points than ab . And such are the objections against the present problem.

But there are also cases of the construction of this problem, which are to be distinguished from the objections. For case is not the same with objection; since the former shews the same thing differently, but the latter leads the objection to an inconvenience. But other expositors, not distinguishing these from one another, bring all into the same, so that it is uncertain, whether they enunciate to us in their writings, cases, or objections. We therefore distinguishing these, having enumerated the objections, shall now describe the cases of the problem.



Let there be then an infinite right line ab , and a given point c . Now it may be said that there is no farther place in the other part of the perpendicular right line, but in that only where the point c lies. Taking, therefore, in the right line ab a point d , with the centre c , and interval cd , let us describe the circumference of a circle $d e f$, and bisecting df in h , let us connect the lines cd , ch , cf . Because, therefore, dh is equal to hf , but ch is common, and cd is equal to cf , (for they are from the centre,) hence, the successive angles at the point h , are equal. They are, therefore, right. And hence, ch is a perpendicular to df .



But if any one should also say that the described circle does not cut the right line ab , but touch it as the circle de , by taking the point e externally, and using the centre c , and interval ce , as in the preceding, we shall obtain the object of our enquiry. And thus much we have said concerning the cases of the problem, for the sake of exercising the attention of the reader.

But if we are desirous of adding contemplation likewise to these two problems, a right line erected at right angles, seems to imitate a life tending on high from inferior concerns, ascending purely, and without contamination, and abiding inflexibly with regard to natures subordinate to its own. But a perpendicular is the image of a life perpendicularly descending, and the least of all replete with generative infinity. For a right angle is the symbol of an energy inflexible, and restrained in the comprehension of equality, bound, and finite. From whence, indeed, Timæus also calls the *other circle* in the divine soul, possessing the reasons of sensible natures, *right*; for in our souls it is bent with flexions of every kind, and suffers various contortions and perturbations from the unceasing whirls of generation: but among *wholes* it resides immaculate, uncontaminated, firm, and indeclinable, prior to sensible forms. But if likewise an infinite right line is the symbol of the whole of generation, which is moved infinitely and indeterminately, and besides this, of matter itself, which is deprived of bound and form: and if a point placed externally bears an image of an essence impartible, and separate from material natures, doubtless the deduced perpendicular will imitate that life which proceeds into generation with an undefiled progress from unity, and an impartible essence. But if a perpendicular cannot be shewn without circles, this also will be the symbol of an inflexibility inherent in life, through the medium of intellect. For life, indeed, since it subsists by itself as motion, is indeterminate: but it becomes terminated, and is filled with a pure and immaculate power, by participating and adhering to the circulations of intellect.

PROPOSITION XIII. THEOREM VI.

When a right line standing upon a right line forms angles, it either forms two right, or angles equal to two right.

Euclid again passes on to theorems, consequent to things exhibited by problems. For after a perpendicular had been drawn to a right line, and a right line erected at right angles, it remained, to enquire if it should not be a perpendicular, what angles it would form, and how it would be affected to the line upon which it stands. This then he proves universally, *that every right line standing upon a certain line, and forming angles, either forms two right, if its state be indeclinable, firm, and never verging: or angles equal to two right, if it declines in one part, but is more distant from its subject line, in the other part*. For as much as it takes away from a right angle by its declination in one part, so much it adds by its distance in the other. But it is requisite to take notice, that in this proposition also, the Geometrician employs diligent care. For he does not simply say that every right line, standing upon a right line, forms either two right angles, or angles equal to two right, but he adds, *if it forms angles*. For what if standing on the extremity of a right line, it should form one angle, will it happen that this may be equal to two right? This certainly is impossible. Since every rectilineal angle is less than two right, as also every solid angle is less than four right. Hence, though you should receive that which appears to be the greatest of all obtuse angles, this also will increase, as that which does not yet receive the measure of two right angles. It is

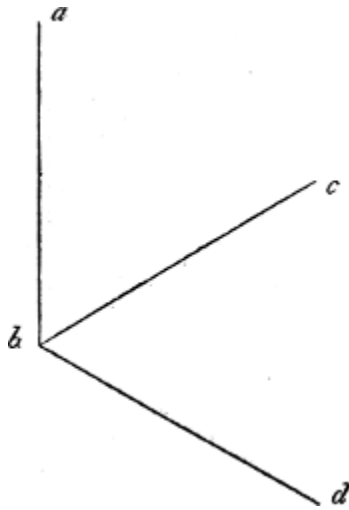
requisite, therefore, that the right line should stand in such a manner, that it may form angles. And these observations regard the productive diligence of science.

But what does he mean by adding the particle, *either two right, or equal to two right*? For when he has constituted two right, he forms angles equal to two right; since right angles are equal to each other. Shall we say that one of the equal angles is also common, but that the other of the equals is only proper? But we are accustomed when both *proper* and *common* is verified, to express every particular from that which is proper, but when we cannot effect this, we are content with that which is common for the explication of the subject concerns. This then, the equality of the successive angles, is common to right angles, but is not predicated of these alone: but this, that they are right, is peculiar to their equality. Hence, the assertion, *equal to two right*, alone signifies the inequality of the angles. For in these it is alone verified, but by no means in such as are equal. And this also the institutor of the Elements divides in opposition to two right. For since it is predicated by itself, it has a power of signifying that the angles on each side are unequal. But through these observations we may also perceive, that equality is the measure and bound of inequality. For though the increase and decrease of an obtuse and acute angle is indeterminate and infinite, yet it is said to receive limitation, and bound from a right angle; and each of them, indeed, separately, recedes from a similitude to the right; but both, according to one harmonizing union, are reduced to its bound. But as they can by no means perfectly equal the simplicity of a right angle, they receive an equality to it when doubled, the duad being the exemplar of their infinity, as of itself endued with an infinite nature. And this seems to procure a manifest image of the progression of primary causes; and of their abiding according to one boundary, in a manner perpetually the same, about the infinity of generation. For how could otherwise generation, which participates of the more and the less, and is carried in indefinite whirls, agree with intelligibles, and be equalled with them in a certain respect, unless by participating their natures, whilst they advance with prolific powers, and only multiply themselves in their progressions? For things which abide in their own simplicity and impartibility, are entirely separated from generable natures. And thus much is assumed from the present theorem, and applied to the knowledge of universals.

PROPOSITION XIV. THEOREM VI.

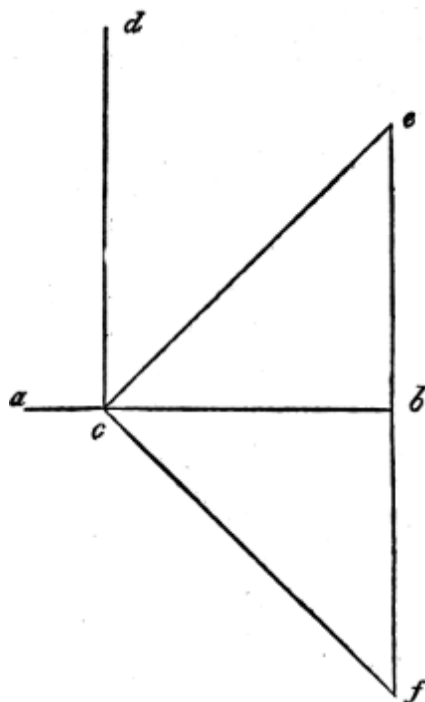
If to any right line, and at a point in it, two right lines being placed in a consequent order, and not towards the same parts, make the successive angles equal to two right, those right lines shall be in a direct position to each other.

The present theorem is the converse of the foregoing: for such as are converse are always consequent to preceding theorems. Since, therefore, the former had constituted a right line upon a right line, and had shewn that it made the successive angles either two right, or equal to two right; in the present theorem he receives the equality of the angles to two right, which are formed at some right line, but he shews that it is one right line which produces their equality. Hence, that which was a *datum* in the former, is in the present theorem an object of enquiry; and is shewn by a deduction to an impossibility. For after this manner the converse of theorems ought to be exhibited; but in problems they should receive principal demonstrations. But in this theorem we may also perceive the greatest and most admirable diligence of this proposition producing science. For in the first place, after he had said, *if to any right line*, he adds, *and at a point in it*; for what if the two extremes of the right line existing, one of the right lines should be drawn from the one extreme, but the other from the remaining one, and should form angles at the right line, equal to two right, would they on this account have a direct position? And how can this take place in lines drawn from different points of the right line? It is on this account also, that he adds, *and at a point in it*, since he is willing that both should be in the same point. But in the second place, because it is possible that the right lines which are drawn, may be at the same point, and not consequent (since we may receive infinite right lines placed at the same point) he adds the particle, *in a consequent order*. And in the third place, because the word *consequent* may be considered as well at the same parts as on both sides: but because it is impossible that lines which are consequent at the same parts should be mutually in a direct position, this indeed he explains, but affords us an opportunity of considering that consequent right lines are to be received in position on both sides; since these also can be shewn to be in a right line.



Let there be placed at the right line $a b$, and at a point in it b , towards the same parts, two right lines $b c$, $b d$, these, therefore, shall be consequent to each other. For no other right line is situated between them. But those things are *successive*, between which there is nothing similar. Thus we call the columns *consequent*, between which there is no other column: for though the air intervenes, yet nothing of the same kind is situated in the middle. Because, therefore, they lie towards the same parts, they are by no means in a direct position, although they form two angles equal to two right; I mean the angles at the point b . For nothing hinders but that the angle $a b d$, may contain in itself, one right, and a third part of a right angle: and that the angle $a b c$, may be two thirds of a right angle. And thus much concerning the proposition.

But one petition is employed in the construction, viz. the second, which begs to produce a right line ⁹⁶ straight forwards, as in the demonstration he uses the preceding theorem, and two axioms; i.e. the one which says, *things equal to the same, are equal to one another*; and also the one which affirms, *that if from equal things equals are taken away, the remainders shall be equal*. But at the collection of the impossibility, he employs the axiom, which says, *the whole is greater than its part*. For it is equal one common angle being taken away, which is impossible. But that it is possible to the same right line, and at a point in it, two right lines in a consequent position, and yet, towards the same parts, may form angles belonging to that one right line, equal to two right, we may shew with Porphyry, as follows. Let there be a certain right line $a b$, and any point in it c , and let $c d$ be raised at right angles to $a b$, and let the angle $d c b$ be bisected, by the line $c e$. Then from the point e , to the line $a b$, let there be drawn the perpendicular $e b$, and let $e b$ be produced, and place $f b$ equal to $e b$, and connect $c f$. Because, therefore, $e b$ is equal to $b f$, but $b c$ is *common*, and they contain equal angles (for they are right), hence, the base $e c$, is equal to the base $c f$. All, therefore, are equal to all. Hence, the angle $e c b$, is equal to the angle $f c b$. But the angle $e c b$ is the half of a right angle: because the right angle $d c b$ was bisected by the line $e c$.

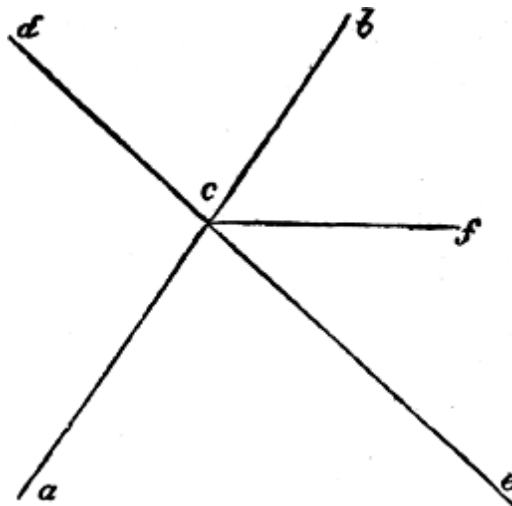


Hence, also the angle fcb , is the half of one right. The angle, therefore, dcb , is equal to one right, and the half of a right angle. But the angle dce , also, is the half of a right angle. Hence to the right line cd , and to a point in it c , two right lines are consequently placed towards the same parts, viz. ce and cf , forming angles equal to two right, ce causing the half of a right angle, and cf one and a half. Lest, therefore, we should enquire after things impossible to be effected, viz. how the right lines ce , cf , forming angles at the right line dc , equal to two right, can be in a direct position to one another, the Geometrician adds the particle *not towards the same parts*. It is requisite, therefore, that the right lines which form angles equal to two right, should be placed on both sides of the right line, being raised, indeed, from one point, but drawn to different parts of the right line.

PROPOSITION XV. THEOREM VIII.

If two right lines cut one another, they will form the angles at the vertex equal.

We must call *successive* angles different from such as are *vertical*. For these last originate from the section of two right lines: but the former from the mere dissection of the one by the other. Thus, if a right line remaining itself without section, but cutting another in its extremity, forms two angles, we denominate these *successive* angles. But if the two right lines mutually cut each other, they form *vertical* angles. And they are so called, because they have their vertices conjoined in the same point. But their vertices are the points, at which the planes, while they are contracted, form angles. This, therefore, is what the present theorem evinces, that when two right lines mutually cut each other, the vertical angles are equal. And it was first invented (according to Eudemus) by Thales: but was thought worthy of a demonstration producing science by the institutor of the Elements. But it is not exhibited from all the particulars requisite to a perfect proposition. For construction is wanting in the present theorem: but demonstration, which must be necessarily inherent, depends on the thirteenth theorem. But he uses two axioms, one of which is, *that things equal to the same, are equal among themselves*: and the other, *if from equal things equals are taken away, the remainders will be equal*. The theorem, indeed, of Euclid, is manifest, but another such is converted to the present theorem. *If to any right line, and at a point in it, two right lines, not assumed towards the same parts, make the vertical angles equal, those right lines shall be in a direct position to each other*.



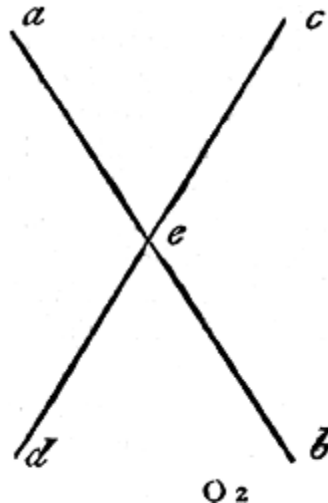
For let there be a certain right line ab , and any point in it c , and at the point c , let two right lines cd , ce , not towards the same parts be assumed, forming equal angles acd , bce . I say that cd , ce , are in a right line. For since the right line cd , insists upon the right line ab , it forms angles equal to two right, i.e. dca , dcb . But the angle dca , is equal to the angle bce . Therefore, the angles dcb , bce , are equal to two right. Because, therefore, to a certain right line bc , and at a point in it c , two consequent right lines cd , ce , not placed towards the same parts, form the successive angles equal to two right, those right lines cd , ce , are in a direct position to each other. The converse, therefore, to the present theorem, is exhibited. But the Geometrician seems to have neglected this, because it is easy to evince its truth, by the same method of deduction to an impossibility as we employed in exhibiting the fourteenth proposition. For the same things being supposed, I say that the right line cd , is in a direct position to ce . For if it be not, let cf be taken in a right line with cd . Because, therefore, two right lines ab , df , intersect each other, they will form the angles at the vertex equal. Hence, the angles acd , bcf , are equal. But acd , bce , were also equal. The angle, therefore, bce , is equal to the angle bcf , the greater to the less, which is impossible. Hence, no other right line, besides cd , is in a direct position to ce . The right lines, therefore, cd , ce , are in a direct position to each other, the angles at the vertex being supposed equal. Since then, there is the same demonstration which was pre-assumed in the fourteenth theorem, would it not have been superfluous to have produced this conversion? But for the sake of exercise, we have proved it as well by a deduction to an impossible, as by an ostensive method. However, this fifteenth theorem seems to rest upon the similitude of the parts of right lines, and their situation in their extremities. Because lines with these conditions, and mutually cutting each other, must necessarily possess similar inclinations on both sides to each other. Since circumferences, and universally non-right lines cutting one another, do not necessarily form the vertical angles equal, but sometimes equal, and sometimes unequal. For if two equal circles cut each other through the centres, or even not through the centres, they will form the lunular angles at the vertex equal: but not likewise the remaining angles, viz. those on both sides concave, and on both sides convex, but the one will be greater than the other. But in right lines, the situation in the extremities, causes the distance of one segment, to be equal to the distance of another.

C O R O L L A R Y .

From hence it is manifest that if two right lines cut each other, they will make four angles equal to four right.

Corollary is one of the geometrical appellations, but it has a two-fold signification. For they denote immediate corollaries, whatever theorems are proved together with the demonstrations of others, becoming as it were the unexpected gain and emolument of the investigator: and likewise, whatever is the object of enquiry, but is indigent of invention, and is neither investigated for the sake of generation alone, nor of simple contemplation. For that the angles at the bases of isosceles triangles are equal, it is requisite to contemplate, and the knowledge of things in existence is of this kind. But to bisect an angle, or constitute a triangle, to cut off, or place an equal right line, all these demand that something may be performed. And again, to find the centre of a given circle, or two commensurable magnitudes being given to find their greatest common measure, with every thing of this kind, are, after a manner, situated between problems and theorems. For neither is the origin of *objects of enquiry*

inherent in these, nor contemplation alone, but invention. Since it is requisite to place the object of enquiry conspicuously and before our eyes. Such then are whatever corollaries Euclid wrote, for he constructed a book of corollaries. But we must now omit to speak of corollaries of this kind. However, such as occur in the elementary institution, appear at the same time with the demonstrations of other things, but they themselves are not principally investigated, as is evident in that which is proposed at present. For the design of the proposition is to enquire whether if two right lines mutually cutting each other, the angles at the vertex are equal. But whilst this is evinced, it is at the same time demonstrated, that the four angles which are formed, are equal to four right.



For when we say let there be two right lines, *a b*, *c d*, cutting each other in the point *e*: because *a e* stands upon *c d*, it makes the successive angles equal to two right. And again, because *b e* stands upon *c d*, it also makes the successive angles equal to two right; then together with the object of enquiry we demonstrate, that the angles about the point *e*, are equal to four right. A corollary, therefore, is a theorem, unexpectedly emerging from the demonstration of another problem, or theorem. For we seem to fall upon corollaries, as it were, by a certain chance; and they offer themselves to our inspection, without being proposed, or investigated by us. Hence, we assimilate these also to gains. And perhaps those skilled in mathematical concerns, have imposed on them this appellation, shewing the vulgar, who rejoice in apparent gain, that these are the true gifts of divinity, and true gains, and not the objects of their sordid estimation. For this indeed produces that faculty resident in our nature, and adds the prolific power of science, to principal enquiries, manifesting the copious riches of theorems. And such is the property of corollaries.

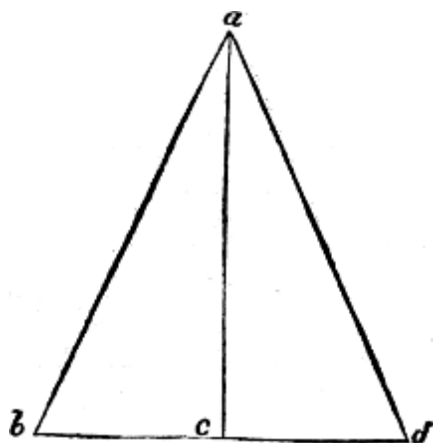
But they are to be divided in the first place, according to sciences. For of corollaries, some are geometrical, but others arithmetical. Thus the present corollary is geometrical: but that which is added at the end of the second theorem of the seventh book of the arithmetical elements, is arithmetical. But afterwards they must be divided according to the principal objects of enquiry. For some things are consequent to problems, but others to theorems. Thus, the present is consequent to a theorem: but that which is placed in the second of the seventh book, is consequent to a problem. But in the third place, they must be divided according to their ostensions. For some are exhibited, together with ostensive methods, but others together with deductions to an impossible. Thus the present is shewn by a direct ostension: but that which is at the same time exhibited in the first of the third book, appears, together with a deduction to an impossible. But corollaries may also be divided in many other modes, but these may suffice our present purpose. The present corollary, however, teaching us that the place about one point is distributed into angles equal to four right, is subservient to that admirable theorem, which shews that the following three multangles about one point, can alone fill place, viz. the equilateral triangle, the quadrangle, and an equilateral, and equiangular sexangle. But the equilateral triangle must be six times assumed; since six two-thirds, form four right angles. But the sexangle must be three times formed; for every sexangular angle is equal to one right, and a third part of a right. And a quadrangle must be four times assumed: for every quadrangular angle is right. Hence, six equilateral triangles conjoined according to their angles, fill four right angles, as also three sexangles, and four quadrangles. But all other multangles, however composed, according to angles, are either deficient from four right, or exceed four right angles^[21]; while these alone, according to the aforesaid numbers, are equal to four right. And this theorem is Pythagoric. But by the present corollary, if even more than two right lines should cut each other in one point, as for instance, three or four, or any other number, all the angles which

they form, may be shewn to be equal to four right. For they will vindicate to themselves the place of four right angles. But it is manifest that the angles always become double to the number of right lines. And thus two right lines intersecting each other, there will be four angles equal to four right: but from the intersection of three lines, there will be six angles; and from four, eight, and so on, in infinitum. For the multitude of the right lines is always doubled: but the angles increase according to multitude, and are diminished according to magnitude, because it is the same four right angles, which is perpetually divided.

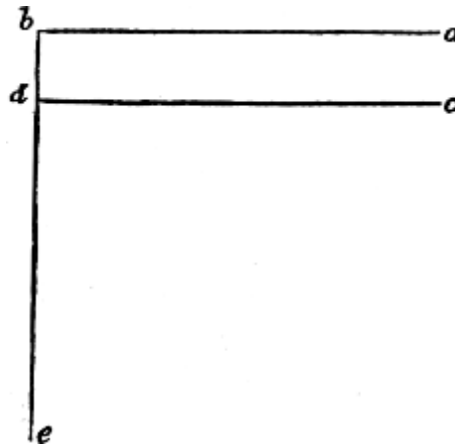
PROPOSITION XVI. THEOREM IX.

In every triangle having one side produced, the external angle is greater than either of the internal and opposite angles.

Those who enunciated this proposition, and at the same time omitted the particle, *having one side produced*, perhaps afforded an occasion of objection to many others, as well as to Philip, (according to the narration of the mechanist Heron.) But such as were desirous of entirely removing this calumny, enunciated the theorem, with the proposed addition, corresponding with the general manner of the geometrician. For in the fifth theorem, being desirous to shew, that the angles under the base of an isosceles triangle are equal, he adds, when the equal right lines are produced, the angles under the base are equal. Hence we infer, that though this proposition might be defective and imperfect in various copies, yet it was perfect and written entire, by the institutor of the Elements. What then does the proposition assert? *That in every triangle, if you produce one of its sides, you will find the angle constituted external to the triangle, greater than either of the internal and opposite angles.* For a little after, this angle will be shewn equal to both, but it is proved to be greater than either in the present; and he necessarily compares it with the opposite angles, and not with the successive angle. For to this last it may be both equal and less: but it is greater than either of the former. Thus, if this triangle should be right angled, and you conceive one of the sides comprehending the right angle to be produced, the external will be equal to the successive angle. But if it should happen to be obtuse-angled, the internal angle may be greater than the external; and it is on this account that he does not compare the external with the successive angle, but with the opposite angles. For of the angles within a triangle, the successive angle borders on the external, but the two others are opposite. Hence, the external angle is greater than either of the successive, but may not exceed the successive angle to which it is proximate. But some conjoining these two theorems, I mean the present, and the following, enunciate the proposition thus. *In every triangle having one side produced, the external angle is greater than either of the internal and opposite angles; and any two of the internal angles, are less than two right.* But there is occasion for the connection of these theorems, because the geometrician himself, a little after, enunciates the proposition after this manner, in equal angles, for he says: *In any triangle having one of its sides produced, the external angle is equal to the two interior, and opposite angles; and the three internal angles of a triangle, are equal to two right.* Hence, they think it proper in the present similar case, to connect the objects of investigation, and to make the proposition a composite. But if the datum be enunciated with this addition, it also will be a composite, (since it is requisite to understand two things, viz. the subject triangle, and one side produced:) and if the datum be given without this, it will be a *composite* in capacity, but *simple* in energy; for this must be received at the same time as a datum; since while we suppose an external angle, we must pre-suppose the side as produced.



But we may assume from the present theorem, that it is impossible from the same point, for three equal right lines to fall on the same right line. Thus let there be drawn from one point a , three equal right lines, $a b$, $a c$, $a d$, to the right line $b d$. Because, therefore, $a b$ is equal to $a c$, the angles at the base are equal. Hence, the angle $a b c$, is equal to the angle $a c b$. Again, because $a b$ is equal to $a d$, the angle $a b d$, is equal to the angle $a d b$. But the angle $a b c$, was equal to the angle $a c b$. Hence, the angle $a c b$, is equal to the angle $a d b$, the external, to the internal and opposite, which is impossible. From the same point therefore, to the same right line, three equal right lines cannot be drawn. But by the present theorem, we can also demonstrate, that if a right line falling on two right lines, makes the external angle equal to the internal and opposite, those right lines will by no means make a triangle, nor coincide, because the same thing would be both greater and equal, which is impossible.

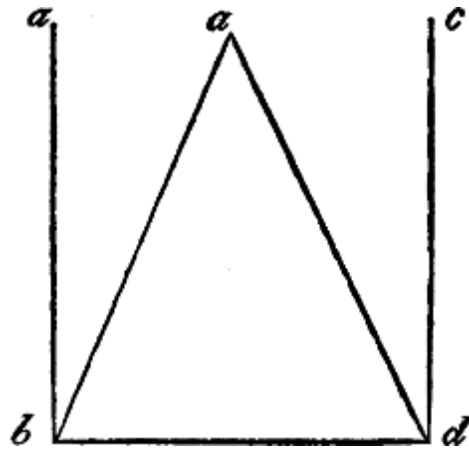


Thus for example, let $a b$, $c d$, be right lines, and let the right line $e b$ falling on them make the equal angles $a b d$, $c d e$, the right lines $a b$, $c d$, will not coincide. For if they coincide, the equal angles remaining, the angle $c d e$, will be equal to the angle $a b d$. And since it is external, it will be greater than the internal and opposite angle. Hence, it is necessary, if they coincide, that the angles remain no longer equal, but that the angle at the point d , be augmented. For whether $a b$ remaining immovable, we conceive that $c d$ is moved towards it, so as to coincide in the point c , we shall produce a greater distance in the angle $c d e$; since $c d$ approaches to $a b$, in the same proportion as it recedes from $d e$. Or whether $c d$, abiding, we conceive that $a b$ is moved towards it, in a similar manner, we shall by this means diminish the angle $a b d$; for it is at the same time carried towards $c d$, and to $b d$. Or whether we conceive both of them tending to each other, we shall find that $a b$ by tending to $c d$, contracts the angle $a b e$; and that $c d$, by receding from $d e$, on account of the motion to the line $a b$, increases the angle $c d e$. Hence, it is necessary, if it be a triangle, and if the right lines $a b$, $c d$, coincide, that the external angle must be also greater than the internal and opposite angle. For either the internal angle remaining, the external is increased, or the external abiding, the internal is diminished, or the internal is contracted, and the external is more dilated. But the cause of these consequences is the motion of the right lines, the one tending to those parts, where it diminishes the internal angle, but the other to the parts where it increases the external. And from this the reader should consider, how the origin of things produces the true causes of enquiries, which we have previously surveyed.

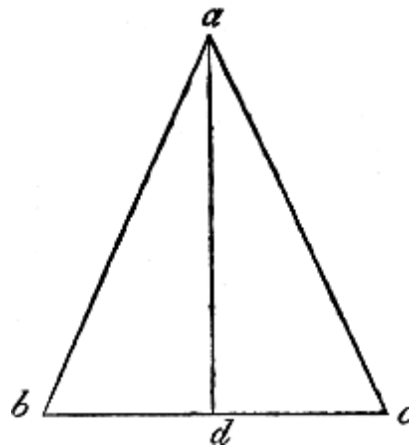
PROPOSITION XVII. THEOREM X.

The two angles of every triangle, taken all possible ways, are less than two right.

In the present theorem he shews indeterminately, that any two angles of a triangle, are less than two right (107) but in the following theorems he determines how much they are less, and that they are deficient by the remaining angle of the triangle: for its three angles are equal to two right; and on this account the two remaining angles are less than two right. And, indeed, the demonstration of the elementary institutor proceeds in a manifest order; for it uses the preceding theorem. But if is necessary, as in the last proposition, by regarding the origin of triangles, to find the cause of the present symptom. Let then the right lines $a b$, $c d$, be at right angles to $b d$.



If these lines then are to form a triangle, it is requisite they should incline to each other. But their inclination diminishes the internal angles, on which account they become less than two right: for they were right before their inclination. In like manner, if we conceive right lines standing at right angles, on the side $a b$, the same consequences will ensue respecting the inclination of the right lines; and the angles at the points a, b , will be less than two right; and so of the other side. This then is the cause of the proposition, and not the external angle being greater than either of the internal, and opposite angles: since it is not necessary that the side should be produced, nor that any angle should be constituted external to the triangle; but it is necessary that any two of the internal angles should be less than two right. Hence, it is necessary, as I have said, that the cause of this theorem should be the inclination of the right lines diminishing the angles at the base. But as the institutor of the elements exhibits the object of enquiry, by the external angle, we may accomplish this, without producing any one of the sides.

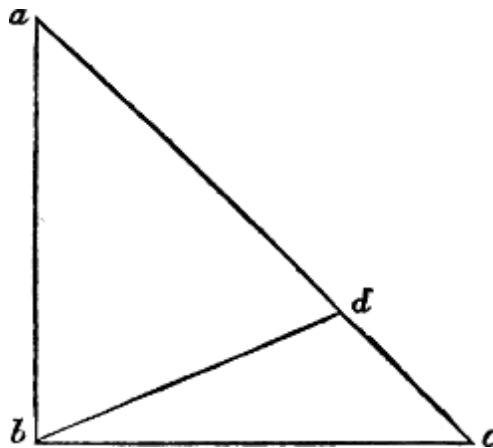


Thus let there be a triangle $a b c$, and let there be taken in the side $b c$, any point d , and let $a d$ be connected. Because, therefore, one side of the triangle $a b d$, is produced, viz. $b d$, the external angle $a d c$, is greater than the internal $a b d$. Again, because one side of the triangle $a d c$ is produced, viz. $c d$, the external angle $a d b$, is greater than the internal $a c d$. But the angles about the right line $a d$, are equal to two right, by the thirteenth of this. Hence, the angles $a b c, a c b$, are less than two right. In like manner, we may shew, that the angles $b a c$, and $b c a$, are less than two right, by taking a point in the side $a c$, and by connecting the point b with the assumed point. And again, we may affirm, that the angles $c a b, a b c$, are less than two right, by taking a point in the side $a b$, and by connecting a right line, from the point c , and the received point. And thus the thing proposed, is exhibited by the same theorem, without producing any side of the triangle. Hence, it is possible, that by this, the theorem may be proved, which asserts, *that from the same point, two perpendiculars cannot be drawn, to one right line*. For let there be drawn, if possible, from the point a , two perpendiculars $a b, a c$, to the right line $b c$. Then the angles $a b c, a c b$, are right. But because $a b c$ is a triangle, two of its angles are less than two right. The angles, therefore, $a b c, a c b$, are less than two right. But they are also equal to two right, because they are perpendiculars, which is impossible. Hence, from the same point, to the same right line, two perpendiculars cannot be drawn.

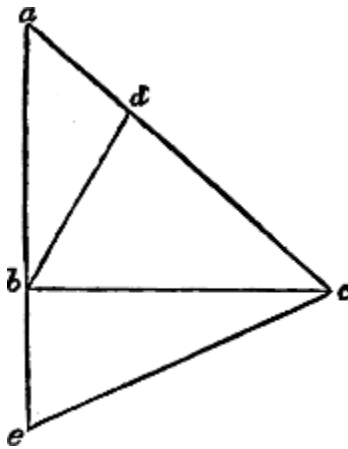
PROPOSITION XVIII. THEOREM XI.

The greater side of every triangle, subtends the greater angle.

That the equality of the sides in every triangle, forms the equality of the angles which they subtend, and that in like manner the equality of the angles shews the equality of their subtending sides, we learn from the fifth and sixth theorems. But that the equality of those angles, which are subtended by the sides, follows the inequality of the sides, and the contrary we now learn by the present eighteenth and nineteenth theorems. For the one shews that the greater angle is contained under the greater side, but the other, that the greater side subtends the greater angle; because these are mutually converted, but the same symptoms are contemplated in things contrary, as in the fifth and sixth theorems. But it is manifest, that we proportionally assume the greater and less side, in scalene triangles, that we distinguish the greatest, middle, and least, and the angles in a similar manner: but in isosceles triangles, the greater and less, simply assumed, are sufficient; for there is one side which is unequal to two, because it is either greater or less, as these theorems cannot take place in equilateral triangles. And here you may observe, that the theorems which exhibit the equality of angles or sides, agree with equilateral and isosceles triangles: but these which exhibit inequality to such as are isosceles and scalene. But the cause of this is, because of triangles, some are produced from equality alone, others from inequality alone, and others from the conjunction of both, which are partly constituted from equality, and partly through inequality. And some are allied to *bound*, others to *infinity*, and others are generated from the mixture of both. Hence the ternary permeates through all geometrical forms, as through lines, angles, and figures; and among figures, through such as are trilateral, quadrilateral, and all the rest in a consequent order. But *bound*, likewise, must be considered as inherent in geometrical forms, as well through similitude, as equality; and *infinite*, both by dissimilitude, and inequality; and that which is mixt, sometimes from the junction of similitudes, and dissimilitudes, and sometimes from the union of equalities, and inequalities. But the reason of this also, is because geometrical forms regard both quantity and quality. And we have assigned these, because, when we have determined these two, it will be manifest to us, that when the institutor of the Elements says, *of every triangle*, he does not also speak of the equilateral, but of that which has a greater and less side: for it is necessary to consider the object of enquiry, as consequent to the preceding datum; and that the triangle which has a greater and less side, contains a greater angle, under the greater side.

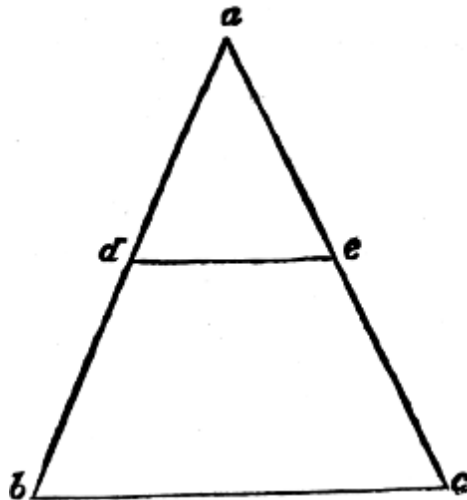


But because the geometrician, when in the construction he receives the triangle abc , and the side ac greater than the side ab , in order that he may shew, that the angle at the point b is greater than at the point c , from the side ac , he cuts off a right line ad , equal to the side ab : on this account it may be said that it is necessary to make the ablation at the point c , let us therefore exhibit the thing proposed upon this hypothesis, according to Porphyry, as follows. Let dc be equal to ab , and produce ab to the point e , and place be equal to da . The whole, therefore, ae , is equal to the whole ac . Connect ec .

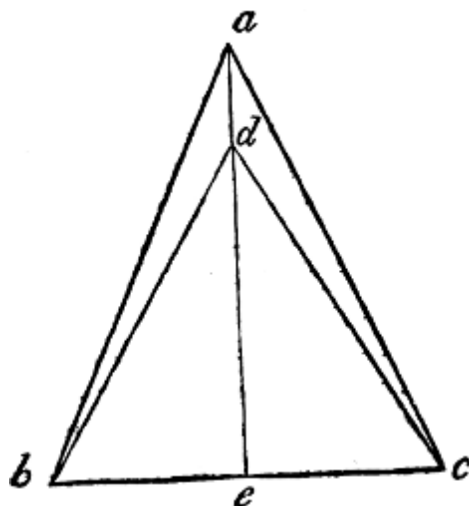


Because, therefore, $a e$ is equal to $a c$, the angle, also, $a e c$, is equal to the angle $a c e$, (by the fifth). Hence, the angle $a e c$ is greater than the angle $a c b$. But the angle, also, $a b c$ is greater than the angle $a e c$; because one side of the triangle $c b e$ is produced, viz. $b e$, and so the angle $a b c$, since it is external, is greater than the internal and opposite angle. Much more, therefore, is the angle $a b c$, greater than the angle $a c b$, which was to be shewn. And such are the geometrical exhibitions of the present theorem.

But it is manifest that the cause of this symptom is the amplification, or diminution according to magnitude, of the side subtending the angle. For when it is greater, it more amplifies the angle; but when less, at the same time it diminishes, and gives a greater contraction to the angle. And this takes place on account of the right line being situated in its extremities: for through its being placed in its extremities, it changes likewise the magnitudes of the angles, according to the increase and decrease which it receives. And this we affirm in one triangle, since it is possible that the same angle may be subtended by a greater or less right line; and that the same right line may subtend a greater and less angle.



For let the triangle happen to be an isosceles one, $a b c$, and let there be taken in the side $a b$, a point d , and let $a e$ be taken equal to $a d$, and connect $d e$. The right lines therefore, $d e$, $b c$, subtend the angle at the point a , of which the one is greater, but the other less. And in the same manner infinite right lines, greater and less subtending the angle a .



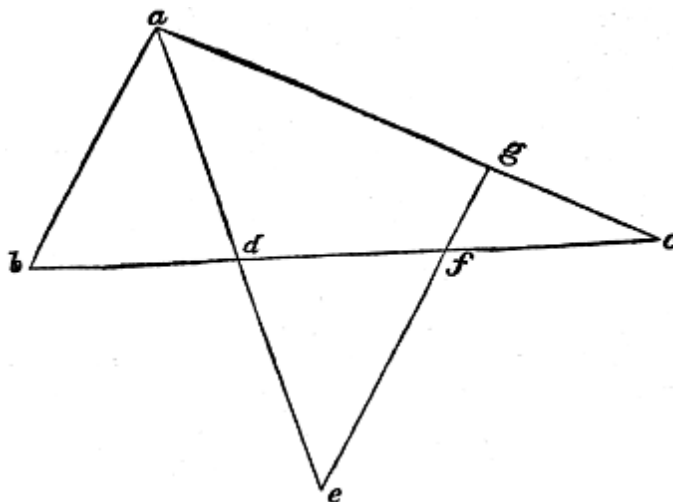
Again, let the triangle $a b c$, be isosceles, and let $b c$ be less than $b a$, $a c$, and construct upon $b c$, an equilateral triangle $b d c$, and connect $a d$, and produce it to e . Because, therefore, the angle $b d e$, of the triangle $a b d$, is external, it is greater than the angle $b a d$. In like manner the angle $c d e$, is greater than the angle $c a d$. The whole, therefore, $b d c$, is greater than the whole $b a c$, and the same right line subtends both, viz. the greater and the less angle. But it is shewn, that likewise greater and less right lines subtend the same angle. But in one and the same triangle, one right line subtends one angle, and the greater always the greater, and the less always the less, the cause of which we have contemplated.

PROPOSITION XIX. THEOREM XII.

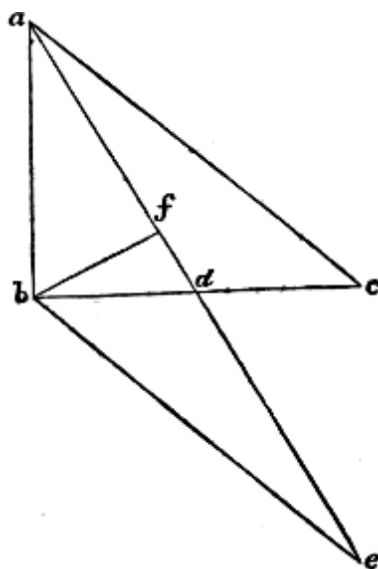
The greater side of every triangle subtends the greater angle.

This is the converse of the preceding theorem; and the datum, as well as the object of enquiry, is simple in each. Add too, that what was conclusion there, is hypothesis here: and what was hypothesis there is conclusion in this. But the former precedes, because it has the inequality of the sides given; and this follows, because it supposes unequal angles. For *sides*, indeed, seem to contain right-lined figures, but the *angles* appear to be contained; and the mode of demonstration in the former is ostensive, but in this it concludes the thing proposed by a deduction to an impossibility. The geometrician, therefore, by division, reasons concerning that which is impossible: for the angles being unequal, *I say*, (says he) *that the sides also subtending the unequal angles are unequal; and the greater subtends the greater given angle*. For if that which subtends the greater angle is not greater, it is either equal, or less. But if it be equal, the angles also which they subtend, are equal by the fifth. But if less, the angle also which it subtends, is less by the preceding: for it was shewn that the greater side subtends the greater angle, and the less the lesser. But the angles have a contrary position; and hence, the one side is greater than the other.

But it is possible that we may exhibit the thing proposed, without this division. For if the angle of a triangle be bisected, and the right line drawn to the base, cutting the angle, divides it into unequal parts, the sides containing that angle will be unequal, and the greater will be that which coincides with the greater segment of the base, but the less that which coincides with the lesser.



Let there be a triangle abc , and let the angle at a be bisected, by the right line ad , and let ad cut the base bc , into unequal parts, and let cd be greater than bd . I say that the side ac is greater than the side ab . Produce ad to the point e , and place de equal to ad . And because dc is greater than db , place df equal to bd , and connect ef , and produce it to the point g . Because, therefore, ad is equal to de , and bd to df , the two are equal to the two, and they comprehend equal angles at the vertex. Hence, the base ba , is equal to the base ef , and all, therefore, are equal to all. On this account also the angle def , is equal to the angle dab . But this is not unequal to dag . Hence, the side ag , is equal to the side eg , by the sixth. The side, therefore, ac , is greater than the side ef . But the side fe , is equal to the side ab ; and hence, the side ac , is greater than the side ab , which was to be demonstrated.



This being pre-assumed, we can shew that the greater side subtends the greater angle. Let there be a triangle abc , having the angle at the point b , greater than the angle at the point c . I say that the side ac , is greater than the side ab . Let bc be bisected in the point d , and connect ad , and draw de , equal to ad , and connect be . Because, therefore, bd , is equal to dc , and ad , to de , the two are equal to the two, and they comprehend equal angles at the vertex. Hence, the base be , is equal to the base ac , and all are equal to all. Hence too, the angle dbe , is equal to the angle at the point c , but less than the angle abd . The angle, therefore abe , is bisected by the right line bf . Hence, ef , is greater than fa . Because, therefore, the angle at the point b , of the triangle abe , is bisected by the right line bf , and ef is greater than fa , it follows from what has been previously shewn, that the side be , is greater than the side ba . But be has been shewn to be equal to ac . The side, therefore, ac , is greater than the side ab ; and the object of enquiry is exhibited. And it is manifest that the institutor of the Elements, avoiding a variety of demonstration, refrains from this mode of demonstrating, and employs a method of proof, which leads from division to an impossibility, because he was willing to fabricate the converse to the preceding, without any

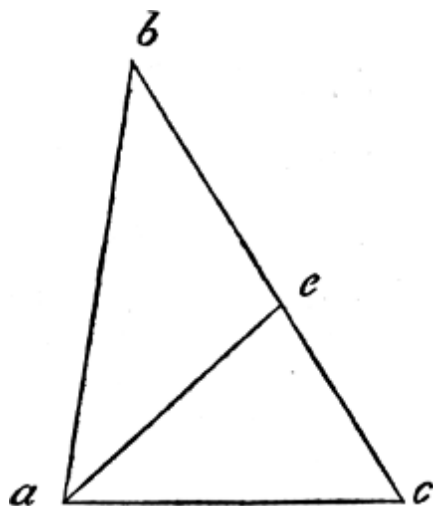
intervening medium. For the eighth theorem, indeed, which is the converse of the fourth, brings great disturbance, because it makes conversion difficult to be known. For it is more excellent to exhibit converse theorems, by preserving the continuity through an impossible, than to destroy the continuity by a principal demonstration. And hence, Euclid shews almost all converse theorems by a deduction to an impossibility.

PROPOSITION XX. THEOREM XIII.

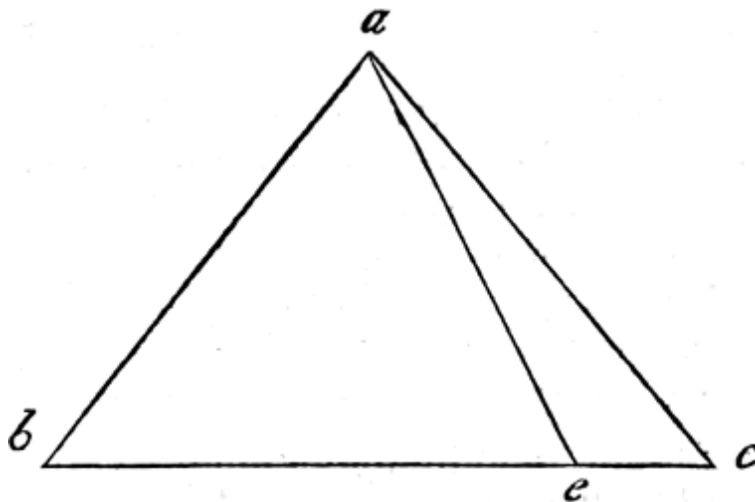
Two sides of every triangle, however taken, are greater than the remaining one.

The Epicureans oppose the present theorem, asserting that it is manifest even to an ass; and that it requires no demonstration: and besides this, that it is alike the employment of the ignorant, to consider things manifest as worthy of proof, and to assent to such as are of themselves manifest and unknown; for he who confounds these, seems to be ignorant of the difference between demonstrable and indemonstrable. But that the present theorem is known even to an ass, they evince from hence, that grass being placed in one extremity of the sides, the ass seeking his food, wanders over one side, and not over two. Against these we reply, that the present theorem is indeed manifest to sense, but not to reason producing science: for this is the case in a variety of concerns. Thus for example, we are indubitably certain from sense, that fire warms, but it is the business of science to convince us how it warms; whether by an incorporeal power, or by corporeal sections; whether by spherical, or pyramidal particles. Again, that we are moved is evident to sense, but it is difficult to assign a rational cause how we are moved; whether over an impartible, or over an interval: but how can we run through infinite, since every magnitude is divisible in infinitum? Let, therefore, the present theorem, that the two sides of a triangle are greater than the remainder, be manifest to sense, yet it belongs to science to inform us how this is effected. And thus much may suffice against the Epicureans.

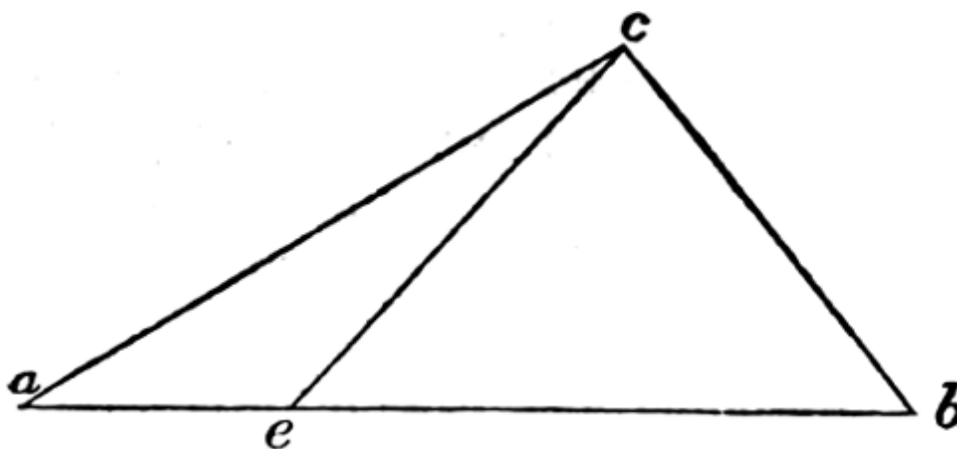
But it is requisite to relate the other demonstrations of the present theorem, such as Heron, and the familiars of Porphyry have fabricated, without producing the right line, after the manner of Euclid.



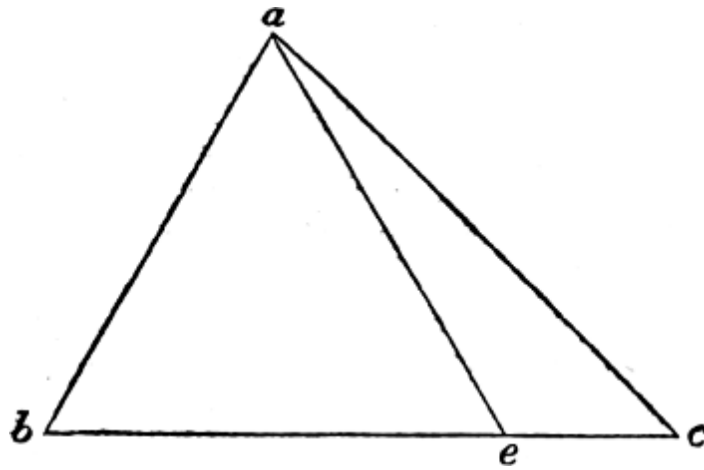
Let there be a triangle abc , it is requisite, therefore, to shew, that the sides ab , ac , are greater than the side bc . Bisect the angle at a , by the right line ae . Because, therefore, the angle aec , is external to the triangle abe , it is greater than the angle bae . But the angle bae , was placed equal to the angle eca . The angle, therefore, aec , is greater than the angle eca . Hence, the side also ac , is greater than the side ce . And for the same reason the side ab , is greater than the side be . For the angle aeb is external to the triangle aec , and is greater than the angle cae ; that is than the angle eab . And on this account the side ab , is greater than the side be . The sides, therefore, ab , ac , are greater than the whole side bc . And the like may be shewn of the other sides.



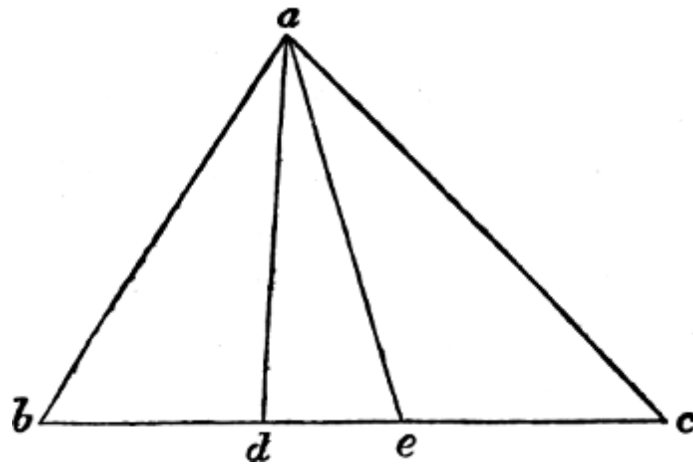
Let there again be a triangle abc . If therefore the triangle abc , be equilateral, two sides will be doubtless greater than the remaining one: for when there are three equal quantities, any two are double of the remainder. But if it be isosceles, it will have a base either less, or greater than each of the equal sides. If therefore the base be less, the two sides are given greater than the remainder. But if the base be greater, let it be bc , and cut off from it a part equal to either of the sides, which let be be , and connect ae . Because, therefore, the angle aec , is external to the triangle aeb , it is greater than the angle bae . On the same account the angle aeb , is greater than the angle cae . Hence, the angles about the point e , are greater than the whole angle about the point a , of which bea is equal to bae , since ab is equal to be . The remainder, therefore, aec , is greater than the remainder cae . Hence, the side ac , is greater than the side ce . But the side ab , was also equal to the side be . The sides, therefore, ab , ac , are greater than the side bc .



But if the triangle abc , be scalene, let the greatest side be ab , the middle ac , and the least cb . The greatest side, therefore, assumed with either of the others, exceeds the remainder: for by itself it is greater than either. But if we are desirous of shewing that the sides ac , cb , are greater than the greatest side ab , we must employ the same construction as in the isosceles triangle, cutting off from the greater side, a part equal to one of the other sides, and connecting the line ce , and using the external angles of the triangles.



Let there be again any triangle $a b c$. I say that the sides $a b$, $a c$, are greater than the side $b c$. For if they are not greater, they are either equal or less. Let them be equal, and cut off $b e$, equal to $a b$. The remainder, therefore, $e c$, is equal to $a c$. Because then, $a b$, $b e$, are equal, they subtend equal angles; and this is likewise true of $a c$, $e c$, because they are equal. Hence, the angles at the point e , are equal to the angles at the point a , which is impossible. Again let the sides $a b$, $a c$, be less than $b c$, and cut off $b d$, equal to $a b$, and $e c$ to $a c$.



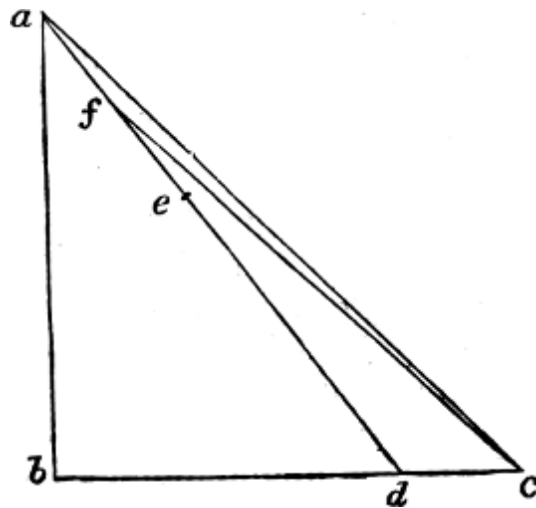
Because, therefore, $a b$ is equal to $b d$, the angle $b d a$, is not unequal to the angle $b a d$. And because $a c$ is equal to $c e$, the angle $c e a$, is equal to the angle $e a c$. Hence, the two angles $b d a$, $c e a$, are equal to the two $b a d$, and $e a c$. Again, because the angle $b d a$ is external to the triangle $a d c$, it is greater than the angle $e a c$: for it is greater than $c a d$. By a similar reason also, because the angle $c e a$, is external to the triangle $a b e$, it is greater than the angle $b a d$: for it is greater than the angle $b a e$. Hence, the angles $b d a$, $c e a$, are greater than the two $b a d$, $e a c$. But they were also equal to them, which is impossible. The sides, therefore, $a b$, $a c$, are neither equal to, nor less than the side $b c$, but greater. And the like may be exhibited in others.

PROPOSITION XXIV. THEOREM XIV.

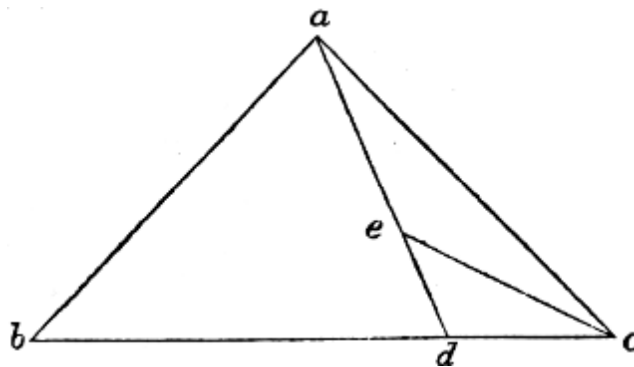
If upon one side of a triangle, two right lines beginning from the extremities, are internally constituted, the constituted right lines will be less than the other sides of the triangle, but they will contain a greater angle.

That which is expressed by the proposition, is, indeed, manifest; and the demonstration adopted by the elementary institutor, is evident; and the theorem is consequent to the first principles, since it depends on two theorems, the one previously exhibited, and the sixteenth. For in order to shew, that the lines internally constituted, are less than the external, the theorem is required, which says, *the two sides of every triangle, are greater than the remaining one*: but for the purpose of confirming that the angle comprehended by them is greater than that comprehended by the external sides, that theorem procures the greatest utility, which says, *the external angle of every triangle, is greater than the internal and opposite angle*. But you will receive at the same time,

conviction of geometrical diligence, and a commemoration of things admirable in the mathematical disciplines, if we shall shew that *it is possible within a certain triangle, upon one of its sides, not upon the whole, but upon some one of its parts, to constitute two right lines greater than the external right lines*^[22]; and again, others comprehending a less angle, and comprehended in the angle made by the external lines. For this being exhibited, it will at the same time be manifest, that the institutor of the Elements necessarily adds, that the internally constituted lines must begin from the extremities of the common basis; and must be constituted upon one whole side, and not upon any one of its parts: but likewise, as I have said, one of the admirable things which geometry contains, will be manifest. For is it not, indeed, admirable, that the lines constituted upon the whole side, should be less than the external sides: but that those constituted upon a part should be greater? Let there be then a right angled triangle abc , having the angle at the point b right, and take in the side bc , any point d , and connect ad . Hence, ad is greater than ab .

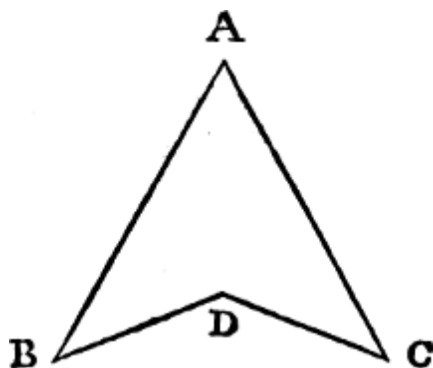


Take from ad , a part equal to ab , which let be de , and bisect ea , in the point f , and connect fc . Because, therefore, afc is a triangle, the lines af, fc , are greater than ac . But af is equal to fe . The right lines therefore, fe, fc , are greater than ac . But de is equal to ab . Hence, the right lines fc, fd , are greater than the right lines ab, ac , and they are internally constituted.



Let there be again an isosceles triangle abc , having the base bc , greater than either of the equal sides. Then from bc , cut off bd , equal to ab , and connect ad , and take in ad , any point e , and connect ec . Because, therefore, ab , is equal to bd , the angle bda , is also equal to the angle bad . And because the angle bda is external to the triangle edc , it is greater than the internal and opposite dec . Hence, the angle bda , is greater than the angle dec . Much more, therefore, is the angle bdc , greater than the angle dec ; and bdc is contained by the external lines, but dec by internal lines. Within a triangle, therefore, right lines de, ec , comprehending a lesser angle, are constituted within the angle comprehended by the external lines; and the thing proposed is shewn without employing the parallel lines of expositors. Hence, it is necessary that the constituted right lines should begin from the extremities of the basis: for those which are constituted upon any one of its parts, are shewn to be sometimes greater than the external lines, and to comprehend a lesser angle. But when they are constituted in this manner, beginning from the extremities, the species of triangles, called (ἀκιδοειδῆ) or, similar to the point of a spear^[23],

presents itself to our view; and is one of the admirable things contained in geometry, viz. to find a quadrilateral triangle. As for example, the triangle A B C. For it is contained by the four sides B A, A C, B D, D C; but it has three angles, one at B, the other at A, and the other at C. And hence, the present figure is a quadrilateral triangle.

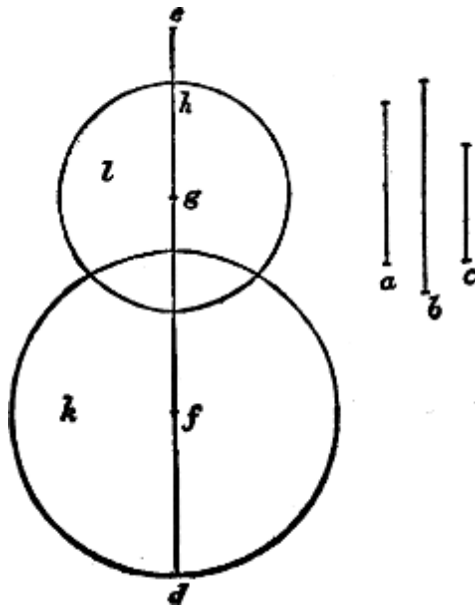


PROPOSITION XXII. PROBLEM VIII.

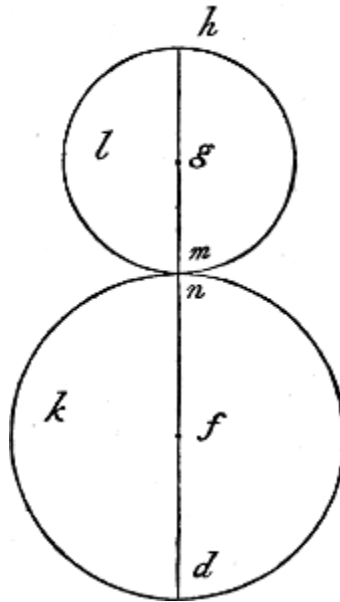
To construct a triangle from three right lines, which are equal to three given right lines.

But it is requisite that two of the lines must be greater than the remaining one, in whatever manner they may be taken.

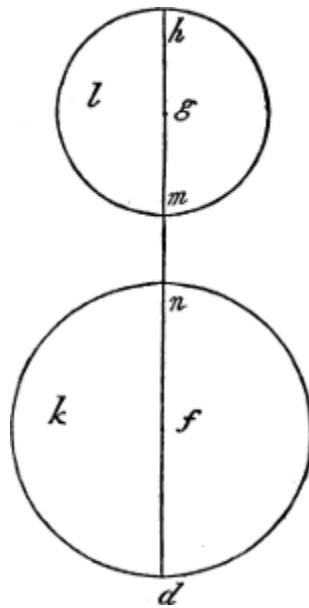
We again pass to problems, and Euclid commands us to construct a triangle from three proposed right lines, two of which are greater than the remaining one, equal to given right lines. Because he knew this in the first place, that it was impossible to construct a triangle from those same lines, which had already received the declared position: but that this was possible to be effected from their equals. In the next place, he knew it was necessary that two of the right lines about to complete the triangle, should be greater than the remaining one: for the two sides of every triangle are greater than the remaining one, however assumed, as we have shewn. On this account he adds, *that it is necessary the first right lines remaining, to construct a triangle from three equal to them: but that it is requisite, any two, however taken, should be greater than the remainder, or there will not be a triangle from three lines equal to the given right lines.* But by this means he also destroys all the objections which are urged against the construction, and which may be perfectly dissolved by this addition. Hence, the present problem ranks among things determined, and not among such as are indetermined: For of problems as well as of theorems, some are indeterminate, but others without termination. Thus if we should simply say, *from three right lines which are equal to three given right lines, to construct a triangle,* the problem is indeterminate and impossible. But if we add, *two of which, however assumed, are greater than the remainder,* the problem is determined and possible. For as the division of theorems takes place, according to true and false, so that of problems according to a possible and impossible enunciation. But that the objections which are urged against the construction, may be from hence dissolved, we shall learn from a little inspection: for we shall follow the words of the geometrician. Let there be three right lines, $a b c$, of which any two, however taken, are greater than the third, and let it be required to accomplish the thing proposed. Let there be placed a certain right line $d e$, on one part finite, as at the point d : and on the other part infinite. Then place $d f$ equal to a , but $f g$ to b : and $g h$ to c .



And from the centre f , but interval fd , let a circle k be described. Again, with the centre g , but interval gh , let the circle l be designed; and the circles will intersect each other. For this is assumed by the institutor of the Elements. But it may be asked how this takes place? For perhaps they either only touch each other, or they do not even touch. Since it is necessary that they should suffer some one of three cases, I mean that they should either intersect or touch, or be distant from each other. I say, therefore, that they necessarily intersect each other. For let them in the first place, touch each other.



Because, therefore, the point f is the centre of the circle k , df is equal to fn . And because the point g is the centre of the circle l , hg is equal to gm . The two, therefore, df, gm , are equal to one, viz. to fg . But they were placed greater than one, as also a , together with c , is greater than b . They are therefore equal to it, and at the same time greater, which is impossible. Again, if it be possible, let the circles be distant from each other, as k and l .



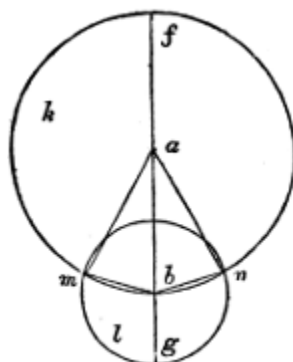
Because, therefore, the point f , is the centre of the circle k , df , is equal to fn . And because the point g , is the centre of the circle l , hg is equal to gm . The whole, therefore, fg , is greater than the two, df , hg : for fg , exceeds df , gm , by nm . But it was supposed that df , hg , were greater than fg , in the same manner as a and c are greater than b . For df was placed equal to a , but fg , to b , and hg to c . It is necessary, therefore, that the circles k l , should intersect each other. Hence, the institutor of the Elements very properly receives them cutting one another: since of the three right lines, he supposes two greater than the third, however, they may be assumed, but neither equal to, nor less than one. But it is necessary that when the circles touch, two of the lines should be equal to the third; and that when they are distant from each other, two should be greater than the remainder.

PROPOSITION XXIII. PROBLEM IX.

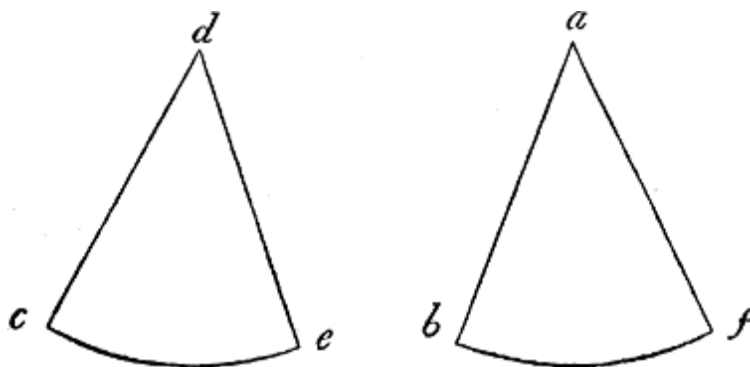
[126]

On a given right line, and at a given point in it, to constitute an angle equal to a given right lined angle.

This also is a problem, whose invention according to Eudemus is rather the gain of Oenopides than of Euclid: but it requires the construction of an angle, on a given right line; and at a given point in it equal to another right lined angle. This, then, Euclid necessarily adds, that the given angle must be rectilineal; because it is impossible that an angle can be construed on a right line equal to every angle. For it has been shewn^[24] that there are only two curve-lined angles equal to right-lined angles, viz. the angle of a lunular figure, which we have proved equal to every right-lined angle; and the angle of that figure similar to an axe^[25], which is equal to two thirds of a right angle. But a lunular figure of this kind, which is called (πελεκκοειδής) Pelecoides, is formed from two circles cutting each other through their centres. However, the construction of an angle on a certain right line, causes the constituted angle to become determinate, and not indifferent in species, but forms it either right-lined, or mixt. But since no mixt can be equal to a right-lined angle, it is manifest that this must be perfectly rectilineal. The institutor of the Elements, therefore, simply using the present problem, and constructing a triangle from three right lines, equal to three given lines, accomplishes the thing proposed. But you may receive a more exquisite construction of the triangle, by the following method.



Let there be a given right line ab , and a given point in it a , and a given right-lined angle cde . It is required, therefore, to accomplish the problem. Connect ce , and produce ab on both sides to the points f, g . Then place fa , equal to cd , and de to ab , and bg to ec . And with the centre a , but interval af , describe the circle k . And again, as in the preceding, with the centre b , but interval bg , describe the circle l . The circles, therefore, will cut each other, as we have shewn in the last proposition. Let them cut each other in the points m, n , and from these points draw right lines to the centres as in the figure. Because, therefore, fa , is equal to am , and an , but cd , is equal to fa ; am , and an , will be each equal to cd . Again, because bg is equal to bm , and bn , but ge is not unequal to ce ; bm , and bn , will be also equal to ce . But ab is equal to de . The two therefore, ab, am , are not unequal to the two de, dc , and the base bm , is equal to the base ce . Hence, the angle mab , is equal to the angle at the point d . And again, the two na, ab , are equal to the two cd, de , and the base nb , is equal to the base ce . The angle, therefore, nab , is equal to the angle cde , and the thing proposed is doubly accomplished: for we have not only constituted one, but two angles, equal to the given angle, on each side of the right line ab ; so that in whatever part we may desire the construction to be made, it will be indubitable, and without contradiction. And this we have added to the construction of the elementary institutor.

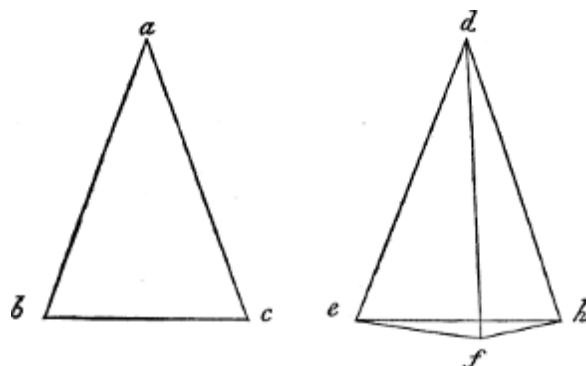


But we cannot praise the method of Apollonius, because it requires the assistance of the third book^[170] for he receives any angle cde , and a right line ab , and with the centre d , but interval cd , he describes a circumference ce . In like manner with the centre a , but interval ab , he describes a circumference bf ; and intercepting a circumference ce , equal to bf , he connects the right line af , and affirms that the angles a and d , insisting on equal circumferences, are equal. But it is necessary to pre-assume that ab is equal to cd , in order that the circles may be also equal. We therefore think that a demonstration of this kind requiring posterior propositions, is foreign from an elementary institution; and we give the preference to that of the geometrician, as consequent to principles.

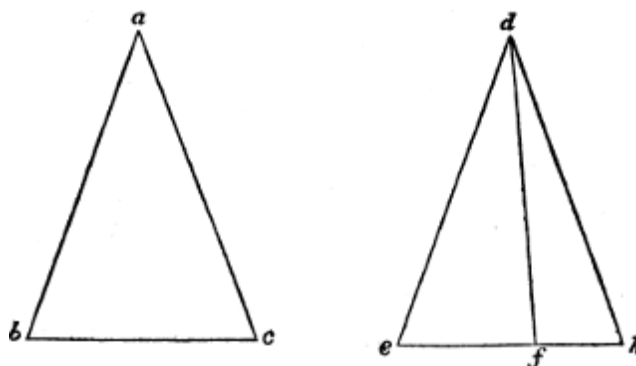
PROPOSITION XXIV. THEOREM XV.

If two triangles have two sides equal to two, each to each, but the one angle contained by the equal right lines greater than the other: they shall also have the base of the one greater than the base of the other.

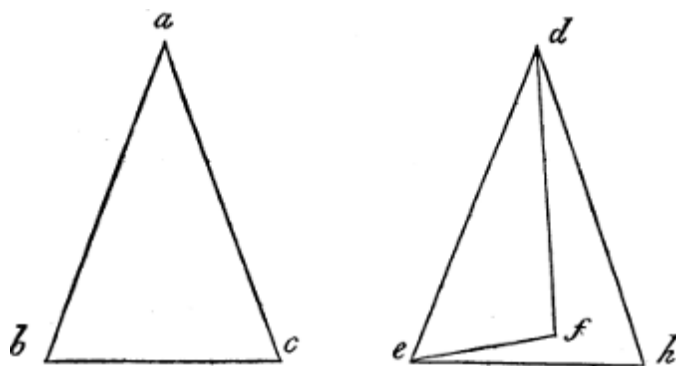
Euclid again passes on to theorems, and speaks concerning inequality in two triangles, in a manner similar to his discourse concerning equality. For supposing two triangles, having two sides equal to two, each to each, he sometimes places the vertical angle equal in each, and sometimes unequal; and he proceeds in a similar manner with respect to the base. Besides this, he demonstrates that the equality of the bases is consequent to the equality of the vertical angle, and that the equality of the vertical angles, is consequent to the equality of the bases: but he now shews that the inequality of the one, follows the inequality of the other. The present theorem, therefore, is opposite to the fourth: for that, indeed, supposes the vertical angles of the triangles equal, but this supposes them unequal. And that demonstrates the equality of their bases; but this proves them unequal, in the same manner as their angles. It precedes, however, the following theorem: for that deduces its proof of inequality from the bases to the angles subtending the bases: but this, on the contrary, reasons from the angles to the bases, which are under the angles. Hence it is, after this manner, the converse of its consequent proposition, but opposite to the eighth theorem. For the one from the equality of the bases, demonstrates the equality of the vertical angles, but the other from the inequality of the bases, shews that the vertical angles are unequal. It is, however, common to these four (two of which are conversant with equality, I mean the fourth, and the eighth, but two about inequality, the present and the following; and two begin from angles, viz. the fourth, and the object of investigation in the present, but two from bases, viz. the eighth, and the following proposition); it is common, I say, to all these four, as well to the fourth and the eighth, as to the twenty-fourth and twenty-fifth, to have two sides equal to two, each to each. For these being unequal, all enquiry is superfluous, and subject to deception. And thus much for a universal speculation concerning the present theorem.



But let us now consider the construction of the elementary institutor, and add to it where deficient. For Euclid receiving two triangles, abc , def , having the sides ab , ac , equal to the sides de , df , each to each, and the angle at the point a , being greater than the angle at the point d , and willing to shew that the base bc , is greater than the base ef , on the right line ed , and at a point in it d , constitutes an angle edh , equal to the angle at the point a . For the angle at the point a , is greater than the angle at the point d , and he connects dh , equal to ac . The right line, therefore, eh , produced to the point h , either falls above, or upon, or beneath the line ef . The institutor of the Elements, indeed, considers it as lying above the line. But let it be upon the right line. Again, therefore, we may exhibit the same.

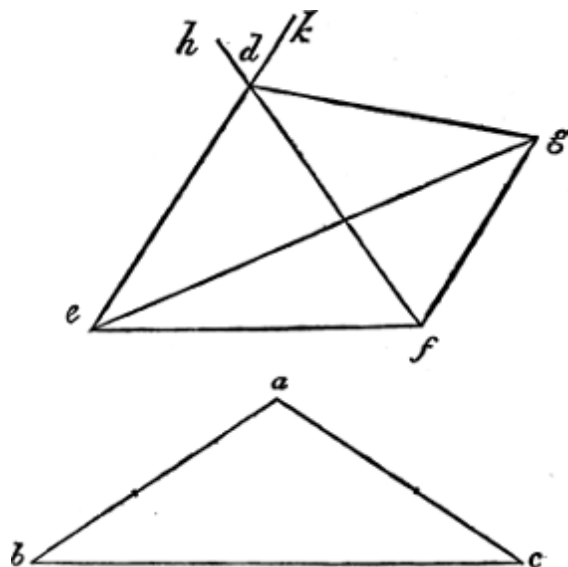


For the two ab, ac , are equal to the two de, dh , and they contain equal angles. Hence, the base bc , is equal to the base eh . But eh is greater than ef ; and on this account bc is greater than ef . Again, let it be placed beneath ef . Connecting, therefore, eh , we must say, that since ab, ac , are equal to de, dh , and they comprehend equal angles, bc is also equal to eh . Because, therefore, within the triangle deh , two right lines df, fe , are constructed on the side de , they are less than the external sides. But dh , is equal to df : for it is equal to ac . Hence he is greater than ef . But he is equal to bc . And therefore, bc is greater than ef . The theorem, therefore, is exhibited according to every position.



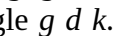
Why then, as is the fourth theorem, he at the same time demonstrated that the areas of triangles are equal, does he not add in the present, that besides the inequality of the bases, the areas also are unequal? Against this doubt we must say, that there is not the same proportion in equal, as in unequal angles and bases. For when the angles and bases are equal, the equality also of the triangles follows: but when they are unequal, it is not necessary that the inequality of the areas should be consequent; since the triangles may as well be equal, as unequal; and that may be greater, and likewise less, which contains the greater angle, and the greater base. On this account, therefore, the institutor of the Elements leaves the comparison of the triangles; to which we may add, that the contemplation of these, requires the doctrine of parallels.

But if it be requisite, that anticipating things which are afterwards exhibited, we at present make a comparison of areas, we must say, that if the angles a, d , are equal to two right, the triangles may be shewn to be equal: but when they are greater than two right, the lesser triangle will be that which contains the greater angle; and when they are less than two right, this will be the case with the greater triangle.



For let the construction in the element be given, and produce ed, fd , to the points k, h ; and let us suppose the angles bac, edf , equal to two right. Because, therefore, the angle bac , is equal to the angle edg , the angles edg, edf , are equal to two right. But the angles edg, kdg , are also equal to two right. Let the common angle edg be taken away, and the remainder edf , will be equal to the remainder kdg . But edf is equal to hdk ; for they are vertical angles. Hence, the angle kdg , is equal to the angle hdk . And because the angle gdh , is external to the

B1351 Because,

 $g, g \mid k,$ 

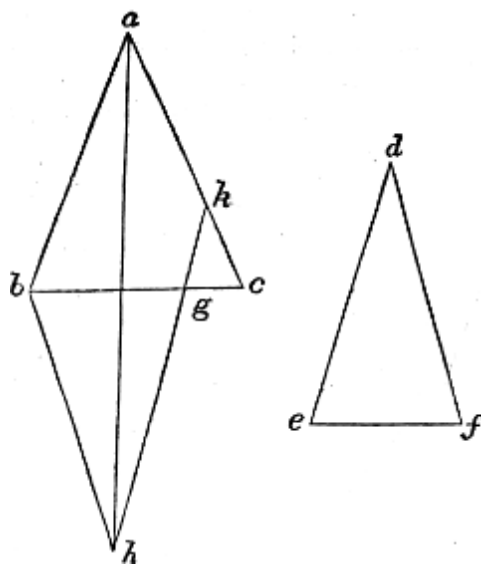
gle at the

It is shewn, therefore, that the triangle $a b c$, is both equal to, and is also greater and less than the triangle $d e f$, the angles at the points a and d , being either equal to, or greater or less than two right. And thus, all the hypotheses may be accomplished. For what if the angle at the point a , should be one right, and the half of a right angle, but the angle at the point b , the half of one right, would not those two angles be equal to two right? But what if the angle at the point a , should be one right, and the half of a right, but the angle at the point b , two thirds of one right, would they not be greater than two right angles? And lastly, if the angle at the point a , should be one right, and the half of a right angle, but the angle at the point b , a third part of a right angle, would they not be less than two right, and the angle a be greater than the angle d ? All these comparisons, therefore, are produced by the assistance of parallels; and hence, they are necessarily not found in the present elementary institution.

PROPOSITION XXV. THEOREM XVI.

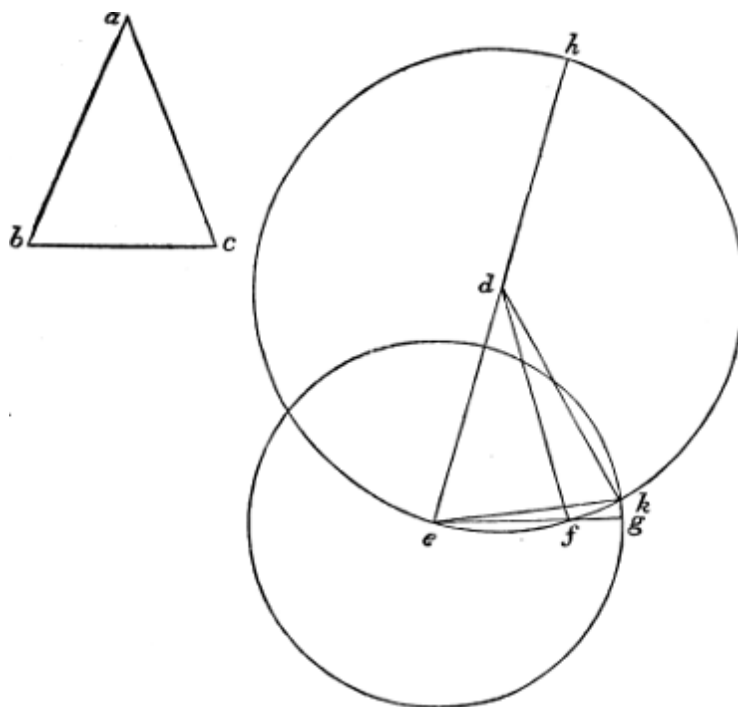
If two triangles have two sides equal to two, each to each, but have the base of the one greater than the base of the other; they shall likewise have the angle contained by the equal sides in the one, greater than the angle contained by the equal sides in the other.

The present theorem is the opposite to the eighth, but the converse of the preceding. For the institution of the Elements produces theorems concerning the equality and inequality of angles and bases, according to *conjunction*; in each of the *conjunctions*, receiving some as precedents, but others as converse. And in such as are precedent indeed, he employs direct ostensions: but in such as are converse, he uses deductions to an impossibility. After this manner he proceeds in some particular triangle, sometimes from the equality of the sides which it contains, shewing the consequent equality of the angles which they subtend: but sometimes from their inequality evincing inequality. And again, on the contrary, affirming that equality of sides is consequent to equality of angles, but inequality to inequality. However, that we may proceed to the thing proposed, we refer those who are desirous of learning how the geometrician shews when this is manifest, to his books on this subject. But we shall briefly relate the demonstrations which others produce of this proposition; and in the first place, that which Menelaus Alexandrinus invented and delivered.



Let there be two triangles $a b c$, $d e f$, having the two sides $a b$, $a c$, equal to the two $d e$, $d f$, each to each, and the base $b c$, greater than the base $e f$, I say that the angle at the point a , is greater than the angle at the point d . For let there be cut from the base $b c$, a line $b g$, equal to the base $e f$, and construct at the point b , an angle $g b h$, equal to the angle $d e f$, and place $b h$ equal to $d e$. Lastly, connect $h g$, and produce it to the point k , and connect $a h$. Because, therefore, $b g$ is equal to $e f$, but $b h$ to $d e$, the two are equal to the two, and they contain equal angles. Hence, $g h$ is equal to $d f$, and the angle $b h g$, is not unequal to the angle $e d f$. And because $g h$ is equal to $d f$, but $d f$ to $a c$, $g h$, also, is equal to $a c$. Hence $h k$ is greater than $a c$, and consequently is much greater than $a k$. The angle, therefore, $k a h$, is greater than the angle $k h a$. Again, because $b h$, is equal to $a b$, for it is equal to $d e$, the angle $b h a$, is equal to the angle $b a h$. Hence, the whole angle $b h k$, is less than the whole, $b a c$, but is shewn to

be equal to the angle at the point d . The angle, therefore, $b a c$, is greater than the angle at the point d . And such is the demonstration of Menelaus.



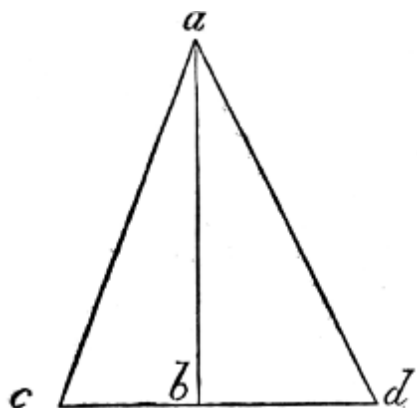
But Heron, the mechanist, shews the same thing, in the following manner, without leading to an impossibility, as is the case with the demonstration of Euclid. Let there be two triangles $a b c$, $d e f$, with the same hypotheses as above. And because $b c$ is greater than $e f$, let $e f$ be produced, and place $e g$ equal to $b c$; and in like manner extend $d e$, and place $d h$ equal to $d f$. The circle, therefore, which is described with the centre d , and interval $d f$, will pass also through the point h . Let it be described as $f k h$. And because $a c$, $a b$, are together greater than $b c$, but these are equal to $e h$, and $b c$ is equal to $g e$, hence the circle which is described with the centre e , but interval $e g$, will cut $e h$. Let it cut $e h$, as the circle $g k$, and connect from the common section of the circles to the centres, the right lines $k d$, $k e$. Because, therefore, the point d , is the centre of the circle $h k f$, $d k$ is equal to $d h$, i.e. to $a c$. Again, because the point e , is the centre of the circle $g k$, the line $e k$, is equal to $e g$, i.e. to $b c$. Hence, since the two $a b$, $a c$, are equal to the two $d e$, $d k$, and the base $b c$, is equal to the base $e k$, the angle, also, $b a c$, is equal to the angle $e d k$. And thus the angle $b a c$, is greater than the angle $f d e$.

PROPOSITION XV. THEOREM XVII.

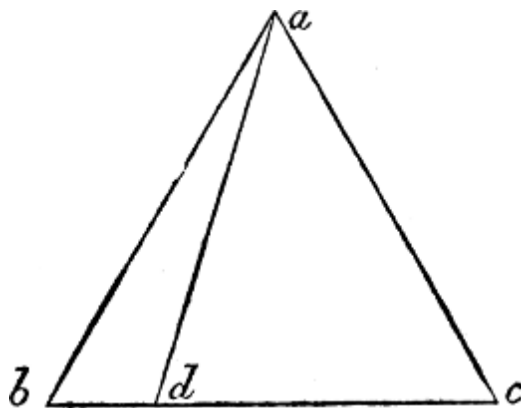
If two triangles have two angles equal to two, each to each, and one side equal to one side, either that which is adjacent to the equal angles, or that which subtends one of the equal angles: then they shall have the remaining sides equal to the remaining sides, each to each, and the remaining angle equal to the remaining angle.

It is necessary, that he who wishes to compare triangles with each other, according to sides, angles, and areas, should either, by receiving the sides alone equal, enquire after the equality of angles; or by assuming the angles alone equal, investigate the equality of the sides; or by mingling the angles and sides, scrutinize the equality of angles and sides. Since, therefore, Euclid alone receives the angles equal, he could not likewise shew that the sides of the triangles are equal. For the least triangles are equiangular with the greatest, though at the same time they are excelled by them, both according to sides and comprehended space: but the angles of the former are separately equal to the angles of the latter. However, as he supposes the sides alone to be equal, he demonstrates that all are equal, by the eighth theorem, in which there are two triangles having two sides equal to two, each to each, and the base to the base, and these are shewn to be equiangular, and to possess a power of comprehending equal spaces. And the institutor of the Elements omits this addition, as necessarily following from the fourth, and requiring no demonstration. But when receiving sides and angles, he ought to receive either one side equal to one, and one angle equal to one; or one side, and two angles of the triangles, equal to two; or on the contrary, one

angle and two sides; or one angle and three sides; or one side and three angles; or more than one side, and more than one angle. But when he had received one angle, and one side, he could by no means shew the thing proposed. I mean, the equality of the rest. For it is possible that two triangles which are equal, according to one side only, and one angle, may be entirely unequal as to the rest.

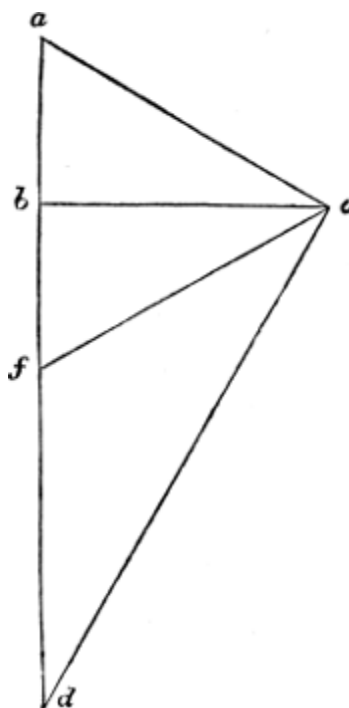


Thus let there be a right line ab , perpendicularly erected upon the right line cd , but let bd be greater than bc , and connect ac , ad . In these triangles, therefore, there is one common side, and one angle equal to one, but all the rest are unequal. But it is lawful to receive one side, and two angles, and to prove the rest equal, and this he performs by the present theorem: though again, to suppose one side, and three equal angles, is superfluous; since from the equality of two alone, the equality of the rest is exhibited. Again, receiving one angle, and two equal sides, he demonstrates that the rest are equal in the fourth theorem. But it is superfluous to receive one angle, and three equal sides: for two equals being alone assumed, conclude the equality of the rest. Besides, it is superfluous to assume two sides, and two equal angles; or two sides, and three equal angles; or two angles and three sides; or three angles and three sides. For the consequents to fewer hypotheses attend likewise a greater multitude, while the hypotheses are received with proper conditions. Hence, three hypotheses requiring demonstration, present themselves to our view, one, which alone receives three sides; and another which assumes one side, and two angles, which the geometrician now proposes; and a third, the opposite to this. On this account, we have only these three theorems, concerning the equality of triangles, which are conversant in sides and angles; since all the other hypotheses are either invalid for the purpose of shewing the object of enquiry; or they are valid indeed, but superfluous, because the same things may be readily procured by fewer hypotheses. As, therefore, when he assumed two sides equal to two, and one angle equal to one, he did not, indeed, assume every angle, but (as it was proposed by him) that contained by equal right lines, in the same manner when he assumes two angles equal to two, and one side to one, he does not assume any side, but either that which is adjacent to the equal angles, or that which subtends one of the equal angles. For neither is it possible in the fourth theorem, by assuming any equal angle, nor in the present by assuming any side, to shew the equality of the rest.



Thus for example, an equilateral triangle abc , being given, let the side bc be divided into unequal parts, by the line ad . Hence, there will be formed two triangles, having two sides ab , ad , equal to the two ac , ad , and one angle at the point b , equal to one angle at the point c , but the remaining sides will not also be equal, as for instance, the side bd , to the side dc : for they are unequal. But neither are the remaining angles equal: the reason

of which is, because we receive an angle equal to an angle, but not the angle which is contained by equal sides. After the same manner, indeed, the present theorem also will appear dubious, unless we assume, according to the aforesaid condition, an equal side subtending one of the equal angles, or adjacent to the equal angles.



For let there be a right angled triangle abc , having the angle at the point b right, and the side bc , greater than the side ba , and let there be constructed on the right line bc , and at a point in it c , an angle bcd , equal to the angle bac , and let bd, cd , produced, coincide in the point d . There are two triangles, therefore abc, bcd , having one side bc common, and two angles equal to two, viz. abc , to cbd (for they are right), and bac to bcd , according to construction. Hence, as it appears the triangles are equal, and yet it may be shewn that the triangle bdc , is greater than the triangle abc . But the reason of this is, because in the triangle abc , we assume the common side bc , subtending one of the equal angles, viz. the angle at the point a : but in the triangle bcd , we assume the equal side, adjacent to the equal angles. It was requisite, therefore, in each, either to subtend one of the equal angles, or to be adjacent to the equal angles. But not observing this, we affirmed that triangle to be equal, which is necessarily greater: for is not the triangle bcd , greater than the triangle abc ? To be convinced of this, let there be constructed on the right line bc , and at a given point in it c , an angle $fc b$, equal to the angle acb : for the angle bcd , as well as the angle at the point a , is greater than the angle acb . Because, therefore, there are two triangles, abc, bcf , having two angles abc, bca , equal to two cbf, bcf , each to each, and one side common, adjacent to the equal angles, viz. bc , the triangles are equal. But the triangle bcd is greater than the triangle bcf , and consequently it is also greater than the triangle abc . But it was formerly shewn to be equal, on account of the assumption of any side: And thus much the diligence of Porphyry has supplied us on the present occasion. But Eudemus, in his Geometrical Narrations, refers the present theorem to Thales. For he says it is necessary to use this theorem in determining the distance of ships at sea, according to the method employed by Thales in this investigation. But from the preceding division we may briefly assume all the contemplation concerning the equality of triangles, and are enabled to relate the causes of things omitted, confuting those hypotheses, as either false, or superfluous. And thus far we determine the limits of the first section of the elementary institutor, because he forms the constructions and comparisons of triangles, according to equal and unequal. And by construction, indeed, he delivers their essence: but by comparison, their identity and diversity. For there are three things which are conversant about being, *essence*, *same*, and *different*^[26], as well in quantities, as in qualities, according to the propriety of subjects. From these, therefore, as images it may be shewn, that every thing is the *same* with itself, and *differs* from itself, on account of the multitude which it contains; and that all things are the *same* with one another, and *different* from themselves. For both, in every triangle, and in more triangles than one, equality and inequality has been found to reside.

B O O K IV.

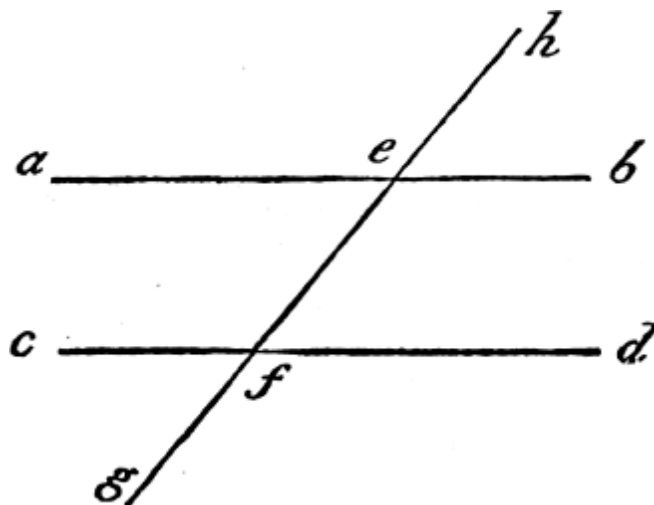
Whatever can be said in an elementary institution, concerning the origin and equality of triangles, we may learn from the preceding discourse. But after this, the narration of Euclid is concerning quadrilateral figures, and he particularly teaches us concerning parallelograms, together with the contemplation of these delivering the doctrine of trapeziums. For a quadrilateral figure, (as we have formerly observed in our discourse on hypotheses,) is divided into parallelogram and trapezium; and a parallelogram into other certain species, and in like manner a trapezium. But because a parallelogram, on account of its participation of equality, possesses disposition and order, but a trapezium has neither the same, nor a similar order; Euclid's principal discourse, is with propriety, concerning parallelograms, but he also contemplates together with these a trapezium. For from the section of parallelograms, the origin of trapeziums will appear, as will be manifest as we proceed. But because again, it is not possible that any thing can be said of the construction or equality of parallelograms, without the consideration of parallels, (for as it is manifest from the very name, that is, a parallelogram, which is circumscribed by parallel right lines in an opposite position,) hence, he necessarily assumes from parallels the beginning of his doctrine, but having advanced a little from these, he enters on the doctrine of parallelograms, employing one middle theorem, between the elementary institution of each, because he appears to contemplate a certain symptom inherent in parallels: but he delivers the first origin of a parallelogram. For such is the proposition, which says, that *right lines which join equal and parallel right lines towards the same parts, are themselves equal and parallel*. For in this theorem, indeed, a certain accident to equal and parallel right lines is considered: but from the connection a parallelogram appears, having its sides opposite and parallel. And from hence it is manifest that the discourse concerning parallels, was necessarily pre-assumed. But three things are to be assumed, essentially inherent in parallels, which they essentially express, and are converted with them, not only the three together, but every one separately assumed from the rest. Of these, one is, *that when a right line cuts parallel lines, the alternate angles are equal*; but the second, *that when a right line cuts parallel lines, the internal angles are equal to two right*; and the third, *that in consequence of a right line cutting parallel lines, the external is equal to the internal and opposite angle*. For when any one of these symptoms is demonstrated, we have sufficient authority to affirm that the right lines are parallel. But other mathematicians, also, have been accustomed to discourse after this manner concerning lines, delivering the symptoms of every species. For Apollonius, in each of the conical lines, shews what a *symptom* is, as also Nicomedes in his Treatise on Conchoids, and Hippias in his Quadratics, and Perseus in his Spirals. Since after their origin, that which is essentially inherent in these lines, and according to what it is inherent, being assumed, distinguishes a constructed form from all others. After the same manner, therefore, the institutor of the Elements, first of all investigates, the symptoms of parallels.

PROPOSITION XXVII. THEOREM XVIII.

If a right line falling upon two right lines, makes the alternate angles equal to each other, those right lines shall be parallel to each other.

In the present theorem it was not pre-assumed as evident that the right lines are in one plane, but this⁴⁶ ought rather to be previously admitted in all theorems which are considered in a plane. This, however, is added, because it does not universally follow, that when the alternate angles are equal, the right lines will be parallel, unless they are in the same plane. For nothing hinders, but that a right line falling on right lines disposed in the shape of the letter X, one of which is situated in one plane, but the other in a different one, may make the alternate angles equal; and yet the right lines thus disposed will not be parallel. It was pre-assumed^[27], therefore, that in a treatise on planes, we conceive every thing described in one and the same plane: and on this account, he does not require this addition in the present proposition. But it is requisite to know that the geometrician considers the particle *alternate*, in a two-fold respect, sometimes, indeed, according to a certain situation, but sometimes according to a certain consequence of proportions. And according to this last signification, the particle *alternate* is used in the fifth book, and in such as are arithmetical: but agreeable to the former, both in this, and in all the other books concerning parallel right lines, and that which falls upon these. For he calls the angles alternate, which are not formed at the same parts, and are not successive to each other, which are distinct, indeed, from the incident line,

but both of them exist within parallels, and differ in this, that the one has an upward, but the other a downward position.



I say, for example, that when a right line ef , falls on the right lines ab , and cd , he calls the angles aef , dfe , and also the angles cfe , bef , *alternate*, or *altern*, because they have an alternate, or changed order, according to their position. But this too must be known, that from such a situation of right lines, all the symptoms become by division, six; three of which the geometrician alone receives; and three he omits. For we either assume the angles at the same parts, or not at the same. And if at the same parts, either both within the right lines, which shews them to be parallels; or both without, or one without, and the other within. And if not at the same parts, again, after the same manner, they are either both without the right lines, cutting the lines it is necessary to receive; or within; or one within, and the other without. But what we have said will become manifest by the same description as above. For let there be certain right lines ab , cd , and let a right line ef , fall upon them, and let it be produced to the points h and g . If then you assume angles at the same parts, you will either place them both within, as bef , and efd , or as aef , and efc ; or both without, as heb , and dfg , or as hea , and cfg ; or one within, and the other without, as heb , and efd , or as gfd , and feb , or as hea , and efc , or as gfc , and aef : for these last are received in a quadruple respect. But if you assume the angles not at the same parts, you will either place both within, as aef , and efd , or as cfe , and feb ; or both without, as ah , and dfg , or as heb , and cfg ; or one within, and the other without, and this again in a quadruple respect. For they will either be the angles ah , and efd ; or heb , and efc ; or gfc and feb ; or gfd , and fea . And besides these, there is no other assumption.

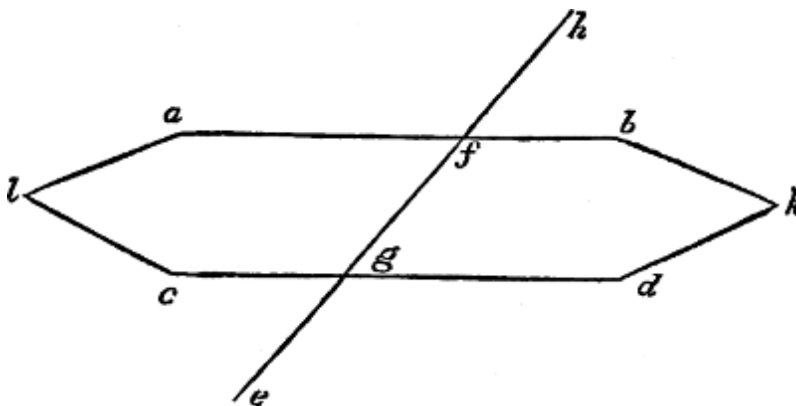
As, therefore, angles are assumed according to six modes, the geometrician combines three assumptions alone; and these consequent symptoms, are naturally adapted to express parallels. But of these three assumptions, one belongs to those angles which are not at the same parts, viz. to those which are only assumed within; and these he calls *alternate*, so that those, which are both external, and those, one of which is external, but the other internal, are omitted: but two of these assumptions belong to angles at the same parts, to those, indeed, which are both internal, which he says are equal to two right, and to those, one of which is internal, but the other external, which he says are equal, leaving indeed one assumption which supposes both the angles to be external. We therefore affirm that the same things will be consequent to the three omitted hypotheses. Thus, in the preceding figure, let both the external angles heb , dfg , be at the same parts, I say that these are equal to two right angles. For the angle dfe , is equal to the angle heb , and the angle bef , to the angle dfg . But if the angles bef , efd , are equal to two right, the angles dfg , heb , are equal to two right. Let again the angles ah , efd , not be towards the same parts, of which the one is within, but the other external, I say that these also are equal to two right angles. For if the angle ah , is equal to the angle bef , but the angles bef , and efd , are equal to two right, the angles, also, ah , and efd , are equal to two right. Again, let them not be at the same parts, but both without the right lines as ah , dfg . I say that these are equal to one another. For if the angles ah , and bef , are equal to each other, but the angle dfg , is equal to the angle bef , hence the angle ah , is not unequal to the angle dfg . If, therefore, the things assumed by the geometrician, in three hypotheses are verified, all the same follow in the remaining three as indisputably true. Besides this too is to be observed, that in such as the geometrician receives these, according to two assumptions, the angles are supposed equal to each other, but when according to one assumption, equal to two right: but in these last on the contrary, according to two assumptions, they are supposed

equal to two right angles, but according to one equal to each other. For since all the assumptions are six, it happens, indeed, from three, that the angles are equal to two right, but from the other three, that they are equal to each other. Hence, those which are omitted are not undeservedly contrary to the assumptions which are reckoned worthy of relation. But the geometrician appears to have chosen such hypotheses as either abound in affirmation, or are more simple, and, on this account of those angles which are not at the same parts, he assumed alone the internal, which he calls alternate: but of those at the same parts, he assumes as well the internal, as well as one internal and the other external, but he avoids the rest, either because they are more declared by negation, or because they are more various. However, whether this or some other be the cause, the number of the consequents to those hypotheses is from hence sufficiently manifest.

PROPOSITION XXVIII. THEOREM XIX.

If a right line falling upon two right lines, makes the external equal to the internal angle, placed opposite, and at the same parts, or makes the angles internally situated, and at the same parts equal to two right, those right lines shall be parallel to each other.

The preceding theorem receiving the angles, not at the same parts, but situated within right lines, shews that the right lines are parallel among themselves: but the present theorem proposes the two remaining hypotheses, of which one separates the angles according to the particles *without* and *within*, but the other supposes them both within, and exhibits the same conclusion. But it may seem, perhaps, that the institutor of the Elements has inconveniently distributed the theorems. For it was necessary either to receive three hypotheses in a divided manner, and to make three theorems; or to collect all into one theorem, as Æneas Hierapolites does, who wrote a compendium of the Elements; or willing to divide them into two, to make an orderly division, and to assume the hypotheses separately, which contain equal angles, and separately that in which the angles are equal to two right. But in the present propositions, in one theorem he supposes the alternate angles equal, but in the other, the external to the internal, and the internal angles situated at the same parts equal to two right. What then is the cause of this division? Does he regard the equality of the angles to each other, or to two right, and on this account does not separate the proposed theorems from each other; or does he respect the angles being received at the same, or not at the same parts? For the preceding theorem does not respect angles at the same parts, since such as these are alternate: but the present regards such as are situated at the same parts, as is perspicuous from the proposition. But how the institutor of the Elements shews, that from the internal angles being equal to two right, the right lines are parallel, appears from his writings on this subject. Ptolemy, however, in the theorems in which he proposes to demonstrate that right lines produced from angles less than two right, coincide at the same parts, in which the angles less than two right are situated, shewing before all his theorems, that from the internal angles being equal to two right, the right lines are parallel, proves it in the following manner.



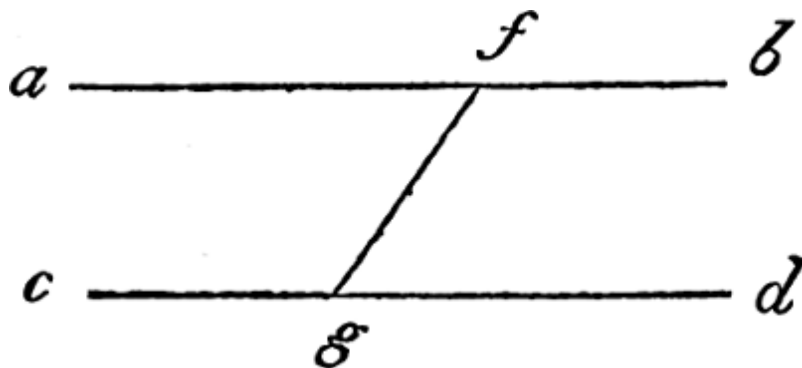
Let there be two right lines ab, cd , and let a certain right line $efgh$, so cut them, that it may make the angles bfg , and fgd , equal to two right, I say that those right lines are parallel, that is, will never coincide. For if it be possible, let them coincide while the right lines bf, gd , are produced in the point k . Because, therefore, the right line ef , stands upon the right line ab , it makes the angles afe, bfe , equal to two right. In like manner because fg stands upon cd , it makes the angles cgf, dgf , equal to two right. Hence, the four angles bfe, afe, cgf, dgf , are equal to four right, two of which bfg, fgd , are supposed equal to two right. The remainders, therefore, $afg, c g f$, are equal to two right. If then the right lines fb, gd , when produced, coincide, the internal angles being

equal to two right, $f a$, and $g c$, also, shall coincide when produced: for the angles $a f g$, $c g f$, are also equal to two right. Either therefore the right lines shall coincide in both parts, or in neither, since these, as well as the former, are equal to two right. Let the right lines then $f a$, $g c$, coincide in the point l . But if this be admitted two right lines $l a f k$, $l c g k$, will comprehend space, which is impossible. It is not therefore possible, that the internal angles being equal to two right, the right lines can coincide. They are therefore parallel.

PROPOSITION XXIX. THEOREM XX.

A right line falling upon parallel right lines, makes the alternate angles equal to each other; and the external equal to the internal angle, oppositely situated, and at the same parts; and the internal angles at the same parts equal to two right.

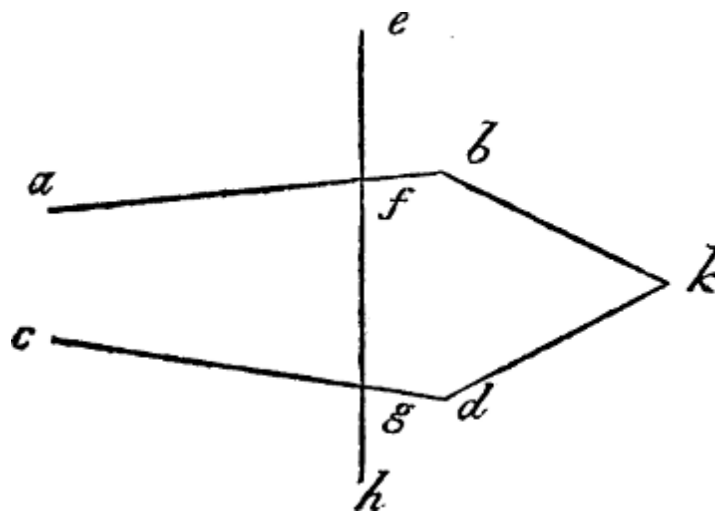
The present theorem is converted in both the preceding. For that which is *the object of investigation*,^[154] each of them, forms the hypothesis: but what are *data* in the preceding, he proposes to shew in the present. And this difference of converse theorems is not to be passed over in silence. I mean that every thing which is converted, is either converted as one to one, as the sixth proposition to the fifth; or as one to a many, as the present to the preceding; or as many to one, as will shortly be manifest^[28]. But in the present theorem, the institutor of the Elements first employs the petition, which says: *If a right line falling upon two right lines, makes the angles situated internally, and at the same parts less than two right, those right lines whilst they are infinitely produced, will coincide at those parts in which the angles less than two right are situated.* But in our exposition of things prior to theorems^[29], we have asserted, that this petition is not allowed by all to be indemonstrably evident. For how can this be the case when its converse is delivered among the theorems as demonstrable? For the theorem which says that the two internal angles of every triangle are less than two right, is the converse of this petition. Besides, the perpetual inclination of right lines, more and more, while they are produced, is not a certain sign of coincidence, because other lines are found perpetually inclining, and never coinciding, as we have already observed. Formerly, therefore, some, when they had pre-ordained this as a theorem, considered that which is assumed by the institutor of the Elements as a petition, to be worthy of demonstration. But this seems to be shewn by Ptolemy himself, in a book entitled: *That right lines which are produced from less than two right angles, coincide.* And this he proves by pre-assuming many things, which as far as to the present theorem, are already demonstrated by the elementary institutor; and he supposes that all are true (lest we should also superadd another confusion) and that this, as a small assumption, may be exhibited from the preceding. But this also is one of the things previously exhibited, which says, *that the right lines produced from two angles equal to two right, will never coincide.* I say, therefore, that the converse also is true, which says, *that right lines being parallel, if they are cut by one right line, the angles situated internally, and at the same parts, shall be equal to two right angles.*



For it is necessary that a line cutting parallels, should either make the angles internally situated, and towards the same parts, equal to two right, or less, or greater than two right. Let the lines then, $a b$, $c d$, be parallel, and let the right line, $g f$, fall upon them, I say that it will not make the angles internal, and at the same parts greater than two right. For if the angles $a f g$, $c g f$, are greater than two right, the remainders $b f g$, $d g f$, are less than two right. But the same are also greater than two right. For $a f$, and $c g$, are not more parallel than $f b$, and $g d$. Hence, if the line which falls upon $a f$, $c g$, makes the internal angles greater than two right, that also which falls upon $f b$, $g d$, will make the internal greater than two right. But they are also less than two right (since the four, $a f g$, $c g f$, $b f g$, $d g f$, are equal to four right) which is impossible. In like manner we may plainly shew, that the right line which falls upon parallels, does not make the angles internal, and at the same parts, less than two right. But if it makes

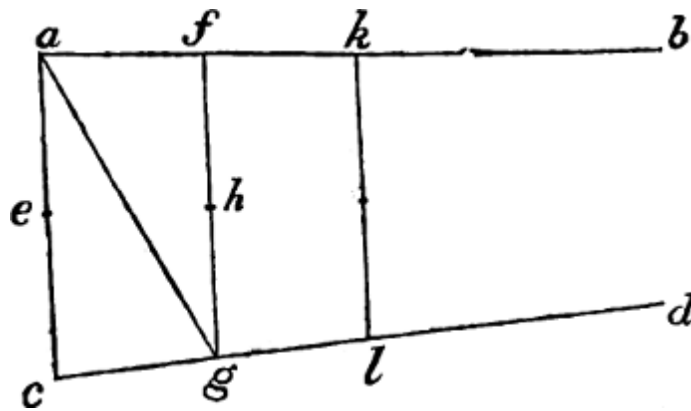
them neither greater nor less than two right, it remains that the incident line must make the angles internal, and at the same parts equal to two right. This then being previously shewn, the thing proposed, is doubtless demonstrated. For I say, that if a right line falling upon two right lines, makes the angles situated internally, and at the same parts, less than two right, if those right lines are produced they will coincide at those parts in which the angles less than two right are situated. For let them not coincide. But if they are non-coincident at those parts in which the angles less than two right are situated, much more will they be non-coincident at the other parts, in which the angles greater than two right are situated. Hence, the right lines will be non-coincident at both parts; and if this be true, they will be parallel. But it was shewn that the right line which falls on parallels, makes the angles internal and at the same parts equal to two right. The same, therefore, are both equal to, and less than two right, which is impossible.

Ptolemy having previously shewn this, and proceeding to the thing proposed, wishes to add something more accurate, and to shew that if a right line falling upon two right lines, makes the angles internal, and at the same parts, less than two right, the lines are not only coincident as has been shewn, but likewise that their coincidence takes place at those parts, in which the angles less than two right, and not at those in which the angles greater than two right are situated.

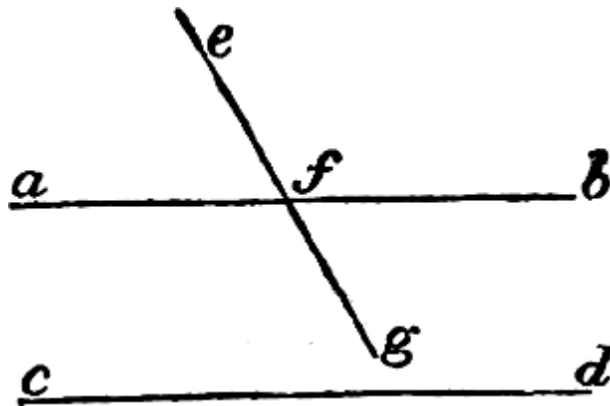


For let there be two right lines ab , cd , and let a right line $efgh$, falling upon them make the angles afg , and cgf , less than two right. The remainders, therefore, are greater than two right; and thus it is shewn that the right lines coincide. But if they coincide, they will either coincide at the points a and c , or at the points b and d . Let them coincide at the points b and d in the point k . Because, therefore, the angles afg , and cgf , are less than two right, but the angles afg , bfg , are equal to two right, by taking away the common angle afg , the angle cgf , will be less than the angle bfg . The external angle, therefore, of the triangle gfk , is less than the internal and opposite angle, which is impossible. Hence then, they do not coincide at these parts. But they do coincide; and consequently they will be coincident at the other parts, in which the angles less than two right are situated. And thus far Ptolemy.

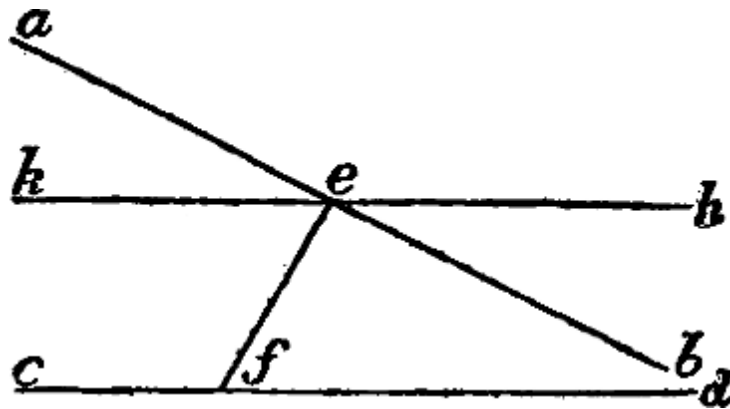
But it is necessary to scrutinize this demonstration, lest perhaps there should be any perverse and capricious reasoning in the assumed hypotheses, in those, I say, in which he affirms, that a right line cutting non-coincident right lines, by forming four internal angles, forms the angles at the same parts on each side, either equal to two right, or greater, or less than two right. For the division is not perfect; since nothing hinders our calling those lines non-coincident, which are produced from angles less than two right, denominating, indeed, the two angles at the same parts, greater than two right, but the two at the remaining parts less than two right and not admitting in these, one and the same proportion. But the division being imperfect, the thing proposed is by no means demonstrated. Besides this, also, is not to be passed over in silence against his demonstration, that he does not essentially shew that which is impossible. For it is not because a certain right line cutting parallels, makes the angles at the same parts on each side, greater or less than two right, that an absurdity on this account follows these hypotheses. Nevertheless, because the four angles within the lines which are cut, are equal to four right, on this account each of these hypotheses is impossible; since, if parallel right lines are not assumed, yet, when the same hypotheses are assumed, the same consequences will be the result. And such are our animadversions against the demonstration of Ptolemy: for the imbecility of his demonstration appears from what has been said.



Let us now consider those, who say it is impossible that lines produced from angles less than two right, should coincide. For when they have assumed two right lines ab, cd , and a right line ac , falling upon them, and making the two internal angles less than two right, they say it is possible that the right lines ab, cd , may be shewn to be non-coincident. For let ac be bisected in e , and cut off from ab , a part af , equal to ae : but from cd , a part cg , equal to ec . It is manifest, therefore, that the right lines af, cg , will not coincide in the points f and g . For if they coincide, these two in the triangle will be equal to ac , which is impossible. Let again fg be connected, and bisected in h , and cut off equal parts. These, therefore, will not coincide on the same account, and this will be the case, in infinitum, by connecting the non-coincident points; and bisecting the connecting line, and by cutting from the right lines, lines equal to the halves of the connecting lines; for by this means they say, that the right lines ab, cd , will never coincide. To such as these we reply, that they indeed affirm that which is true, but not so much as they imagine. For it is not true that the point of coincidence is simply determined by this means, nor is it true that the lines by no means coincide. Thus, when the angles bac , and dca , are determined, the lines ab , and cd , will not coincide in the points f and g , yet nothing hinders their coinciding in the points k and l , though fk and gl should be equal to fh , and hg . For when ak and cl coincide, the angles kfh , lgh , will not remain the same, and a certain part of the right line fg , will be left external to the right lines ak and cl ; and so again the two lines fk , and gl , are so much greater than the base, as the interior parts of the right line fg , which they intercept. Besides this also is to be said to such as affirm the non-coincidence of lines extended from angles less than two right, that they destroy what they are unwilling to destroy. For let the same description be given. Whether, therefore, is it possible, or impossible to connect a right line from the point a , to the point g ? For if it be impossible, besides destroying the fifth petition, they also destroy that which says, *that a right line may be drawn from every point to every point*: but if possible let it be connected. Because, therefore, the angles fag, gca , are less than two right, it is manifest that the angles also, gac, gca , are much less than two right. The right lines, therefore, ag, cg , will coincide in the point g , being produced from angles less than two right. Hence, it is not possible to affirm indeterminately, that lines produced from angles less than two right, will not coincide. It is however manifest, that some right lines produced from angles less than two right will coincide, though the present discourse seems to investigate this in all. For it may be said, that when the diminution of two right lines is indefinite, the lines will remain non-coincident according to such a diminution: but will coincide according to another less than this. But he who desires to behold a demonstration of this affair, must be informed that it is requisite for this purpose to pre-assume such an axiom as is employed by Aristotle^[30] in proving the world to be finite, viz. *If from one point two right lines forming an angle are produced in infinitum, the distance of the lines infinitely produced will exceed every finite magnitude*. For he shews that when infinite right lines are produced from the centre to the circumference, the interval also contained between them will be infinite: since, if it be only finite, it is possible that the distance may be increased; and on this account the right lines will not be infinite. Right lines, therefore, infinitely produced, are distant from each other by an interval greater than every finite magnitude.



This being pre-supposed, I say that if any right line cuts the one of parallel right lines, it will also cut the other. For let $a b$ and $c d$ be parallels, and let the right line $e f g$ cut $a b$. I say that it will also cut $c d$. For since there are two right lines, which are produced infinitely from the point f , viz. $b f$, and $f g$, they shall have a distance greater than every magnitude. Hence, they shall exceed the quantity of the interval contained between the parallel lines. Since, therefore, their distance from each other is greater than that of the parallels, $f g$ shall cut $c d$. But this being demonstrated, we can exhibit the thing proposed in a consequent order. For let there be two right lines $a b$, $c d$, and let a right line $e f$, fall upon them, making the angles $b e f$, $d f e$, less than two right.



I say that the right lines will coincide in those parts, in which the angles less than two right are situated. For since the angles $b e f$, $d f e$, are less than two right, let the angle $h e b$, be equal to the excess of two right angles above these angles, and produce $h e$, to the point k . Because, therefore, a right line $e f$, falls upon the right lines $h k$, $c d$, and makes the internal angles equal to two right, viz. the angles $h e f$, $d f e$, the right lines $h k$, $c d$, are parallel; and $a b$ cuts $h k$. It will therefore also cut $c d$, by the assumption previously exhibited. Hence, the right lines $a b$, $c d$, will coincide in those parts, in which the angles less than two right are situated. And on this account the thing proposed, is evinced^[31].

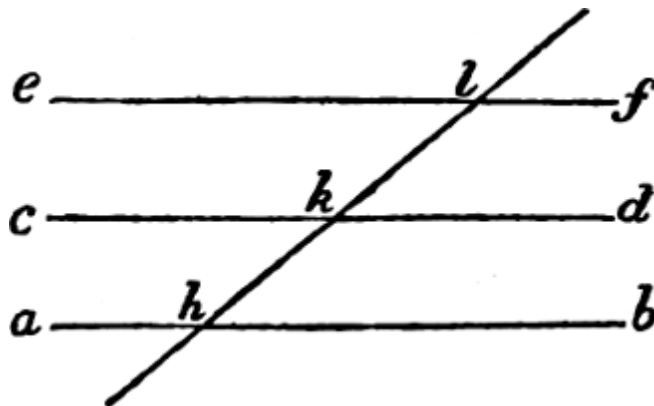
PROPOSITION XXX. THEOREM XXI.

Right lines parallel to the same right line, are parallel to each other.

The geometrician in these discourses which are conversant with relation, is accustomed to shew identity permeating through all quantities, having the same relation to the same. Thus among the axioms also he says, *things equal to the same, are equal to each other*: and in the following books he says, *things similar to the same, are similar to each other*, and *things having the same proportion to the same, have the same proportion to each other*. After this manner, therefore, he now also demonstrates, *that right lines parallel to the same, are parallel to each other*. But it happens that this is not true in all respects. For quantities double of the same, are not also double of each other: nor are those which are sesquialter of the same, sesquialter likewise to each other, but it appears to take place in those alone, which are univocally converted in equality, similitude, identity, and parallel position. For that which is parallel to a parallel, is itself also parallel. As that which is equal to an equal, is itself equal; and that which is similar to a similar, is itself similar. For the relation of parallels to each other, is

similitude of position. He affirms, therefore, and shews, in the present theorem, that lines parallel to the same, are entirely so related, that they are also parallel to each other. And he also exhibits the parallels with an external position, and likewise a medium, to which these have a similar relation, that what he asserts may become manifest from a common conception. For if they coincide with each other on either side, and coincide with that which is situated in the middle, they will no longer be parallel to it.

But it is possible that he who changes the position may shew the same thing, and by the same methods which the geometrician employs in exhibiting his proposition.



For instance, he assumes both $c d$, and $e f$, parallel to $a b$, both of them situated above, and $a b$ being beneath, and not in the middle. For a right line $h k l$, falling upon them, makes each of the angles $h k d$, $k l f$, equal to $a h k$, because they are alternate; and on this account it makes the angles $h k d$, $k l f$, equal to each other. The right lines, therefore, $c d$, $e f$, are parallel. But if any one should say that $a h$, $h b$, are parallel to $c d$, and are therefore parallel to each other, we reply that $a h$, $h b$, are parts of one parallel, and are not two parallel lines. For parallels are conceived to be infinitely produced, but $a h$, when produced, falls upon $h b$. It is therefore the same with $h b$, and not a different line. Hence, all the parts of a parallel, are parallel both to the right line, to which the whole was parallel, and to its parts. As for example, $a h$ is as well parallel to $k d$, as $h b$, to $c k$. For if they are infinitely produced, they will never coincide. And these remarks must be considered as not foreign from the purpose, both on account of sophistical importunities, and the juvenile habits of mathematical auditors. For the vulgar rejoice to find captious reasonings of this kind, and to procure vain molestation to the possessors of science. But it is not requisite to convert the present theorem, and to shew that lines parallel to each other, are also parallel to the same. For if we again suppose one line parallel to some other, the remainder also of these shall be parallel to it, and they will be parallel to the same, and we shall again return to the same proposition.

PROPOSITION XXXI. PROBLEM X.

Through a given point to draw a right line parallel to a given right line.

It is requisite that we should not only learn the essential accidents of parallels, in the discourses of the elementary institutor, but also that we should relate their origin, and know how one right line becomes parallel to another: for origin every where renders the essence of subjects more known to us. And this the institutor of the Elements effects by the present problem. For having received a point and a right line, he draws through that point a line parallel to a right line. But we ought to pre-assume as necessary, that the point should entirely be placed external to the right line: for we must not place it in the right line, because it is said, *through a given point*; since no other, besides the given line, can be that which is drawn parallel through the point. Since, therefore, the point and the right line is divided, it indicates that the point is to be received external to the right line, which he manifests in a perpendicular by addition, commanding, *upon a given infinite right line, and from a given point which is not in it, to let fall a perpendicular*. One thing, therefore, which is common to both these problems, is, the external position of the point: but the other, that from the same point two perpendiculars cannot be let fall to the same right line, and that through the same point, two lines cannot be drawn parallel to the same right line. Hence, the institutor of the Elements commands in the singular number *to draw a right line*, in the former problem, *a perpendicular*, but in the present *a parallel*. And, *that* indeed, has been shewn, but *this* is manifest, from what is previously demonstrated. For if through the same point two parallels are drawn to the same right line, they would be parallel to each other, and coincide in the given point, which is impossible. But it is requisite

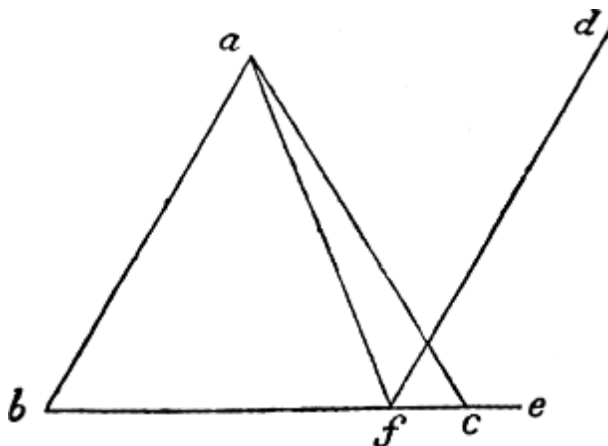
to observe the differences of these two propositions, *from a given point*, and *through a given point*. For sometimes the point is the beginning of the right line which is drawn, and on this account the deduction is made from it: but sometimes the point is in the drawn right line, and on this account the drawing is made through the point. For the particle *through*, was not asserted, because the right line cuts a given point, but because it coincides with it, and terminates its own interval, in respect of that right line, by the distance of the point and the right line. Since as much as the given point is distant from the given right line, so much also is the interval of the parallel between itself and the right line.

PROPOSITION XXXII. THEOREM XXII.

[163]

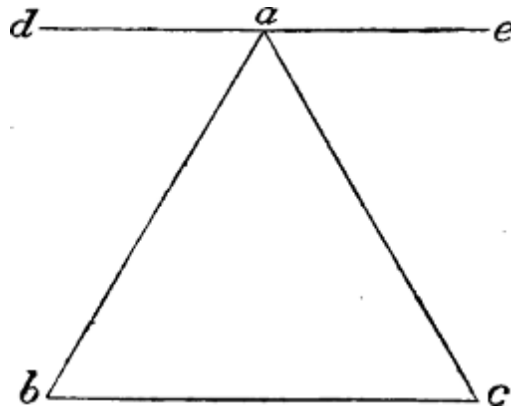
One side of every triangle being produced, the external angle of the triangle is equal to the two internal and opposite angles; and the three internal angles of a triangle are equal to two right angles.

As much as was deficient in the sixteenth and seventeenth theorem, so much Euclid adds in the present. For we not only learn by this theorem that the external angle of a triangle is greater than either of the internal and opposite angles, but likewise how much it is greater; since as it is equal to both, it is greater than either of the remaining angles. Nor do we alone know from this theorem, that any two angles of a triangle, are less than two right, but by how much they are less: for they are deficient by the remaining third. The former, therefore, were more indefinite theorems: but this brings with it, on both sides, a boundary to science. We must not, however, call them on his account superfluous: for they are of the greatest utility in many demonstrations; and the present is proved by their assistance. And besides this, it is necessary that our knowledge, proceeding from the imperfect to the perfect, should pass from indeterminate apprehensions, to determinate and certain propositions. But the institutor of the Elements, by drawing a parallel externally, exhibits each of the objects of investigation. It is, however, possible that the same thing may be shewn without drawing the parallel externally; and this, by only changing the order of the things exhibited. For Euclid first shews, that the external angle is equal to the internal and opposite, and from this he proves the remainder. But we shall demonstrate this by a contrary mode of proceeding.



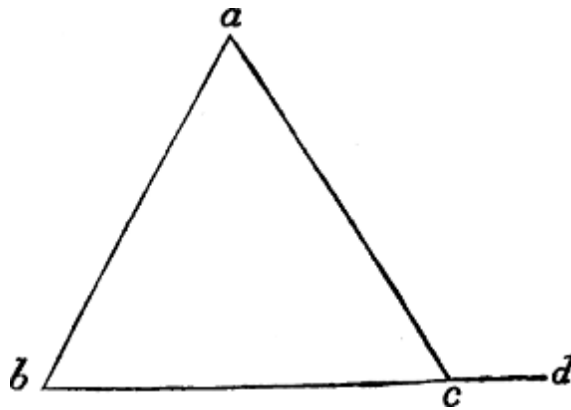
Let there be then a triangle abc , and let the side bc be produced to the point e . Then take a point f in bc , and connect af , and through the point f , let fd be drawn parallel to ab . Because, therefore, fd is parallel to ab , and a right line af , falls upon these parallels, as also a right line bc , hence, the alternate angles are equal, and the external is equal to the internal angle. The whole, therefore, afc , is equal to fab , added to abf . In like manner we may shew by drawing a parallel, that the angle afb , is equal to the angles fac , acf . The two, therefore, afb , afc , are equal to the three angles of the triangle: and hence, the three angles of a triangle are equal to two right, viz. to afb , added to afc . But acf , ace , are also equal to two right angles. Let, therefore, the common angle acf , be taken away; and then the remaining external angle will be equal to the internal and opposite angles. And after this manner may the present theorem be exhibited.

But Eudemus, the Peripatetic, ascribes the invention of this theorem to the Pythagoreans, I mean that every triangle has its internal angles equal to two right, and says that they demonstrate it in the following manner.

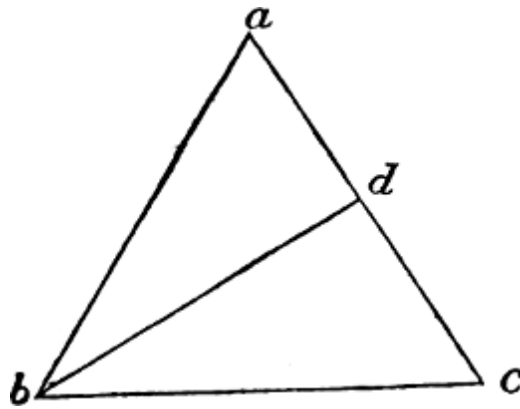


Let there be a triangle $a b c$, and let there be drawn through the point a , a line $d e$, parallel to $b c$. Because, therefore, the right lines $d e, b c$, are parallel, the alternate angles are equal. Hence, the angle $d a b$, is equal to the angle $a b c$; and the angle $e a c$, to the angle $a c b$. Let the common angle $b a c$, be added. The angles, therefore, $d a b, b a c, c a e$, that is, the angles $d a b, b a e$, and that is two right, are equal to the three angles of the triangle. And such is the demonstration of the Pythagoreans.

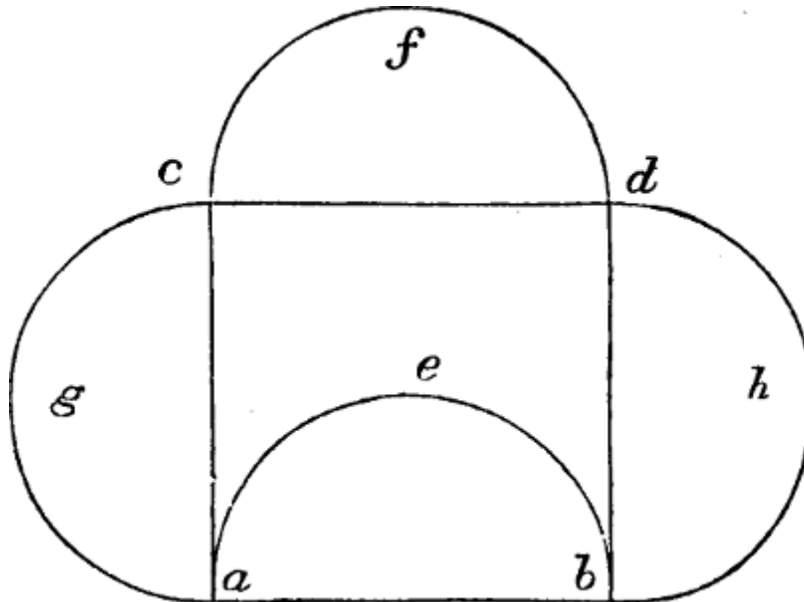
But it is here requisite to deliver such theorems as are converse to the present theorem of the elementary institutor. For two are converted to one, since this is a composite, both, according to *the object of enquiry*, and the *datum*: for the *datum* is two-fold, viz. the triangle, and one of its sides produced; and in like manner *the object of enquiry*. For one part says, that the external angle is equal to the internal and opposite angles: but the other, that the three internal angles are equal to two right. If therefore, we suppose that the external is equal to the internal and opposite angles, we may shew that one side is produced, and that the right line externally situated, is in a direct position with one of the sides of the triangle: but if the three internal angles are equal to two right, we may shew that the given figure is a triangle. And so *the whole object of enquiry*, is converse to *the whole datum*.



Let there be then a triangle $a b c$, and let the external angle $a c d$, be equal to the internal and opposite angles, I say that the side $b c$, is produced to the point d , and that $b c d$, is one right line. For since the angle $a c d$, is equal to the internal and opposite angles, let the common angle $a c b$ be added. The angles, therefore, $a c d, a c b$, are equal to the three angles of the triangle $a b c$. But the three angles of the triangle $a b c$, are equal to two right. And hence, the angles $a c d, a c b$, are equal to two right. But if two right lines being consequently placed, and not at the same parts to any right line, and at a point in it, make the successive angles equal to two right, those right lines shall be in a direct position to each other. The right line, therefore, $b c$, is in a direct position to $c d$.



Let there be again, a certain right lined figure abc , having three angles alone equal to two right, viz. a , b , and c , I say that the figure is a triangle, and that ac , is one right line. For let the right line bd be connected. Because, therefore, the three angles of each of the triangles abd , bdc , are equal to two right, of which the angles of the figure abc , are equal to two right, the remainders adb , $cd b$, are equal to two right, and they are placed about a right line bd . Hence dc , is in the same direction with da ; and so the side ac , is one right line. In like manner we may shew that the side ab , and the side bc , are each of them one right line. And consequently the figure abc , is a triangle. If then a figure having internal angles equal to two right, is right-lined, it is perfectly a triangle: but it does not follow that a figure is a triangle merely because it has internal angles equal to two right. For you will find a figure constructed from circumferences, having its internal angles equal to two right.



For let there be a quadrangle $abcd$, and upon the side ab , let a semicircle aeb , be internally described: but upon the other sides, let the semicircles be externally described, as f , g , h . The figure, therefore, which is comprehended by the semicircles, has two angles gae , ebh , equal to two right, viz. to cab , dba . For this was shewn in the petitions^[32], and these angles alone are in this figure. There is, therefore, a certain figure not a triangle, which has its internal angles equal to two right. And thus much may suffice concerning converse theorems.

But as we have discovered that the three angles of every triangle are equal to two right, we ought to determine a certain method, by which we may find how many angles, of all other multangles, are equal to so many right angles; as for instance, of a quadrangle, quinquangle, and of all consequent multilateral figures. In the first place, therefore, it must be known, that every right-lined figure may be resolved into triangles, since a triangle is the principle of the constitution of all things, which Plato also asserts in the *Timæus*, when he teaches us that the rectitude of a plane basis is composed from triangles. But every figure is resolved into triangles less in number, by the binary, than its proper sides. If a quadrilateral figure, into two triangles: if a figure of five sides, into three:

if of six sides, into four. For two triangles composed together, immediately form a quadrilateral figure. But the number of composite triangles by which the first constituted figure differs from its sides, is the measure of difference to the rest. Hence, every multilateral figure possesses more sides, by the binary, than the triangles into which it may be dissolved. But every triangle has been shewn to contain angles equal to two right. And hence, if the number of the angles be made double to that of the composite triangles, it will afford a multitude of right angles, to which the angles of every multangle will be equal. On this account every quadrilateral figure has angles equal to four right, since it is composed from two triangles: but every figure of five sides has angles equal to six right; and after the same manner of the rest in a consequent order. This one thing, therefore, is to be assumed from the present theorem, concerning all multangular and right-lined figures.

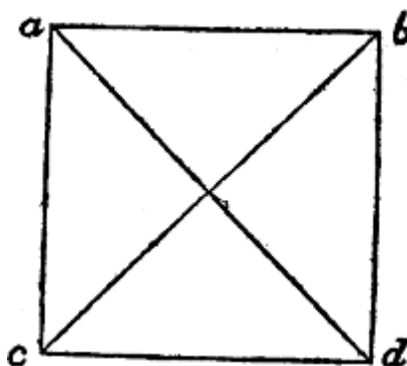
But there is another consequent to this, which is summarily as follows. In every right-lined figure, each of its sides being at the same time produced, the angles externally constituted are equal to four right. For it is requisite, indeed, that the successive right angles should be double of the multitude of the sides; because, in each they are constituted equal to two right. But the right angles equal to the internal angles being taken away, the remaining external angles are equal to four right. As for example, if the figure is a triangle, while every one of its sides is produced, at the same time internal and external angles are constructed equal to six right angles, of which the internal angles are equal to two right, but the remaining external angles to four right. But if the figure be quadrilateral, the angles are in all eight, since they are double of the sides, of which the internal are equal to four right, and the external to the four remaining angles, and the consequences will be similar in infinitum. But after these observations we may also collect, that by this theorem every angle of an equilateral triangle is two thirds of a right angle; but that an isosceles triangle, when the vertical angle is right, has each of its remaining angles the half of one right, as a semiquadrangle; and that a scalene triangle, when it is the half of an equilateral triangle, formed by a perpendicular drawn from any angle to its opposite side, has one angle right, but the other (which likewise belonged to the equilateral triangle) two thirds of a right angle, and the remainder by a necessary consequence, a third part of a right angle. For it is requisite that the three should be equal to two right. But I do not conceive that these remarks are foreign from our purpose, since they prepare us for the doctrine of Timæus. This also must be observed, that the possession of internal angles equal to two right is inherent essentially, and answering to the predication *according to what*, in a triangle. And on this account, Aristotle in his Treatise on Demonstration^[33], employs this as an example, considering it *according to what*. As therefore *to be terminated*, is essentially and primarily inherent in every figure, so likewise the possession of internal angles equal to two right, is essentially and primarily inherent in a triangle, though not in every figure. And the truth of this theorem seems to present itself to us according to common conceptions. For if we conceive a right line, and two right lines standing on its extremities, and inclining to each other, so as to form a triangle, we shall find that in proportion to their inclination they diminish the right angles, which they form with the right line. Hence, obtaining as much angular quantity, by their inclination at the vertex, as they take away from the base, they necessarily form three angles equal to two right.

PROPOSITION XXXIII. THEOREM XXIII.

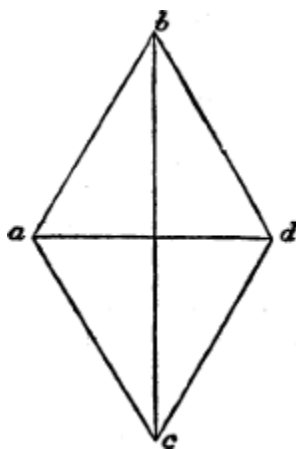
The right lines which join equal and parallel right lines at the same parts, are themselves also equal and parallel.

The present theorem is, as it were, the confine of the consideration of parallels and parallelograms. For it seems to declare a certain symptom of parallel right lines, and delivers the latent origin of parallelograms. For a parallelogram is formed, as well from those equal and parallel right lines, which are drawn in the beginning, as from those which conjoin them, and which are in like manner shewn to be equal and parallel. Hence, the proposition which immediately follows the present, contemplates the properties essentially inherent in these spaces, in a parallelogram as it were already constructed. And these things are indeed manifest. But it is requisite to consider the diligence which this preposition contains. In the first place, indeed, that it is not sufficient, that the lines which are conjoined should be equal: for the lines which connect equals, are not entirely equal, unless they are also parallel. For a triangle being isosceles, and a point being assumed in one of the equal sides, and through this a line being drawn parallel to the basis, equal lines shall indeed conjoin parallels to the basis, and the basis itself, yet these parallels shall not also be equal; and the sides will not be parallel, because they coincide at the vertex of the triangle.

In the second place, he considers that the subject right lines being parallel, is not sufficient to constitute the equality of the lines which conjoin them. For this is evident from the preceding construction of the isosceles triangle; since the drawn right line, and the basis, are parallel, and yet the lines which connect them are not parallel, because they are parts of the sides of the isosceles triangle. The parallel position, therefore, of the lines which are conjoined, is requisite to the equality of the connecting lines: but the equality of the latter is necessary to the parallel position of the former. On this account the institutor of the Elements assumes each, in those which are conjoined, for the purpose of exhibiting, that the connecting lines are as well equal, as parallel to one another. But in the third place, he intimates, that right lines being supposed both equal and parallel, their connecting lines will not be universally equal and parallel. For unless we make the conjunctions at the same parts as in this case, the connecting lines cannot be parallel (since they will cut each other), so they may be sometimes equal, and sometimes not.



For if you assume a quadrangle, or oblong, as $a b c d$, and connect the right lines $a d, b c$, the diameters are indeed equal, but not parallel, and they conjoin the equal and parallel opposite sides of the aforesaid spaces. But if the figure be a Rhombus, or a Rhomboides, the diameters of these, are not only non-parallels, but also unequal.



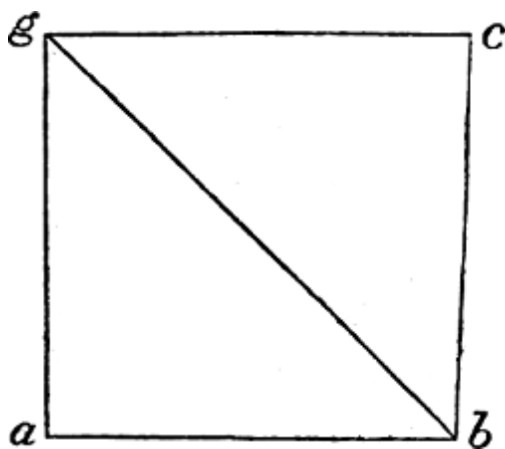
For since $a b$, is equal to $c d$, but $a c$ is common, and the angle $b a c$, is unequal to the angle $a c d$, the bases also are unequal. The institutor of the Elements, therefore, very properly considered, that the lines which conjoin equal and parallel lines, ought to make the conjunction at the same parts, lest $a c, b d$, being supposed equal and parallel, we should assume $a d, b c$, as the connecting lines, and not $a b$, and $c d$. For he shews that these latter are equal and parallel: but that the former are, indeed, never parallel, but equal, as we have observed in a quadrangle and oblong, but never in a rhombus and rhomboides; as the opposite to this has been proved to be true, because they are unequal, on account of the inequality of the angles internal, and situated at the same parts.

PROPOSITION XXXIV. THEOREM XXIV.

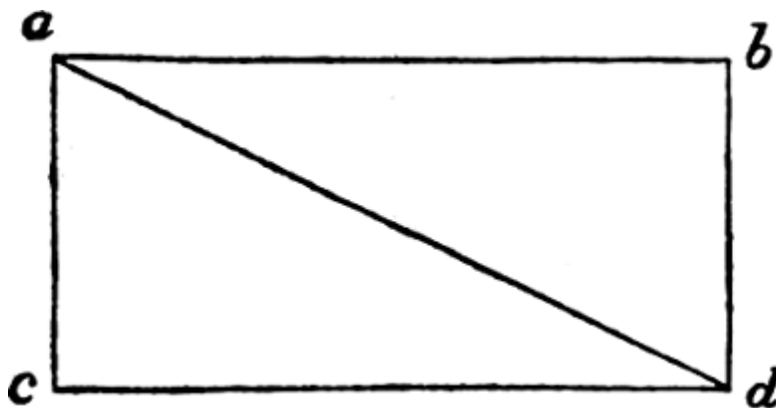
The opposite sides and angles of parallelogrammic spaces are equal to each other and they are bisected by the diameter.

As from the preceding theorem, he had assumed a parallelogram already constructed, he now contemplates its primarily inherent properties, and such things as express its peculiar constitution. But these are the following: *that*

the sides and angles which are opposite, are equal, and that the spaces themselves are bisected by the diameter. For that part of the proposition relates to the spaces, which says: *and they are bisected by the diameter*. So that the area itself, is that whole which is bisected, and not the angles through which the diameter passes. These three properties then, are essentially inherent in parallelograms, the equality of the opposite sides and angles, and the bisection of the spaces by the diameter. And you may observe that the properties of parallelograms are investigated from all these, viz. from the sides, from the angles, and from the areas. But as there are four kinds of parallelograms, which Euclid defines in the hypotheses^[34], viz. a quadrangle, oblong, rhombus, and rhomboides, it deserves to be remarked, that if we divide these four into rectangles, and non-rectangles, we shall find, that not only the diameters bisect these spaces, but that the diameters themselves, are, indeed, in rectangles equal, but in non-rectangles unequal, as was observed in the preceding theorem. But if we divide them into equilateral, and non-equilateral, we shall again find that in the equilateral figures, not only the spaces are bisected by the diameters, but likewise the angles through which they are drawn: but in non-equilaterals this is never the case. For in a quadrangle, and a rhombus, the diameters bisect the angles, and not the spaces only: but in an oblong, and a rhomboides, they alone bisect the spaces.



For let there be a quadrangle, or a rhombus, $g c a b$, and a diameter $g b$. Because, therefore, the sides $g c$, $c b$, are equal to the sides $g a$, $a b$ (for they are equilateral), and the angles $g c b$, $g a b$, are equal (for they are opposite), and the basis also is common, hence, all are equal to all; and on this account the angles $c g a$, $a b c$, are bisected.

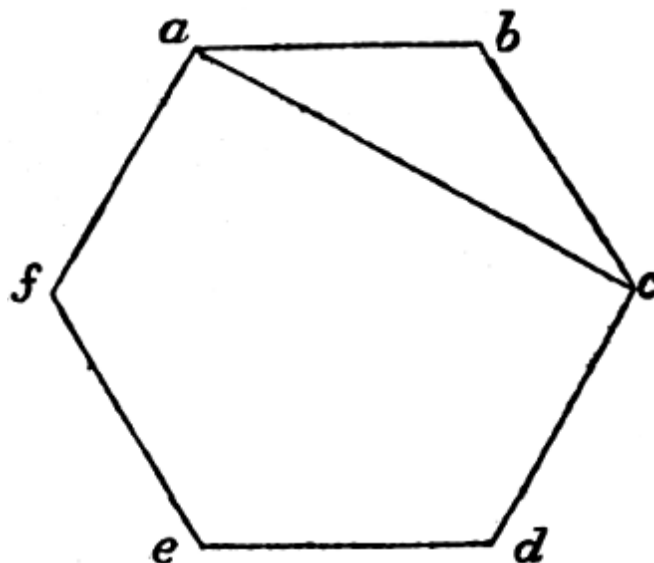


Again, let there be an oblong, or rhomboides given. If, therefore, the angle $b a c$, and the angle $c d b$, is bisected by the diameter, but the angle $c a d$, is equal to the angle $a d b$, the angle also, $b a d$, will be equal to the angle $a d c$. Hence, the side also, $a b$, will be equal to the side $b d$. But they are unequal; and consequently the angle $b a c$, is not bisected by the diameter, nor its equal the angle $c d b$. That I may therefore comprehend the whole in a few words, in a quadrangle the diameters are equal, on account of the rectitude of the angles, and the angles are bisected by the diameters, on account of the equality of the sides, and the areas are bisected by the diagonal, on account of the common property of parallelograms: but in an oblong, the diameters are indeed equal, because it is a rectangle, but the angles are not bisected by the diameters, because it is not equilateral, though the division of spaces into equal parts, is also inherent in this figure, so far as it is a parallelogram: but in a rhombus the diameters are unequal, because it is not a rectangle, but the spaces are not only bisected by these, because it is a

parallelogram, but the angles also, because it is equilateral; and in the remaining figure, i.e. a rhomboides, the diameters are unequal, because it is not a rectangle, and the angles are cut by these into unequal parts, because it is not equilateral, and the spaces alone situated at each part of the diagonals, are equal, because it is a parallelogram. And thus much concerning observations of this kind, which exhibit the diversity found in the four divisions of parallelograms.

But we must not pass over in silence, the artificial consequence appearing in this theorem, that of the 1761, some are universals, but others non-universals. But we shall speak concerning each of these, when we divide *the object of investigation*, which has, indeed, one part universal, but the other non-universal. For though every theorem may seem to be universal, and every thing exhibited by the elementary institutor may appear to be of this kind (as in the present he may not only seem to assert, that in all parallelograms universally, the opposite sides and angles are equal, but likewise that each is bisected by the diameter), yet we must say that some things are universally exhibited, but others not universally. For it is customary to call the *universal* which affirms the truth concerning every thing of which it is predicated, differently from *that universal*, comprehending all things in which the same symptom is inherent. Thus it is universal, that every isosceles triangle has three angles equal to two right, because it is true of all isosceles triangles: and it is universal that every triangle has three angles equal to two right, because it comprehends all things, in which this is essentially inherent. On which account we affirm that the possession of three angles equal to two right, is to be primarily manifested of a triangle. According to this signification, therefore, of theorems, calling some universal, but others non-universal, we must affirm that the present theorem, has, indeed, one of its objects of investigation universal, but the other non-universal. For the possession of opposite sides and angles that are equal, is a universal, since it is alone inherent in parallelograms: but that the diameter bisects the space, is not universal, because it does not comprehend all things in which this symptom is beheld; for this is inherent in a circle and ellipsis. And it appears, indeed, that primary conceptions of such like concerns, are more particular, but that in their progress they comprehend the whole. For when the ancients had contemplated that a diameter bisects an ellipsis, circle, and parallelogram, they afterwards surveyed that which was common in these. But we are deceived (says Aristotle^[35]) when a non-universal is exhibited as universal, because that common something in which the symptom is primarily inherent, is nameless. For we cannot say what that is, which is common to numbers and magnitudes, motions and sounds; and it is likewise difficult to express what is common to an ellipsis, circle, and parallelogram. For one of these figures is right-lined, but the other circular, and the third mixt; and on this account we conceive that he exhibits universally, who demonstrates that a diameter bisects every parallelogram, because we do not at the same time perceive that common something, on account of which, this is true. This then in parallelograms, is not an universal of this kind, on account of the aforesaid cause; but the proposition is universal, which asserts, that every parallelogram has its opposite sides and angles equal. For if any figure is supposed, having its opposite sides and angles equal, it may be shewn to be a parallelogram. Thus let such a figure be $a b c d$ ^[36], and its diameter $a d$. Because, therefore, the sides $a b$, $b d$, are equal to the sides $a c$, $c d$, and the angles comprehended by them are equal, and the base common, all will be equal to all. The angle, therefore, $b a d$, is equal to the angle $a d c$, and the angle $a d b$, to the angle $c a d$. Hence, $a b$, is parallel to $c d$, and $a c$ to $b d$. And on this account the figure $a b c d$, is a parallelogram. And thus much may suffice for observations of this kind.

But the institutor of the Elements seems to have composed the name of parallelograms, by taking an occasion from the preceding theorem. For when he had shewn that right lines, which conjoin equal and parallel right lines at the same parts, were themselves also equal and parallel, it is evident that he pronounces as well the opposite sides which conjoin, as those which are conjoined, to be parallel: but that he very properly calls the figure which is contained by parallels, a parallelogram, in the same manner as he denominates that which is comprehended by right lines rectilineal. And it is evident that the institutor of the Elements places a parallelogram among quadrilateral figures. But it is worthy our observation and enquiry, whether every right-lined figure, which is composed from equal sides, since it is equilateral and equiangular, is to be called a parallelogram. For a figure of this kind also, has its opposite sides equal and parallel, as likewise the opposite angles equal. As for example, a sexangle, and an octangle, and a decangle.



Thus, if you conceive a sexangle $a b c d e f$, and connect a right line $a c$, you may shew that $a f$ is parallel to $c d$. For the angle at the point b , is one right, and the third part of a right angle; and this is true of every angle of a sexangle, since it is equiangular. Besides the side $a b$, is equal to the side $b c$, for it is placed equilateral. Each of the angles, therefore, $b a c$, $b c a$, is a third part of a right angle. Hence, the angles $f a c$, $a c d$, are right angles. And on this account $a f$, is parallel to $c d$. In like manner we may shew that the other opposite sides are parallel, and the same may be evinced in an octangle, and in the remaining figures of this kind. If, therefore, that is a parallelogram which is comprehended by parallels oppositely situated, a parallelogram will likewise subsist among non-quadrilateral figures. But it appears that with the institutor of the Elements a parallelogram is quadrilateral. And this is particularly perspicuous in that theorem, in which he says, *that a parallelogram which has the same base with a triangle, and is between the same parallels, is double of the triangle*: for this is alone true in quadrilateral figures.

PROPOSITION XXXV. THEOREM XXV.

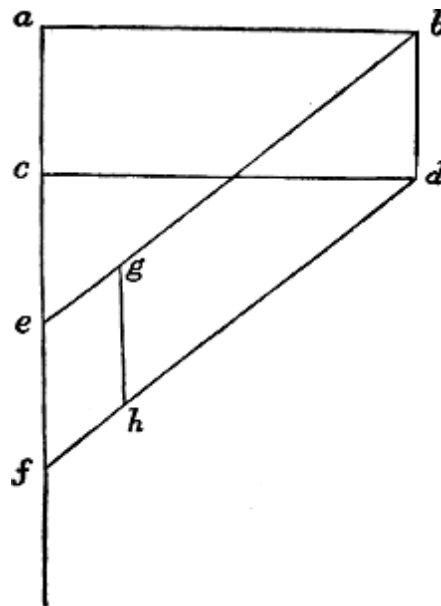
Parallelograms which are upon the same base, and between the same parallels, are equal to each other.

As we have said that of theorems, some are universal, but others particular, and as dividing these we have subjoined, that some are also simple, but others composite, and have shewn the nature of each, so according to another distinction, we assert that some of these are local, but others non-local. But I call those local, to which the same symptom happens in a certain place; and I denominate the place of a line or a superficies, that situation, which produces one and the same symptom. For of local theorems some are constructed in lines, but others in superficies. And because of lines, some are plane, but others solid, the plane being those of which there is a simple conception in a plane, as of a right line: but the solid those whose origin appears from a certain section of a solid figure, as of cylindric, spiric, and conic lines, I should say, that of the local theorems which are constructed in lines, some have a plane, but others a solid place. The present theorem, therefore, is both local, and local in lines, and a plane. For the whole space which lies between the parallels, is the place of the parallelograms constructed upon the same base; and which the institutor of the Elements shews to be equal to each other. But of those local theorems which are called solid, let the following be an example^[37]. *The parallelograms which are inscribed within the asymptotes and the hyperbola, are equal*: for it is evident that the hyperbola is a solid line.

But Chrysippus, as we are informed by Geminus, assimilates theorems of this kind to ideas. For he¹⁷⁸ ideas comprehend the origin of infinites, in terminated limits, so in these also there is a comprehension of infinites, in terminated places, and by this boundary equality appears, since the altitude of the parallels remaining the same, if infinite parallelograms are conceived upon the same base, they may all be shewn to be equal to each other. The present, therefore, is with the institutor of the Elements, the first local theorem. And he appears, when, agreeable to an elementary mode, he had distinguished theorems by a variety, according to all possible divisions, with great propriety not to have omitted, considering their idea of this kind. Nevertheless, as his discourse, for the present, is concerning right lines, he delivers local plane theorems in right lines: but in the third book, as he treats concerning things which may be contemplated of circles, and their symptoms, he likewise teaches the particulars,

which are constructed in circumferences belonging to local, and at the same time, plane theorems. And such, among these, is the theorem, which says, *that angles in the same segment, are equal to one another*. Also this which asserts, *that the angles in a semicircle are right*. For if infinite angles are constructed in a circumference, the same base remaining, they are all shewn to be equal; but if that which is comprehended by the base and the circumference, is a semicircle, they are all shewn to be right. And these, indeed, correspond in proportion to triangles and parallelograms upon the same base, and between the same parallels. And such is the species of theorems called local, by the ancient mathematicians.

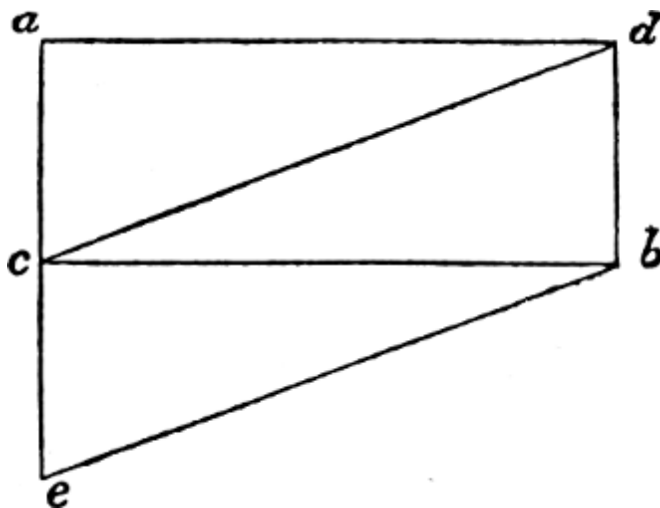
But perhaps it may seem perfectly worthy of admiration, to such as are unskilled in contemplation^[38] of this kind, that parallelograms constructed upon the same base, and between the same parallels, should be equal to each other. For it may be asked, how is this possible, since the longitude of the spaces, constructed on the same base, increases in infinitum? Since as much as we produce the parallels, by so much we may also increase the longitudes of the parallelograms. But some one may not improperly enquire how, while this takes place, the equality of the spaces remains. For if the breadth is the same (since the base is one), but the length is greater, will not the space also be greater? The present theorem, therefore, and that which follows concerning triangles, are among the number of mathematical theorems, which are denominated admirable. For mathematicians in theorems, as the Stoics in arguments, have established a *place*, which is called admirable, and they place the present among theorems of this kind. The vulgar, therefore, are immediately astonished, when they hear that the multiplication of length does not destroy the equality of spaces on the same base. We must nevertheless assert, that equality and inequality possess the greatest power in increasing or diminishing the spaces of angles. For in proportion as we make angles unequal, in such proportion we diminish the space, if the length and breadth remain the same. Hence, the increase of length is necessary, that we may preserve equality. Thus, for example, let there be a parallelogram $a b c d$, and let the side $a c$ be produced in infinitum, and let it be a right-angled parallelogram; and lastly, on the base $b d$, construct another parallelogram $b e f d$.



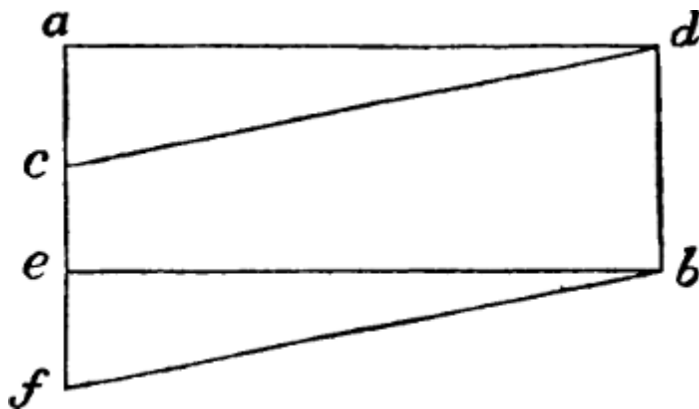
That the length, therefore, is increased is evident: for the side $b e$, is greater than the side $a b$, since the angle at the point a , is right. But this necessarily takes place, as the angles of the parallelogram $b e f d$, are unequal, and some of them are acute, but others obtuse: and this happens, because the side $b e$, approaches after a manner to the side $b d$, and contracts the space. For let $b g$ be taken equal to $a b$, and through g , draw $g h$ parallel to $b d$. The length, therefore, of the parallelogram $b d g h$, is equal to the length of the parallelogram $a b c d$, and the breadth is the same, and yet one space is less than the other; for it is less than $b e f d$. Hence, the inequality of angles diminishes the area, but the increment of length adding as much as the inequality of angles takes away, preserves the equality of the spaces. But the boundary of the increase of length, is the place of the parallel lines. For when both the parallelograms are rectangular, and have an equal ambit, the quadrangle is shewn to be greater than the oblong^[38]: but when they are both equilateral, and have consequently an equal ambit, that which is rectangular, is shewn to be greater than that which is non-rectangular^[39]. For the rectitude of angles, and the equality of sides, possesses universal power in the augmentation of spaces. It is on this account that a quadrangle is the greatest of

all figures with an equal ambit, and a rhomboides the least. And these observations we shall demonstrate in another place^[40]: for they more properly belong to the hypotheses of the second book.

But with respect to the present theorem, it is requisite to know, that when Euclid calls parallelograms equal, he means the spaces, and not the sides: for he now discourses of areas. And we must likewise observe, that he first mentions trapeziums in the demonstration of this theorem: from whence also it is manifest, that he does not improperly teach us concerning a trapezium, in the definitions, when he informs us that it is indeed of a quadrilateral species, but is not a parallelogram. For the figure which has not its opposite sides and angles equal, falls from the order of parallelograms. The institutor of the Elements, therefore, as he had chosen a more difficult case, demonstrates the thing proposed. But if any one should say, let the parallelograms $a b c d$, and $b d c e$, be upon the same base $d b$, so that the side $c d$ may be the diameter of the parallelogram $a b$, we can shew that according to this position they are equal. For the triangle $b c d$, is the half of each parallelogram: because $c d$ is the diameter of $a b$, but $c b$ of $d e$; and diameters bisect parallelograms.



Hence, $a b$ is equal to the parallelogram $d e$. Again, if any one should suppose that the side $a c$, of the parallelogram $a b$, is cut by the side $d c$, and that the parallelograms are situated as $a d b e$, $b d c f$, we can shew that these also are equal.

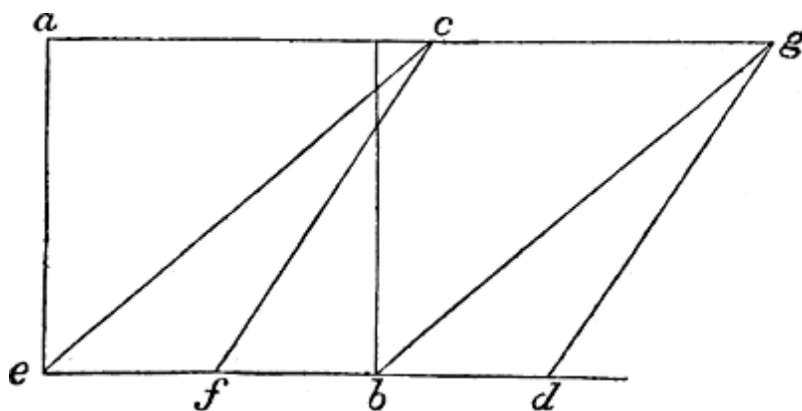


For since the side $a e$, is equal to the side $c f$ (each because opposite being equal to $d b$), let the common right line $c e$ be taken away. Hence, $a c$ is equal to $e f$. But $a d$, also, is equal to $e b$, and the angle $c a d$, to the angle $f e b$. For $a d$ is parallel to $e b$; and hence, the base $c d$, is equal to the base $f b$, and the whole triangle $a d c$, is equal to the whole triangle $e b f$. Let the common trapezium $c b$, be added. The whole, therefore, $a b$, is not unequal to the whole $d f$. And here you may observe that these are the only three cases. For the side $d c$, either cuts the side $e b$, according to the position of the elementary institutor; or it falls on the point c , as in the penultimate description: or it cuts the line $a e$, according to the present supposition. And thus the theorem is shewn to be true according to all its cases. Lastly, as there is a two-fold difference of trapeziums, and one kind has neither of its opposite sides parallel, but the other has one side parallel to one, this latter species of trapeziums is alone employed by the geometrician throughout the elements, and in the present description: for $c e$ is parallel to $d b$.

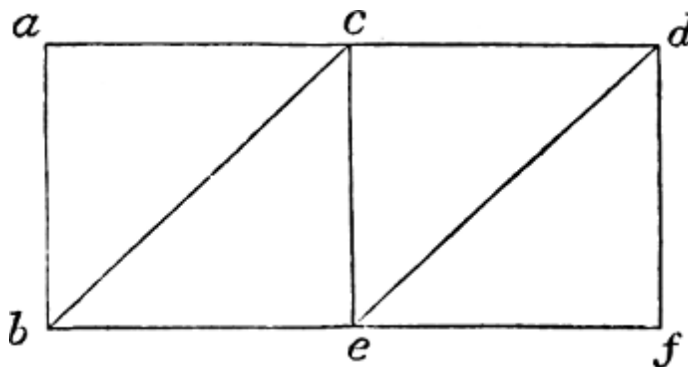
PROPOSITION XXXVI. THEOREM XXVI.

Parallelograms which are upon equal bases, and between the same parallels, are equal to each other.

The preceding theorem assumed, indeed, the same bases, but this receives them equal, and different from each other. But it is common to both, to suppose the parallelograms between the same parallels. It is requisite, therefore, that they should neither fall within, nor without their subject parallel lines. For parallelograms are said to be between the same parallels, when their bases and opposite sides are adapted to the same parallels. As to the rest, the institutor of the Elements, as he had assumed the bases entirely separate, exhibits the theorem. But nothing hinders our receiving them with this hypothesis, so that they may have a common part. For let $a b, c d$, be parallelograms upon equal bases $e b, f d$, having a common part, and constructed between the same parallels, I say that they are equal.



Let the lines $e c, b g$, be connected. Because, therefore, $e f$, is equal to $b d$ (for the base $e b$, was supposed equal to the base $f d$), but the side $c f$, is equal to the side $d g$, and the angle $c f e$, is equal to the angle $g d b$, and hence, $c e$ is equal to $b g$. But it is also parallel to it. Hence, $c b$ is a parallelogram, and has the same base with each of the parallelograms $a b, c d$, and is between the same parallels. The parallelogram, therefore, $a b$, is equal to the parallelogram $c d$.



But if any one should suppose that the bases of the parallelograms have neither a common part, [185] are separate from each other, but (which is the only remaining hypothesis) that they touch each other in one point, as in the parallelograms $a e, e d$, we must say that the base $b e$ is equal to the base $e f$, and to the side $c d$. Hence, also, the right line $c b$, is equal to the right line $d e$, and is parallel to it. For the lines which join equal and parallel lines, are themselves also equal and parallel. Hence, $b d$ is a parallelogram, and is upon the same base, and between the same parallels, with the parallelograms $c b, d e$. The parallelograms, therefore, $c b, d e$, are equal. But according to the first conception of a theorem, we may divide the constructions by asserting that the bases have either a common part, or touch each other, or are distant from each other. It is however possible, that though they may touch each other, as $b e, e f$, yet the whole parallelogram $d e$ may be supposed external to the side $c e$; or one side of the parallelogram $c f$, may be the diameter of the parallelogram $a e$; or the side $c e$, may cut the side $a c$; or the side $a c$, being produced beyond a , the side $c e$, may fall as the diameter of the parallelogram increased towards a , when the side $d f$ becomes the same as a line drawn from a to f , or the side $c e$, may cut the side $a c$, produced beyond a ; or the side $a c$, may be still farther produced beyond a , so that the side $c e$ may fall beyond

the point, to which $a c$ was extended in the preceding case, and the side $d f$, may cut the line produced beyond a [41]. * * *

PROPOSITION XXXVII. THEOREM XXVII.

Triangles which are upon the same base, and between the same parallels, are equal to each other.

The beginning of this Commentary is wanting.

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* * for those being equal, the spaces are unequal; and when these are unequal, those are shewn to be equal. And this is the case with Chorographers, when they reason concerning the magnitudes of cities, from their ambits. But formerly, certain persons deceived their partners, in the distribution of their possessions, deluding them by an excess of ambit, so as to make them believe that they received a greater portion of land, when they received a greater ambit; and that they were gainers, by changing spaces into areas of less ambit. [42] Thus two isosceles triangles being proposed, one of which has each of its equal sides, containing five parts, but the base six: and the other has each of its equal sides five parts, but the base eight; and let these parts be, for instance, cubits, or digits, these triangles will very much deceive the ignorant in their choice. For the ambit of the one is eighteen, and of the other sixteen measures. But a geometrician is not ignorant that the spaces are equal, though the ambits are unequal; since the area of each is twelve measures. For if you draw a perpendicular from the vertex, you will bisect the bases, and cause the half of the one to be three but of the other four measures: but the perpendicular on the contrary, will be there equal to four, but here equal to three; since it is requisite that the square from the quinary, should be equal to the squares from the perpendicular, and the half of the base. But if the base of the one is equal to three, the perpendicular must be four; and if the base of the other is equal to four, its perpendicular must be three. When, therefore, you have multiplied the half of the base with the perpendicular, you will have a space equal to the triangle: but this is the same in each, whether you multiply the quaternary with the ternary, or the ternary with the quaternary. And we have made these observations for the purpose of shewing that the equality of spaces is not to be entirely received from the ambits. Nor should we wonder, that though triangles upon the same base, may be infinitely increased between the same parallels, according to the remaining sides, yet the equality of the spaces immutably remains. But those triangles are said to be between the same parallels, which have their bases upon one of the parallel lines, and fix their vertices on the remainder; and whose vertices being connected, form one right line, parallel to the bases on the same right line.

PROPOSITION XXXVIII. THEOREM XXVII.

Triangles which are upon equal bases, and between the same parallels, are equal to each other.

The present theorem also is local, because it corresponds in proportion with parallelograms, and supposes the situation of triangles upon equal bases. But Euclid seems, to me, to have delivered one demonstration by the first proposition of the sixth book of these four theorems, two of which are exhibited in parallelograms, and two in triangles: and two of which are on the same base, and the other two on equal bases. But that Euclid has performed this is unknown to the vulgar. For after he had shewn that triangles and parallelograms, which are under the same altitude, have the same proportion to each other as their bases, nothing demonstrates all these four theorems more universally, from proportion, than this theorem: since to possess the same altitude, is nothing else than being constituted between the same parallels. For all figures between the same parallels, are under the same altitude, and the contrary: since the altitude is the perpendicular, which extends itself from one parallel to the rest. In that proposition, therefore, it is shewn by proportion, that triangles and parallelograms, under the same altitude, that is, situated between the same parallels, are to each other as their bases, and so when the bases are equal, the spaces are equal; and when those are double, these will be double; and when the bases have any other proportion, the spaces also will have to each other the same proportion. But for the present, because it is not proper that he should use proportion, who has not yet explained its nature, he is content with equality and identity alone: for the identity of bases is collected from equality. Hence, these four theorems are comprehended in that one; not only because he shews by one demonstration, whatever are contained in these four, but likewise, because he adds what was wanting to their perfection, viz. identity of proportion, though the bases are unequal. But that this theorem, also, has many cases, and that it is possible that the bases of the triangles may be assumed, either having the same part as in parallelograms; or possessing no common part, but touching each other according to one point; or

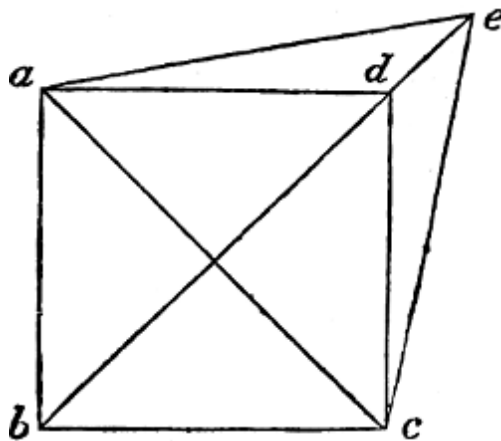
entirely separate, so that a line may intervene between them, is manifest, even to such as are endued with slender capacities. And this too is evident, that according to all cases, however the bases or vertices may be situated, the same method of proceeding must be adopted as in parallelograms; viz. parallels to the sides must be drawn, and produced both ways, and the equality of the triangles exhibited.

PROPOSITION XXXIX. THEOREM XXXIX.

Equal triangles, which are upon the same base, and at the same parts, are between the same parallels.^[188]

When it was proposed to exhibit equality to us, then it was requisite to make four theorems, receiving two in parallelograms, but the other two in triangles, situated either upon the same, or upon equal bases. But now by conversion, we neglect the theorems which are converse in parallelograms, and esteem such as are converse in triangles worthy of relation. And the reason of this is, because the mode of demonstration in parallelograms, is the same indifferently, by a deduction to an impossibility, and the construction is similar. But we are content when we have exhibited the way in more simple figures, I mean triangles, to leave to the more curious the same mode of reasoning in the rest: since it is easy, at the same time, to perceive that there is the same method in these. For when we assume equal parallelograms, upon the same base, or upon equal bases, we must say that they are also between the same parallels. For if they are not, either one of them falls within, when the parallels which are in the other are produced; or without. But which ever case is assumed, when we receive it and its parallels, we may exhibit the same consequences as in triangles, I mean that the whole will be equal to its part: but this is impossible. It is however manifest, that the institutor of the Elements very properly adds the particle, *and at the same parts*. For it is possible that equal triangles, may be assumed upon the same base, one, indeed, at these parts, but the other at different parts, and yet these will not be entirely between the same parallels: for neither will they be contained under the same altitude. And on this account he added the particle.

But since a parallel may be drawn in a two-fold respect, according to an absurd hypothesis, i.e. either^[189] within or without, Euclid draws it within: but we can exhibit the same consequences, by drawing it without. For let the equal triangles abc , dbc , be upon base, and at the same parts, I say that they are between the same parallels, and that the right line connected at their vertices, is parallel to the base. Let the right line ad be connected. But if this is not parallel, let the line, external to this, i.e. ae be parallel, and let bd be produced to the point e , and connect ec .



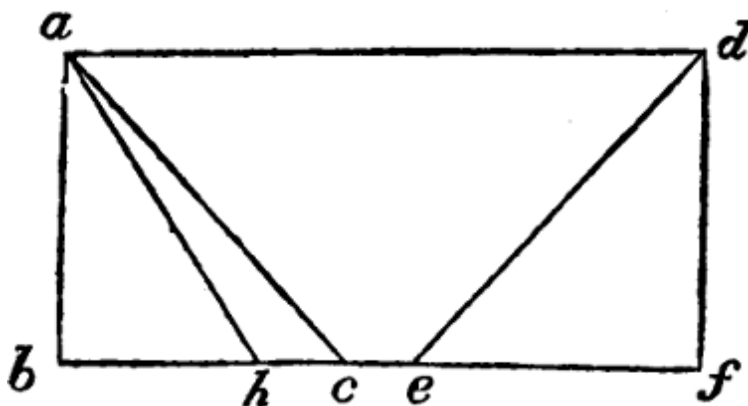
The triangle, therefore, abc , is equal to the triangle ebc , the whole to the part. But this is impossible; and hence, the parallel line does not fall external to ad . But it is shewn by the institutor of the Elements, that neither does it fall within: and hence ad , is parallel to bc . Hence too, equal triangles, which are at the same parts, and upon the same base are parallel to each other. And thus the remaining part of the deduction to an impossibility is demonstrated. But it is worthy of observation, that since the conversion of theorems is triple (for either the whole is converted to the whole, as we have noticed, in the eighteenth and nineteenth theorems; or the whole to the part, as the sixth and fifth; or the part to the part, as the eighth and the fourth: for the whole is not a *datum*, in the one, and an *object of investigation* in the other: nor is the *object of investigation*, a *datum*, but a part) these triangular theorems appear to be of this kind. For, that the triangles are equal, is an object of investigation in the preceding; but this is not a datum alone in these, because it assumes, besides this, a part of that which was hypothesis in those. For to stand upon the same, or upon equal bases, is a datum in these, as well as in those, except that in

these hypotheses he adds something which was neither an object of investigation, nor a datum in these; since the particle *at the same parts*, is over and above extrinsically assumed.

PROPOSITION XL. THEOREM XXX.

Equal triangles which are upon equal bases, and at the same parts, are between the same parallels.

There is the same mode of conversion too in the present theorem, and a similar demonstration; and ~~that~~ part of the deduction to an impossibility, which is omitted by the institutor of the Elements, is demonstrated after the same manner, and there is no occasion for repetition. But since these three conditions are in the aforesaid propositions, *situation upon equal, or on the same bases; position between the same parallels; and equality of triangles and parallelograms*, it is manifest that we may variously convert, by always connecting two, and leaving one. For we either supposed the bases the same, or equal, and triangles and parallelograms between the same parallels, and thus we form four theorems; or we consider the triangles and parallelograms equal, and the bases the same, or equal, and thus we produce another four, two of which the elementary institutor omits, viz. those which respect parallelograms, but the other two relative to triangles, he exhibits; or lastly, when we have assumed them equal, and between the same parallels, we prove the remainder, that they are either upon the same, or upon equal bases, and produce another four, which the institutor of the Elements entirely neglects. For there is the same demonstration in these, except that two of these four are not essentially true. Thus, equal parallelograms or triangles, between the same parallels, are not necessarily upon the same base: but all this is true in these hypotheses, that they are upon the same or equal bases; but the other does not entirely follow the assumed hypotheses. Hence, as all these theorems are ten, the geometrician speaks of six, and neglects four, lest he should labour in vain, by repetition, since the demonstration is the same. For it may be shewn in triangles, that if they are equal, and between the same parallels, they will either be upon the same, or upon equal bases. For let it be denied, and if possible, let the triangles abc , def , have these conditions, upon unequal bases bc , ef .



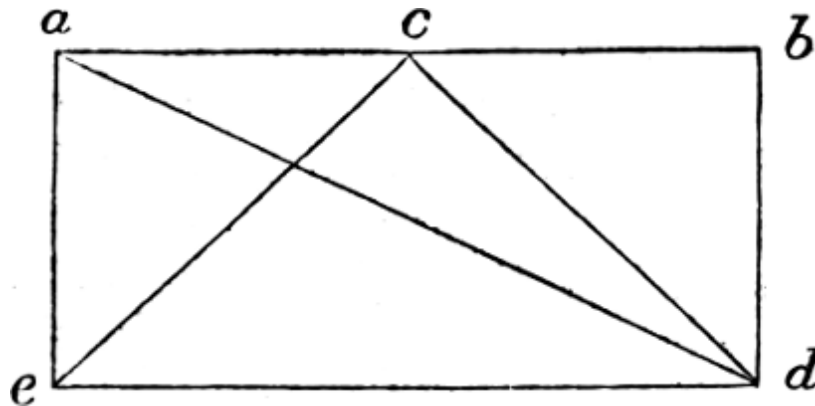
Let too, bc , be the greater, and cut off bh , equal to ef , and connect ah . Because, therefore, the triangles abh , def , are upon equal bases, bh , ef , and between the same parallels, they are equal. But the triangles also, abc , def , are supposed equal. Hence, the triangles abc , abh , are equal, which is impossible. The bases, therefore, of the triangles abc , def , are not unequal. And the mode of demonstration will be the same in parallelograms. Since, therefore, the ostensive method is the same, and the impossibility the same, viz. that the whole is equal to its parts, it is not improperly omitted by the elementary institutor. And thus we have shewn, that there are necessarily ten theorems, and have enumerated what are omitted, and shewn the reason of their omission. But let us now pass to the following propositions.

PROPOSITION XLI. THEOREM XXXI.

If a parallelogram has the same base with a triangle, and is between the same parallels, the parallelogram shall be double the triangle.

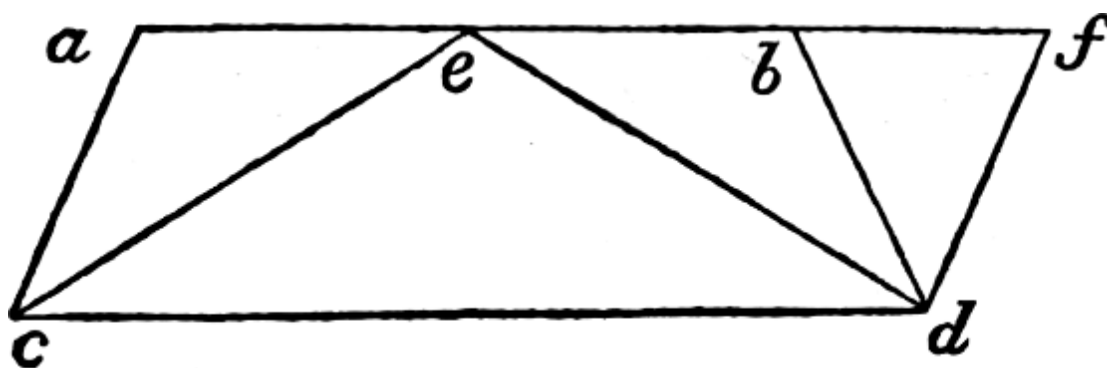
The present theorem also is local, but it mingles the constructions of triangles and parallelograms, ~~and~~ related under the same altitude. As, therefore, we have separately surveyed parallelograms and triangles, so when we assume each of them in conjunction, and with the same condition, we contemplate their proportion to each other. In the former, therefore, an equality of proportion is apparent, since all upon the same bases, and between the

same parallels, have a mutual equality, whether they are triangles, or parallelograms. But in these latter, the first of unequal proportions, I mean the duple, is exhibited: for he demonstrates that a parallelogram is double of a triangle, on the same base, and possessing the same altitude. But the elementary institutor shews the thing proposed, by supposing the vertex of the triangle external to the parallelogram. We can, however, demonstrate the consequence, by assuming the line which is parallel to their common base, in the other side of the parallelogram: for these are two cases of the theorem. Since in consequence of the two having the same base, it is necessary that the vertex of the triangle should either be within, or without the parallelogram. Let there be, therefore, a parallelogram $a b e d$, and a triangle $e c d$, and let a point c be placed between the points a and b , and connect the right line $a d$.

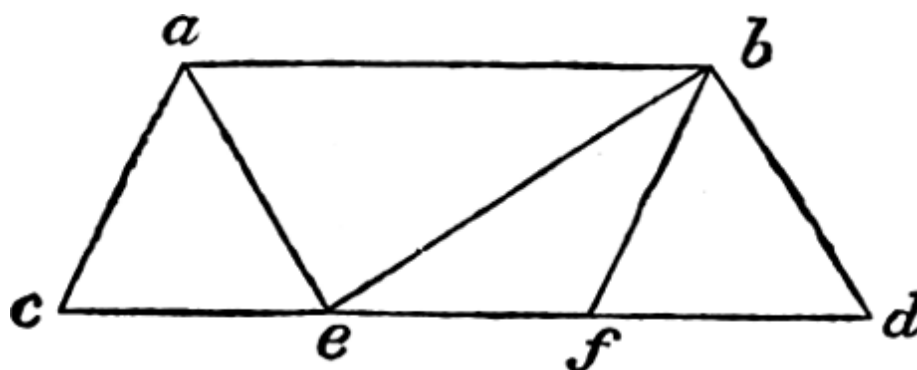


Because, therefore, the parallelogram is double of the triangle $e c d$, but the triangle $a d c$, is equal to the triangle $e d c$, hence, the parallelogram is double of the triangle $e c d$. And hence it is evident that a parallelogram is double of a triangle on the same base. But if the bases are equal, we can shew the same by drawing the diameters of the parallelograms: for if the triangles are equal, the parallelogram which is double of the one, will also be double of the other. But triangles are equal, on account of the equality of bases, and the identity of altitude. The geometrician, therefore, very properly omits this, for the demonstration is the same: since they will either have the same part, or they will be conjoined in one point only, or they will be separate from each other. But in whatever manner they may receive this variety, there is one demonstration according to all the cases.

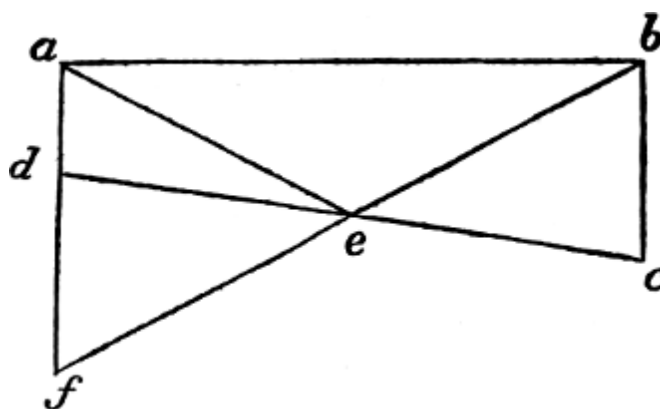
We can likewise demonstrate the converse propositions to this theorem, after the same manner. One of which is: *If a parallelogram is double of a triangle, and they have the same or equal bases, and are at the same parts, they shall be between the same parallels.* For if they are not the whole shall be equal to the part, and the same proportion shall prevail: since it is necessary that the vertex of the triangle should either fall within, or external to the parallels. But in either case, the same impossibility will be the result, by drawing a parallel to the base, through the vertex of the triangle. But the second converse theorem is: *If a parallelogram is double of a triangle, and both are between the same parallels, they will either be situated upon one base, or upon equal bases.* For if they are upon unequal bases, since we have assumed the figures to be equal, we may shew that the whole will be equal to its part. Hence, all these theorems end in this common impossible: and on this account, the institutor of the Elements leaves us to investigate the variety they contain, as he himself, has contracted his speculation to such as are more simple, and of a more primary nature. However, as we have recognized these observations, let us see for the sake of exercise, by not assuming a parallelogram, but a trapezium, two of whose sides only are parallel (because it has the same base with the triangle, while it is situated between the same parallels), let us, I say, consider what proportion it possesses to the triangle. That it has not, therefore, a duple proportion is evident: for if it had a duple ratio, it would be a parallelogram, since it is a quadrilateral figure. But I say that it is either greater than double or less; for since the two sides are parallel, one is greater, but the other less; because if equal, the sides conjoining them will be parallel. If, therefore, the triangle has its greater side for the base, the quadrilateral figure will be less than double of the triangle: but if the lesser side, it will be more than double. For let $a b c d$, be a quadrilateral figure, and let the side $a b$, be less than the side $c d$, and produce the side $a b$, in infinitum, and let the triangle $e c d$ have the same base with the quadrilateral figure, that is $c d$; and lastly, through d , draw $d f$, parallel to $a c$.



Hence, the parallelogram $a c d f$, is double of the triangle $e c d$; and so the quadrilateral figure $a b c d$, is less than double of the triangle.



Again, let the triangle have the base $a b$, and draw $b f$, parallel to $a c$. The parallelogram, therefore, $a b f c$, is double of the triangle. And hence, the quadrilateral figure, $a b c d$, is more than double of the triangle. This being shewn; we affirm, that when there is a quadrilateral figure, whose two opposite sides only, are parallel, if one of the parallel sides is bisected, and right lines are drawn from it to the other side, the quadrilateral figure, is either more or less than double of the triangle resulting from such a construction. But if one of the sides by which the parallel lines are conjoined, is bisected, and certain right lines are drawn from it to the remaining side, the quadrilateral figure, will be perfectly double of the triangle which is produced. And this may be shewn as follows. Let there be a quadrilateral figure $a b c d$, and let the side $a d$, be parallel to the side $b c$, and bisect $d c$, in the point e , and connect the right lines $a e$, $e b$, and produce $b e$, till it coincides with $a d$, in some point, as f . Because, therefore, the angles at the point e , are equal, for they are vertical; likewise, because the angle $f d e$, is equal to the angle $b c e$, the side also $f e$, will be equal to the side $e b$, and the triangle $d e f$, will be equal to the triangle $b c e$. Let the common triangle $a d e$, be added.



The whole triangle, therefore, $a e f$, is equal to the two triangles $a d e$, $b c e$. But the triangle $a e f$, is equal to the triangle $a e b$: for they are upon equal bases, $b e$, $e f$, and between the same parallels, if a line parallel to $b f$, is drawn^[43]. Hence, the triangle $a e b$, is equal to the triangles $a d e$, $b c e$, and the quadrilateral figure $a b c d$, is

double of the triangle $a e b$, which was to be shewn. After the same manner, we may shew, that if the side $a b$, is bisected, and certain right lines are drawn from it, to the side $e d$, the quadrilateral figure will be double of the triangle formed by such a construction. If, therefore, one of the sides by which the parallel lines are conjoined is bisected, and from it certain right lines are drawn to the remaining side, the quadrilateral figure shall be double of the triangle. And these things are demonstrated for the sake of geometrical exercise. Let us now proceed to the subsequent propositions.

PROPOSITION XLII. PROBLEM XI.

To construct a parallelogram equal to a given triangle, in a given rectilineal angle^[44].

* * * *

[196]

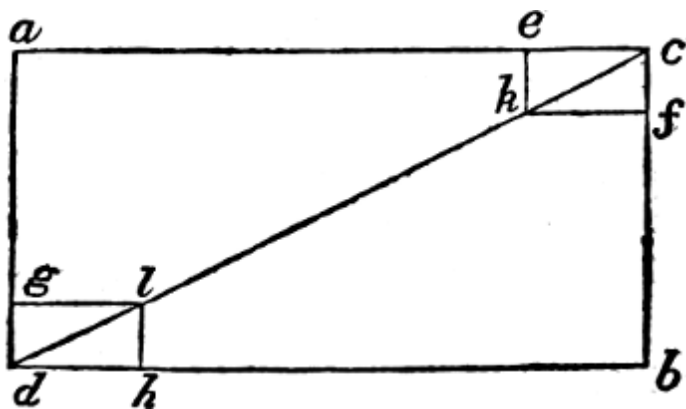
PROPOSITION XLIII. THEOREM XXXII.

The complements of parallelograms, situated about the diameter of every parallelogram, are equal to each other.

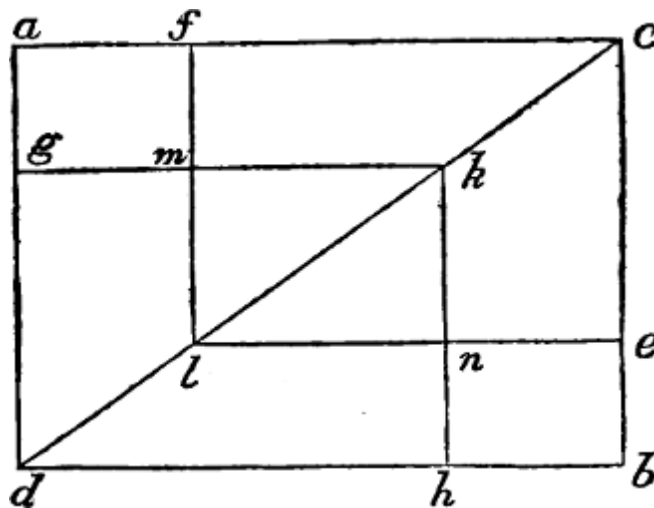
The beginning of this commentary is wanting.

* * * *

that parallelograms are not mutually conjoined according to one point, and that the complements^[47] are not quadrilateral; it is requisite that placing this also as a case, we should regard the same accident.

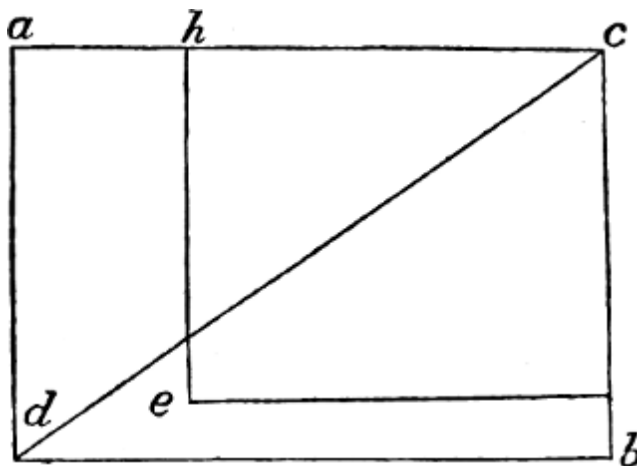


For let there be a parallelogram $a b$, having the parallelograms $c k$, $d l$, about the same diameter, and let a certain right line $k l$, which is a part of the diameter intervene between them. Again, therefore, you may say the same, viz. that the triangle $a c d$, is equal to the triangle $b c d$, and the triangle $e c k$, to the triangle $k c f$; and likewise the triangle $d g l$, to the triangle $d h l$. The remaining figure, therefore, $a g l k e$, of five sides, is equal to the remaining five-sided figure $b f k l h$. But these were the complements. Again, if the parallelograms are neither conjoined according to a point, nor distant from each other, but mutually cut each other, on this hypothesis also, the demonstration will be the same.



For let there be a parallelogram $a b$, and a diameter $c d$, and let parallelograms be constructed about it, one of which is $e c f l$, but the other, by which this also is intersected, $d g k h$. I say that the complements $f g$, $e h$, are equal. For since the whole triangle $d g k$, is equal to the whole triangle $d h k$, but a part of it also, the triangle $k l m$, is equal to the triangle $k l n$; (since $l k$ is a parallelogram); hence the remaining trapezium $d l n h$, is equal to the remaining trapezium $d l m g$. But the triangle $a d c$, is equal to the triangle $b c d$, and the triangle $f c l$, in the parallelogram $e f$, to the triangle $e c l$, and the trapezium $d g m l$, to the trapezium $d h n l$. The remaining quadrilateral figure, therefore $g f$, is not unequal to the remaining quadrilateral figure $e h$. And hence, the theorem is exhibited according to all its cases. But there are three only, and neither more nor less. For the parallelograms consisting about the same diameter, either cut each other or touch each other, according to a point, or are distant from each other by a certain part of the diameter.

But the institutor of the Elements assumes the appellation of *complements*, from the thing itself, so far as these also, besides two parallelograms, fill up the whole: and on this account, it was not of itself thought worthy of being remembered in the definitions. For, indeed, variety is requisite to its declaration, such as the knowledge of a parallelogram, and what those parallelograms are, which are about the diameter of the whole parallelogram; since, when these are explained, this likewise becomes known. But those parallelograms are about the same diameter, which have a part of the whole diameter for their own: and those which have not this condition, are by no means about the same diameter. For when the diameter of the whole parallelogram is cut by the sides of an internal parallelogram, then this parallelogram is not about the same diameter with the whole parallelogram. As for example, in the parallelogram $a b$, let the diameter $c d$, cut the side $e h$, of the parallelogram $c e$. The parallelogram, therefore, $e c$, is not about the same diameter with the parallelogram $c d$.



PROPOSITION XLIV. PROBLEM XII.

To a given right line, to apply a parallelogram equal to a given triangle, in an angle which is equal to a given right lined angle.

According to the Familiars of Eudemus, the inventions respecting the *application*, *excess*, and *defect* of spaces, is ancient, and belongs to the Pythagoric muse. But junior mathematicians receiving names from these, transferred them to the lines which are called conic, because one of these they denominate a parabola, but the other an hyperbola, and the third an ellipsis^[45]; since, indeed these ancient and divine men, in the plane description of spaces on a terminated right line, regarded the things indicated by these appellations. For when a right line being proposed, you adapt a given space to the whole right line, then that space is said to be *applied*; but when you make the longitude of the space greater than that of the right line, then the space is said to *exceed*; but when less, so that some part of the right line is external to the described space, then the space is said to be *deficient*. And after this manner, Euclid, in the sixth book, mentions both excess and defect. But in the present problem he requires application, wishing to apply to a given right line a parallelogram equal to a given triangle; that we may not only have the construction of a parallelogram equal to a given triangle, but also an application to a determinate right line. As for example, a triangle being given, having an area of twelve feet, but a right line being proposed, whose length is four feet, we may apply to the right line a parallelogram equal to the triangle, if when we assume the whole length of four feet, we find how many feet the breadth ought to contain, that the parallelogram may become equal to the triangle. When, therefore, we have discovered that the breadth is three feet, and have multiplied the length with the breadth, the proposed angle being right, we shall obtain the desired space. And such is the verb *to apply*, formerly delivered by the Pythagoreans. But there are three things given in the present problem; one, the right line to which it is to be so applied, that it may become the whole side of that space; but the other is the triangle to which that which is applied ought to be equal; and the third is the angle to which it is requisite that the angle of the space should be equal. And here it is again perspicuous, that when the angle is right, the space which is applied, is either a quadrangle, or an oblong; but when it is either acute or obtuse, the space is either a rhombus, or rhomboides. Besides, this too is manifest, that the right line ought to be finite; since this cannot be accomplished on an infinite line. At the same time, therefore, as he says, *to apply to a given right line*, he indicates that the right line must be necessarily finite. But he uses in the construction of the present problem, the construction of a parallelogram equal to a given triangle; since, as we have observed, application is not the same with construction. For the latter, indeed, constructs both the whole space, and all the sides; but the former, when it has one side given, constitutes on this the space, because it is neither deficient, nor exceeds according to this extension, but uses this one side which comprehends the area. But you may perhaps say, why does he use theorems, when he shews triangles equal to triangles; but problems, when he shews triangles equal to parallelograms? We reply, it is because equality spontaneously arises in things of the same species; but requires origin, and fabrication, in things of a dissimilar species, on account of the mutation subsisting according to species, since it is by itself difficult of invention.

PROPOSITION XLV. PROBLEM XIII.

To construct a parallelogram equal to a given right-lined figure, in a given rectilineal angle.

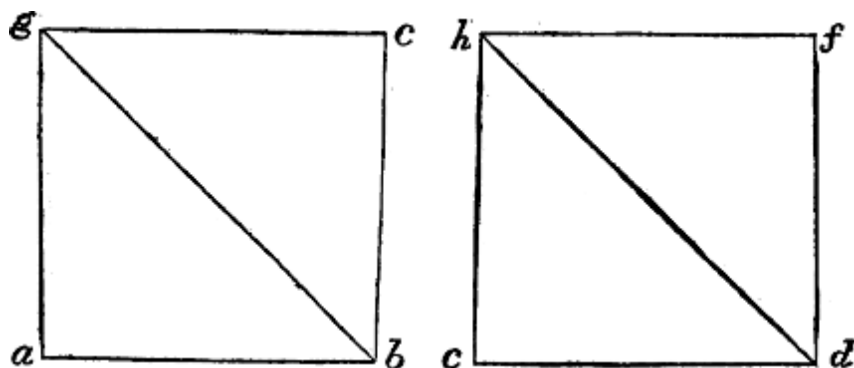
The present is more universal than the two problems, in which he invented as well the construction^[20] as the application of parallelograms equal to a given triangle. For whether a triangle, or a quadrangle, or any other quadrilateral figure is given, we may construct a parallelogram equal to it, by the present theorem; since every right-lined figure, as we have previously observed^[46], may be essentially resolved into triangles, and we have delivered a method of discovering the multitude of triangles. When, therefore, we have resolved a given rectangle into triangles, and have constructed a parallelogram equal to one of them, and have applied to a given right line, parallelograms equal to the rest; then, by assuming that to which we have made the first application, we shall have a parallelogram composed from these parallelograms, equal to the right-lined figure composed from those triangles, and the thing desired, will be accomplished. Hence, though such a rectangle should be a figure of ten sides, yet, by resolving it into eight triangles, and constructing a parallelogram equal to one of them, and seven times applying parallelograms equal to the rest, we shall obtain the object of investigation. But, as it appears to me, the ancients being incited by this problem, sought how to describe a quadrangle equal to a circle. For if a parallelogram can be found equal to any right-lined figure, it deserves to be enquired whether right-lined figures also, can be shewn equal to such as are curve-lined. And Archimedes shews that every circle is equal to a right-

angled triangle, one of whose radii is equal to one of the sides which are about the right angle of the triangle; but whose ambit is equal to the base. However, of this elsewhere: let us now proceed to the consequent propositions.

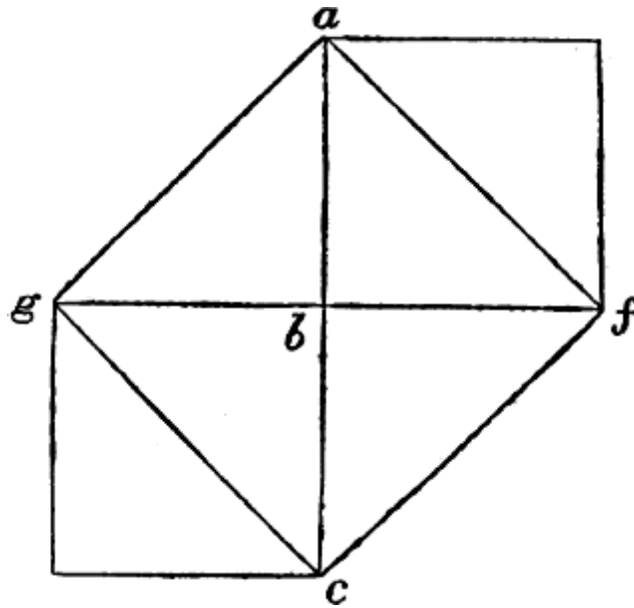
PROPOSITION XLVI. PROBLEM XIV.

To describe a quadrangle from a given right line.

Euclid requires this problem, most particularly, in the construction of the following theorem. But he appears to have been desirous to deliver the origin of the two best rectilineal figures, viz. the equilateral triangle, and the quadrangle; because these right lined figures are required in the constitution of the mundane figures, and particularly of those four, to which origin and dissolution belong. For the icosaedron and the octaedron, and the pyramid, are composed from equilateral triangles; but the cube from quadrangles. And on this account, as it appears to me, he has principally *constructed* the former, but *described* the latter. For he has discovered appellations adapted to these figures: since the equilateral triangle, so far as its composition is various, requires *construction*; but the quadrangle, so far as it originates from one side, requires *description*. For we cannot produce a triangle in the same manner as a quadrangle, by multiplying the number of a given right line into itself; but when we have conjoined right lines produced by other means, with the extremities of the given right line, we construct from these one equilateral triangle: and the description of circles, profits in discovering that point from which it is requisite to connect right lines, to the extremes of the proposed right line. But these observations are indeed perspicuous.



It may, however, be shewn, that the right lines, from which quadrangles are described, being equal, the quadrangles also shall be equal. For let the right lines ab, cd , be equal, and from ab , describe the quadrangle $abcg$, but from cd , the quadrangle $cdhf$, and connect the right lines gb, hd . Because, therefore, the right lines ab, cd , are equal, ag, hc , are also equal; and they comprehend equal angles, and the base gb , is equal to the base hd , and the triangle abg , to the triangle cdh ; and the doubles of these are equal. Hence the quadrangle ac , is not unequal to the quadrangle cf . But the converse of this also is true. For if the quadrangles are equal, the right lines, also, from which they are described, will be equal. Thus let the quadrangles af, cg , be equal, and let them be so placed, that the side ab , may be in a right line with the side bc .



Since therefore, the angles are right, the right line also fb , will be in a direct position, with the right line bg . Let the right lines fc , ag , af , cg , be connected. Because, therefore, the quadrangle af , is equal to the quadrangle cg ; the triangle, also, afb , is equal to the triangle cbg . Let the common triangle bcf , be added. The whole triangle, therefore, acf , is equal to the whole triangle cfg .

Hence, ag is parallel to fc . Again, because, as well afg , as the angle cgb , is the half of a right angle, af , is parallel to cg . The right line, therefore, af , is equal to the right line cg , since they are the opposite sides of a parallelogram. Because, therefore, there are two triangles, afb , bcg , which have the alternate angles equal, since af , cg , are parallel; likewise one side af , equal to the side cg , the side, also, ab , shall be equal to the side bc , and the side bf , to the side bg . And thus it is shewn, that the quadrangles af , cg , being equal, the sides, also, from which they are described are equal.

PROPOSITION XLVII. THEOREM XXXIII.

In right angled triangles, the quadrangle, which is described from the side subtending the right angle, is equal to the quadrangles which are described from the sides comprehending the right angle.

If we attend to the historians of antiquity, we shall find them referring the present theorem to Pythagoras, and asserting that he sacrificed an ox or its invention. For my own part, I admire those who first investigated the truth of this theorem: but I possess a greater admiration for the elementary institutor, not only because he establishes its truth by evident demonstration, but likewise, because he persuades us by scientific reasons, which cannot be confuted of a theorem more universal than this in his sixth book^[47]. For in that he shews universally, *that in right-angled triangles, the figure described from the side subtending the right angle, is equal to the figures described from the sides comprehending the right angle, when they are similar to the former figure, and are similarly described*. For every quadrangle is similar to every quadrangle; but all right-lined figures similar to each other, are not quadrangles: since in triangles, and other multangles, similitude is inherent. Hence, the reason which demonstrates that the figure described from the side subtending the right angle, whether it is quadrangular, or of some other form, is equal to the figures subsisting about the right angle, similar to the former, and similarly described; exhibits something more universal, and which possesses a greater power of producing science, than the reason exhibits, affirming a quadrangle alone, equal to quadrangles. For in the former case, it becomes manifest by an universal ostension, that the rectitude of the angle affords to the figure described from its subtending side, equality, to all the figures, subsisting about its comprehending sides, similar to the former, and similarly described: just as obtuseness is the cause of excess; but acuteness of diminution. But how this theorem is evinced, will be perspicuous, when we comment on it in the sixth book.

But let us now consider the truth of the present theorem, only adding this, that *universal* ought not to be shewn here, by him who has taught nothing concerning the similitude of right-lined figures, and the doctrine of

proportion: for many things which are here exhibited more particularly, are in that theorem shewn more universally by the same method. The institutor of the Elements, therefore, shews the thing proposed in the present, from the common contemplation of parallelograms. But since right-angled triangles are two-fold, i.e. either isosceles, or scalene; in isosceles triangles, we shall never find numbers corresponding with the sides: for there is no quadrangular number, exactly double of another quadrangular number; since the square from the septenary is double of the square from the quinary, by a deficiency of unity. But in scalene triangles it is possible, that numbers may be assumed, so as evidently to evince, that the square from the side subtending the right angle, is equal to the squares from the sides subsisting about the right angle. And of this kind is the triangle in the republic, whose right angle is contained by the ternary, and quaternary, but is subtended by the quinary. The quadrangle, therefore, from the quinary, is equal to the quadrangles from the other numbers: for this is twenty-five; but the quadrangle from the ternary is nine, and from the quaternary sixteen. And thus what we have asserted is perspicuous in numbers.

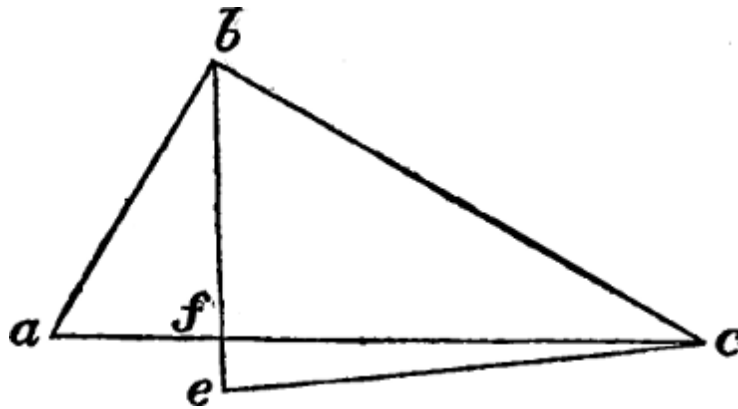
But there are delivered certain methods of inventing triangles of this kind, one of which they refer to Plato, but the other to Pythagoras, as originating from odd numbers. For Pythagoras places a given odd number, as the least of the sides about the right angle, and when he has received the quadrangle produced from this number, and diminished it by unity, he places the half of the remainder, as the greatest of the sides about the right angle; and when he has added unity to this, he produces the remaining side which subtends the right angle. Thus for example, when he has assumed the ternary, and has produced from it a quadrangular number, and from this number nine, has taken unity, he assumes the half of eight, that is four, and to this again he adds unity, and makes five; and thus discovers a right-angled triangle, having one of its sides of three, but the other of four, and the other of five units. But the Platonic method originates from even numbers. For when he has assumed a given even number, he places it as one of the sides about the right angle, and when he has divided this into half, and has produced a quadrangular number from the half, when he has added unity to this quadrangle, he forms the subtending side, but when he has taken unity from the quadrangle, he forms the remaining side about the right angle. Thus for example, when he has assumed the number four, and has multiplied the half of this into itself, and produced four, when he takes away unity he forms the number three, but when he adds unity, he produces the number five; and thus he has the same triangle effected, as by the Pythagoric method. For the square from the number five, is equal to the squares from the numbers three, and four. And thus much for the digression of the present narration. But as the demonstration of the elementary institutor is perspicuous, I do not think, that any thing should be added, because it would be superfluous; but we should be content with what is written. For those who have added any thing more, as the familiars of Hero and Pappus, have been obliged to assume in an affair of no difficulty, some of the propositions of the sixth book; and the cause which regards this affair. We shall therefore pass on to the following theorem.

PROPOSITION LXVIII. THEOREM XXXIV.

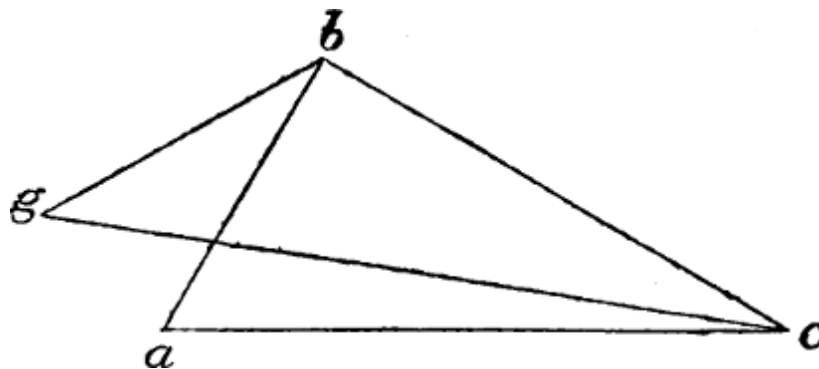
[207]

If the quadrangle described from one side of a triangle, is equal to the quadrangles described from the other two sides of the triangle: then the angle comprehended by the remaining two sides of the triangle, is right.

This theorem is the converse of the preceding, and the whole is converted to the whole. For if the triangle is rectangular, the quadrangle which is described, from the side subtending the right angle, is equal to the quadrangles described from the other sides: and if the square from this, is equal to the squares from the other sides, the triangle is rectangular, because it has the angle right, which is comprehended by the remaining sides. And the demonstration of the Elementary institutor is indeed conspicuous.



But when there is a triangle abc , having the quadrangle, which is described from the side ac , equal to the quadrangles from the sides ab , bc , since in the triangle, a right line from the point b , is raised at right angles to the side bc , if it should be said, that the right line must be raised at right angles, to other parts, and not at those to which the elementary institutor raises it, we assert that this is an impossibility. For it can neither fall within, nor without the triangle; and can be no other than ab . For if possible, let it fall as be . Because, therefore, the angle ebc , is right, the angle cfb , is doubtless acute; and hence, the remaining angle afb , will be obtuse. The side, therefore, ab , is greater than the side bf . Let a line be , be placed equal to ab , and connect ec . Because, therefore, the angle ebc , is right, the quadrangle described from the side ec , is equal to the quadrangles from the sides eb , bc . But eb is equal to ba . The quadrangle, therefore, from the side ec , is equal to the quadrangles from the sides ab , bc . But the quadrangle from the side ac , was also equal to the same. Hence, the quadrangle from the side ec , is equal to that which is described from the side ac ; and so ec is equal to ac . Two right lines, therefore, be , ec , are equal to the two ba , ac , each to each, and are constructed upon the right line bc , which is impossible. And hence, the line raised at right angles, does not fall within the right line ab .



But neither can it fall without, towards other parts of the right line ab . For if possible, let it fall as bg , and let bg be equal to ab , and connect cg . Because, therefore, the angle gbc , is right, the quadrangle described from the side gc , is equal to the quadrangles from the sides bg , bc . But the quadrangle also, from the side ac , was equal to the quadrangles from the sides ab , bc , but ab is equal to gb ; and so gc is equal to ac . But the right line gb , also, is equal to the right line ba , upon one right line bc , which is impossible. Hence, the right line which is raised from the point b , at right angles to bc , neither falls within, nor without the side ab ; and therefore falls upon it. And so the objection is dissolved. But the institutor of the Elements, thus far completes his first book, in which he has delivered many species of conversions; (for he often converts the whole of theorems to the whole, and wholes to parts, and parts to parts) and has invented a great variety of problems; (for he has delivered the sections, positions, constructions, and applications of lines and angles). He likewise touches upon that mathematical place which is called admirable; and sufficiently brings local theorems into our remembrance. Besides, he unfolds the elementary institution of universal and particular theorems, and indicates the difference of indeterminate, and determinate problems; all which, attending him in his progress, we have orderly explained. Lastly, he refers the whole book to one purpose, I mean the elementary institution, of the contemplation respecting the more simple rectilinear figures; and finally, he investigates their constructions, and considers their essential properties. But we, indeed, shall give thanks to the gods, should we be able to comment on the other books, in a similar manner. In the mean time, if other cares should prevent the execution of our design, it is my

opinion, that such as are studious of these contemplations, ought to expound the other books, after the same mode; by investigating that which is every where difficult, and pertinent to the subject, and capable of an easy division. For, indeed, the commentaries which are circulated at the present period, are replete with great and various confusion, because, at the same time, they neither infer any assignation of cause, nor dialectic judgment, nor philosophic contemplation.

END OF THE COMMENTARIES.

THE
H I S T O R Y
OF THE
RESTORATION OF THE PLATONIC THEOLOGY.
By the latter PLATONISTS.

SECT. I.

The Grecian theology, the history of whose restoration by the latter Platonists is the design of the present dissertation, did not originate among the Greeks, but was the progeny of barbarian propagation. This will be evident by considering that Orpheus was a Thracian; Thales, a Phœnician; Hermes Trismegistus, an Egyptian; Zoroaster, a Persian; Anacharsis, a Scythian; and Pherecydes, a Syrian. Yet though Greece was not the parent of theology, she was notwithstanding her benevolent nurse, by whom she was kindly educated, and received the full perfection of her nature. Indeed, though illustrious men flourished in the East, and theology was there particularly cultivated, yet her education was limited and rough, entangled with inexplicable ceremonies, and guarded by the sanctity of inviolable oaths. But when she was removed into the Grecian soil, and experienced the happy temperature of its climate, her genius became both elegant and profound; her person magnificent and graceful; and her ceremonies rational and sublime. Particular nations, indeed, seem to have been distinguished for particular pursuits. Thus the Egyptians appear to have excelled in the powers of invention; and the East, in general, has been remarkable for its attachment to the most recondite and mystic philosophy. Thus the Romans were famous for the arts of eloquence and war; and the Greeks have ever been celebrated as a people by whom every branch of knowledge received its ultimate perfection. They were a nation equally favoured by the graces, the muses, and philosophy; whose celestial union formed the divine genius of Homer, and inspired that elegance and depth with which the works of Plato are replete. They were, in short, the standards of excellence to the ancient, and are the objects of imitation to the enlightened part of the present world; and their theology, as well as their arts, will be admired when modern systems are no more.

It appears at first view strange that this sublime theology should rise to its pristine perfection during the decline of the Roman empire; and at a period when a new religion (I mean the Christian) was continually increasing in reputation, and advancing with rapid steps to a despotic establishment. But if we attentively consider, we shall find that the very causes which apparently threatened its destruction were the natural and proper sources of its renovation. As every part of the universe subsists by perpetual change, it is necessary that philosophy and the sciences, with respect to their appearance or the contrary, should share in the general mutability of things: but at the same time, it is necessary to their preservation to after-ages, that the order of their revolution should be retrograde to that of sensible particulars. Hence we shall often find, that while kingdoms descend in the circle of vicissitude, philosophy ascends, and perhaps attains to her ultimate perfection, at the very period when the most powerful nations become extinct. Thus the falling empire of the Romans was naturally connected with the rising greatness of philosophy; and the foreign ceremonies of a new religion, were the proper means of bringing to light the secret mysteries of the old. We may add too, that the same circumstances produced the great difference between the first and last appearance of this sublime theology. While Greece maintained her independence unconscious of the Roman yoke, and undisturbed by religious invasions, she disdained to expose her genuine wisdom to vulgar inspection, but involved it in the intricate folds of allegory; and concealed it from the profane under the dark veil of impenetrable mystery. But when she lost her liberty and submitted to foreign dominion, when her most ancient rites were threatened with invasion, and her sacred mysteries were treated with contempt, she found it necessary to change the dress of theology and to substitute a simple and elegant garb, instead of one highly marvellous and mystic.

Yet we must not imagine that theology, now stript of her ancient concealments, became the object of open inspection to the profane and vulgar eye. She had not lost her refulgence, though she had changed her

appearance: for the rays of celestial majesty yet beamed from her countenance, with a light awful and terrific to the multitude, but lovely and alluring to the wise. Hence the splendors of divinity no less secured her person from impious curiosity than the dark symbols in which she was formerly involved. The enchanting imagery of a celestial phantasy, and the pure light of an exalted intellect, while they captivated and converted the philosophical part of mankind, were inaccessible to the vulgar, whose mental eye, yet lost in the night of oblivion, was darkened by the splendid vision. However, though the real person of theology was not the object of vulgar inspection, her shadow at least was beheld by the benighted multitude, and became the subject of ridiculous opinions, and idle investigation. Hence some of these astonished with the majesty of her image, fondly fancied she was the progeny of the Jewish religion; and that her sacred mysteries were nothing but corrupt imitations of Mosaic divinity: while others, measuring the obscurity of her real person by the darkness of her shadow, considered her doctrines as delusions, and her sublimest truths as the reveries of a distempered imagination. Thus was true theology perverted and vilified by the multitude, when she appeared in her natural dress to mankind; till, in a few centuries after, indignant of the daring profanation, she ascended to her native heaven, and left the sons of folly involved in the shades of midnight error, and the gross delusions of fancied inspiration.

But let us contemplate her history more minutely, and mark the several particulars which distinguished her appearance on the earth. Let us survey the lives of the great geniuses who so largely participated her celestial light; and who so admirably transfused it in their writings for the benefit of hitherto ungrateful posterity. Let us view with wonder how she rose in majesty, as Rome declined in power, and appeared in full perfection invested with celestial honours, and surrounded with a godlike band of philosophic heroes, while that mighty empire was rapidly diminishing in bulk, and on every side nodding to its dissolution.

We are informed by Proclus^[48], that all the Grecian theology is the progeny of the mystic discipline^[416] of Orpheus; and that Pythagoras was the first who learned the orgies of the gods from Aglaophemus the disciple of Orpheus. This sacred theology was fully displayed by Orpheus, with all the graces of poetical diction, accompanied with the fury of the muses and divine illumination, in a great work entitled, *The Sacred Discourse*, which was divided into twenty-four rhapsodies, and which has unhappily perished in the ruins of time. In this inestimable work, if we may be allowed to conjecture from a treatise of the same name composed by Pythagoras, and often mentioned by Syrianus, all the orders of the gods were celebrated from the highest principle of things, to the last processions of the mundane divinities. But Pythagoras was no doubt deeply indebted for a part of this knowledge to the doctrine of Zoroaster, whose dogmata, according to Apuleius^[49], he embraced, and whose profound mysteries involved in oracular darkness, we may presume he communicated to his initiated disciples. The whole of this recondite theology was afterwards received by Plato from the writings of Archytas, Philolaus, and other Pythagoreans, but was so concealed by poetical embellishments, and mystical traditions, that, like the numbers of Pythagoras, it was alone adapted to the comprehension of a penetrating and sagacious few.

It is, however, a remarkable historical fact that this theology was lost for many centuries among the disciples of Plato, on the death of their divine master. But we are informed by Numenius^[50] the Pythagorean that Plato's successors, Speusippus, Zenocrates, and Polemo, perverted his dogmata, and almost entirely changed the whole of his philosophy. And Aristotle, it is well known, however he might retain some essential doctrines of his master, altered others of the highest importance; and confining himself chiefly to natural disquisitions, ascended but rarely and feebly to theological contemplations. However it was not irrecoverably lost; and it disappeared for a time, only to shine with brighter splendors on its return. Truth, like the light of the sun, may suffer concealment, but cannot be destroyed; for it would rather have its rays broken by resistance than bound to obscurity. About two hundred and fifty two years, therefore, after the Christian religion had made its appearance, this sublime theology was restored by one Ammonius Saccas an Alexandrian. This extraordinary person, was, it seems, at first nothing more than a porter: though by what methods he rose from this servile employment to the summit of philosophy, and what happy circumstances first affected this wonderful change, are enquiries which can never be answered, but whose loss will always be regretted by the liberal few. But though he was not Διογενής, nobly born, his doctrines, as transmitted to us by his disciples, eminently evince his possessing in high perfection all the other endowments of a true philosopher: such as a penetrating genius, a docile sagacity, a tenacious memory, and every other ornament of the soul, requisite, according to Plato, to form the philosophic character. Indeed he must have possessed these qualifications in a most remarkable degree; or he could never have emerged from the obscurity and servility of a porter, to the splendor and liberty of an exalted and divine philosopher. The truth of this

observation is confirmed by the appellation of θεοδιδάκτος, or *divinely-taught*, which was unanimously conferred on him, by his contemporary philosophers.

This great man opened a philosophical school at Alexandria, but with a determination not to commit the more abstruse and theological dogmata of his philosophy to writing. Indeed he was so fearful of profaning these sublime mysteries, by exposing them to vulgar inspection, that he revealed them to his disciples Erennius, Origen, and Plotinus, on the conditions of inviolable secrecy, and under the guard of irrevocable oaths. However, fortunately for posterity, Erennius dissolved the compact, and Origen (different from the Christian father of that name), imitating Erennius, disclosed a part of his master's secrets, in a curious treatise on dæmons, which, among many other valuable productions, is lost in the ruins of time. But the publications of these two great men were but trifling efforts to unveil the mystic wisdom of antiquity: since a perfect revelation was reserved for the divine genius of Plotinus, who considering himself now freed from his engagements, by the examples of his fellow disciples, resolved to bring theology from her dark concealments and to present her to the astonished world, in all the celestial graces and irresistible majesty of her natural appearance. This wonderful man (if he was not something more, since his writings discover a genius superior to the human), who was born to astonish and enlighten mankind, was the first who committed to writing the secrets of theology, free from the obscure enigmas in which she had been enveloped by the sages of antiquity. The celestial vigor and profundity of his genius, render his conceptions indeed, unavoidably abstruse: but he who has once fathomed his depth, will find himself amply compensated for the labour of investigation, by the rewards of uncommon knowledge and inexpressible delight. There is a long and curious life of this high priest of theology, and dæmon of wisdom, extant by his disciple Porphyry, the substance of which, as it will not I presume be unacceptable to the reader, and as it will throw great light on the history of theology, I have selected from that invaluable work.

Plotinus, was an Egyptian by birth, and was a native of Lycopolis, as we are informed by Eunapius^[219] for Porphyry is wholly silent as to this particular. Indeed this is not wonderful, if we consider what Porphyry asserts in the beginning of his life, *that he was ashamed, that his soul was in body*. Hence says he, he would neither tell the race, nor the parents from which he originated, nor would he patiently relate in what country he was born. This I know will be considered by a genuine modern, as either rank enthusiasm, or gross affectation; but he who has perused and fathomed his writings will immediately subscribe to its truth. The same vehement love for intellectual pursuits, and contempt for body, made him disdain to sit for his picture; so that when one of his disciples Amelius, begged that he would permit his likeness to be represented, his answer expressed the true greatness of his mind: as if (says he) it was not sufficient to bear this image, with which nature has surrounded us from the first, you think that a more lasting image of this image should be left as a work worthy to be inspected. However the desire of Amelius was at length accomplished, by the ingenious contrivance of one Carterius a painter, who by frequenting the school of Plotinus, and viewing his countenance with fixed attention, produced at length from his memory a happy likeness of the philosopher. Though he was often afflicted with the colic, he always refused the assistance of clysters, asserting that cures of this kind were not proper to a man advanced in years. Nor would he ever receive the assistance of theriacal antidotes, since he said, his nourishment was not derived from the bodies of even tamer animals. He likewise abstained from baths: but daily used frictions at home. But when a grievous pestilence raged^[51] at Rome, and the servants who were accustomed to rub him, fell victims to the disease, from neglecting cures of this kind, he gradually became a prey to the pestilence. So great was the violence of this distemper, and its effects so dreadful on Plotinus, as Eustochius informed Porphyry who was absent, that through a very great hoarseness, all the clear, and sonorous vigour of his musical voice was lost; and what was still worse, his eyes were darkened, and his hands and feet were covered with ulcers. Hence, becoming incapable of receiving the salutations of his friends, he left the city; and went to Campania, to the estate of one Zethus, an ancient departed friend. Necessaries were here administered to him from the hereditary possessions of Zethus, and were likewise brought from Minturnus, from the fields of Castricius^[52]. But when this divine man drew near to his dissolution, that period which is no less the dread of the vulgar than the transport of the philosopher, and which to Plotinus must be the moment of extatic rapture, Eustochius who dwelt at Puteolus, was not very hasty in his approaches; doubtless not imagining he was on the point of making his triumphant exit from a corporeal life. However when he came into the presence of this departing hero, he was just in time to receive his dying words, and to preserve the sacred sentence to posterity. Listen ye profane with reverence, and treasure in your memories ye wise, the weighty truth it contains! *As yet (says he) I have expected you; and now I consent that my divine part, may return to that divine nature, which flourishes throughout the universe*. Such were the last words of this mighty man, which like those contained in his writings are great and uncommon,

wonderful and sublime. He died at the conclusion of the second year of the emperor Claudius' reign; and was at the time of his death in the sixty-sixth year of his age, according to the information given by Eustochius to Porphyry. The most trifling particulars relative to the life and death of so extraordinary a man merit our attention; and indeed we may presume without being guilty of either superstition or enthusiasm, that scarcely any thing trifling could mark the existence of such a powerful and celestial genius. There is nothing, properly speaking, can be little which has any relation to a character truly great: for such is the power of uncommon genius, that it confers consequence on every thing within the sphere of its attraction, and renders every surrounding circumstance significant and important. Thus immediately on the death of Plotinus, we are informed by Porphyry that a dragon^[53] which had been concealed under his bed, wandered through a hole in the wall, and disappeared. But how great must the grief of Porphyry have been, to be separated from his beloved master, at the time of his death: from a master by whom he had been esteemed beyond the rest of his fellow-disciples; and whose loss no succeeding period was ever likely to repair. Indeed his disciples seem to have been unaccountably dispersed, at this important crisis: for Porphyry was at Lilybæum, Amelius at Apamea in Syria, Castricius at Rome, and Eustochius was alone present at his departure. Porphyry afterwards informs us, in perfect agreement with the genius of Plotinus, that he never would tell to any one, the month, or day in which he was born: because he by no means thought it proper that his nativity should be celebrated with sacrifices and banquets. Indeed we cannot suppose that he who had such a vehement contempt for a corporeal life, would be anxious that his entrance into mortality should be solemnized with festivity; but rather considering himself with Empedocles, as

“Heaven's exile straying from the orb of light,”

he would be disposed to lament his captivity, and mourn the degradation of his nature. However he was not averse to celebrate the nativities of Socrates and Plato; for he assisted at the sacred rites, and invited his friends to a philosophic banquet, where it was required that every guest should recite a written oration, adapted to the occasion of their amicable association.

But the few particulars which this great man condescended to relate of himself, in familiar discourse^[200] the following: When he was eight years of age, and was even under the tuition of a literary preceptor, he used to frequent his nurse, and to uncover her breasts, through an avidity of sucking her milk. And this custom he continued, till being accused of troublesomness, and covered with shame through the reproof, he neglected this extraordinary custom. This story however trifling it may appear, indicates in my opinion the native innocence, and genuine simplicity of manners which marked the character of Plotinus. It is a circumstance, which does not merely point to something uncommon; but it was the harbinger as it were of that purity and sanctity of life, which so eminently formed the conduct, and adorned the writings of our philosopher. But when he was in the twenty-eighth year of his age, being vehemently inflamed with the love of philosophy, he was recommended to the most excellent masters of Alexandria: but he left their schools with sorrow and disappointment. By a fortunate event, however, he told a certain friend, who was well acquainted with the disposition of his mind, the cause of his affliction, and he brought him to the celebrated Ammonius, whose school Plotinus had probably overlooked among the great multitude with which that illustrious city abounded. But when he had entered the school of Ammonius, and had heard him philosophize, he exclaimed in transport to his friend, *this is the man I have been seeking*. From that day he gave himself up to Ammonius with sedulous attention for eleven years; and made such rapid advances in his philosophy, that he determined to study the philosophy of the Persians, and the wisdom particularly cultivated by the Indian sages. For this exalted purpose, when the emperor Gordian marched into Persia, in order to war upon that nation, Plotinus joined himself to the army, being at that time in the nine and thirtieth year of his age. But after Gordian was destroyed about Mesopotamia, Plotinus fled to Antioch, where he received a fortunate shelter from the dangers and devastations of war; and in the reign of the emperor Philip came to Rome, in the fortieth year of his age. It seems therefore that Plotinus was disappointed in his purpose at that time of procuring the Persian and Indian wisdom: it is however certain that he afterwards obtained his desire; and most probably without the inconvenience of a long and dangerous journey. This will be evident from perusing his works; and attending to the latent dogmata they contain.

It was a long time before Plotinus committed his thoughts to writing; and gave the world a copy of his inimitable mind. That light which was shortly to illuminate mankind, as yet shone with solitary splendour; or at best beamed only on a beloved few. It was now destined to emerge from its awful sanctuary, and to display its radiance with unbounded diffusion. But a disciple like Porphyry, was requisite to the full perfection of its appearance. Amelius was indeed laborious, but he was at the same time verbose: he neither appears to have

possessed the inquisitive spirit, nor the elegant genius of Porphyry; and his commentaries were too voluminous to be exquisitely good. Porphyry gives a singular specimen of his endurance of labour, when he informs us, that he committed to writing almost all the dogmata of Numenius, and retained a very considerable part in his memory. He was not however, though an excellent philosopher, calculated to urge Plotinus to write, or to assist him in its prosecution: but this important task was reserved for Porphyry, who in the words of Eunapius, “like a mercurial chain, let down for the benefit of mortals, by the assistance of universal erudition, explained every thing with clearness and precision.” Plotinus indeed began to write in the first year of the emperor Galienus; and he continued just to note such questions as occurred to him, for the ten following years, in the last of which he became acquainted with Porphyry, who was at that time in the thirtieth year of his age. He had then composed one and twenty books, which were in the hands but of a few: for the edition was difficult to be procured, and was not universally known. Besides Plotinus, was neither hasty nor rash in his publications: but he gave those only to the light, which had been approved, by a mature and deliberate, judgment. The one and twenty books we have previously mentioned, after various inscriptions, at length obtained the following titles:

On the beautiful.
 On the immortality of the soul.
 On fate.
 On the essence of the soul.
 On intellect, and ideas, and being.
 On the descent of the soul into body. [221]
 How that which is posterior to the first, proceeds from the first; and concerning *the one*.
 Whether all souls are one.
 Concerning the good itself, or *the one*.
 On the three principal hypostases.
 On the generation and order of things posterior to the first.
 On the two matters, intelligible and sensible.
 Various considerations.
 On the circular motion of the heavens.
 On every one’s peculiar Dæmon.
 On the rational exit, from the present life.
 On quality.
 Whether there are ideas of particulars.
 On virtues.
 On dialectic.
 How the soul is said to be a medium between an impartible and partible essence.

These one and twenty books were finished when Porphyry first became acquainted with Plotinus; and when this great man was fifty-nine years old. During the six years in which Porphyry was his companion as well as disciple, many questions of a very abstruse nature, were discussed in their philosophical conversations, which at the joint request of Porphyry and Amelius, Plotinus committed to writing, and produced from their investigation, two elaborate and admirable books, proving *that true being is totally present in every part of the universe*. He wrote besides two others; one of which asserts, *that the nature superior to being, is without intellection*; and the other *distinguishes primary from secondary intelligence*. He likewise composed at the same period, the following books:

Concerning that which exists in capacity, and energy.
 [54] That incorporeal natures are free from passivity.
 Two books concerning the soul.
 A third concerning the soul, or the manner in which we see.
 On contemplation.
 On intelligible beauty.
 That intelligibles are not external to intellect; and concerning intellect, and the good.
 Against the Gnostics.
 On numbers. [222]

Why things seen at a distance appear small.
 Whether felicity consists in length of time.
 Concerning total mixture.
 How the multitude of ideas subsists, and concerning the good.
 On that which is voluntary.
 On the world.
 On sense and memory.
 Three books on the genera of beings.
 On eternity and time.

But while Porphyry resided in Sicily, Plotinus composed the five following books, which he sent to him for his revision:

On felicity.
 Two books on providence.
 On gnostic essences, and that which is superior to their nature.
 On love.

These books were transmitted to Porphyry in the first year of the emperor Claudius' reign. And about the beginning of the second year, a little before his death, he sent him the following, and the last:

An enquiry into evil.
 Whether the stars operate on sublunary natures.
 What the nature is of man, and animal.
 On the first good, and other goods.

The whole amount therefore of the books written by Plotinus, connecting the preceding with the present, is fifty-four, which Porphyry has divided into six enneads, assigning agreeable to the meaning of the word, nine books to every ennead. But they bear evident marks (says Porphyry) of the different periods, at which they were composed. For the first one and twenty, which were written in the former part of his life, if compared with the next in order seem to possess an inferior power, and to be deficient in strength. But those composed in the middle of his life exhibit the vigour of power, and the summit of perfection. And such with a few exceptions are the four and twenty we have already enumerated. But the last nine, composed in the decline of life, carry the marks of remitted energy, and drooping vigor. And this the four last declare, more evidently than the preceding five. It must however be observed that this difference is only visible, when they are contrasted with one another. To an impartial observer, zealous of truth, and not deeply read in Plotinus, each of his books will appear to be what it really is, uncommonly profound, and inimitably sublime. Each is an oracle of wisdom, and a treasury of invaluable knowledge; and the gradations of excellence consist in the power of composition, and not in the matter from which they are composed.

Plotinus had many auditors, and likewise a multitude of zealous partizans, and philosophic familiars.²³⁴ This indeed must necessarily be the case, if we consider the reputation of philosophy at that golden period, and the extraordinary abilities and celestial genius of its godlike restorer. Among the latter of these, Amelius the Tuscan, and Paulinus the Scythopolitan, a physician, held a distinguished rank. To which may be added Eustochius of Alexandria, a physician, who enjoyed the familiarity of Plotinus to the last, was present at his death, and giving himself entirely to the institutes of Plotinus, assumed the habit of a genuine philosopher. Besides these, Zothicus, a critic and poet, was conversant with Plotinus, who amended the works of Antimachus, and rendered the Atlantic history very poetically in verse: but after this he became blind, and died a short space of time prior to Plotinus. Zethus too, was very familiar with our philosopher, who derived his origin from Arabia, and married the wife of one Theodosius, the familiar of Ammonius. He was deeply skilled in medicine, and very much beloved by Plotinus, who endeavoured to dissuade him from engaging in the administration of public affairs. Such indeed was his familiarity with our philosopher, that, as we have already observed, Plotinus spent the last hours of his life at his rural retreat. Porphyry likewise informs us, that not a few senators were the sedulous auditors of Plotinus. Philosophy indeed, as it is the most noble and liberal of all pursuits, ought never to be separated from noble birth and exalted rank. It is naturally allied with every thing great, and is calculated to confer dignity, even

on greatness itself. It exalts the majesty of the monarch, stamps nobility with true grandeur, and raises the plebeian to immortality. In the age of philosophy, therefore, we cannot wonder that she was revered by the senators of Rome. That illustrious body, even at this declining period, retained a portion of its ancient independence; and the generous ardor of unbounded liberty was not yet extinguished by the frozen hand of despotic usurpation. The Roman manners and *religion* were not yet destroyed; and nobility was not contaminated by the sordid occupations of traffic. *Meekness* was not esteemed a *virtue*, nor *merchandize* an *honour*!!! Among this illustrious body of men, Marcellus Orontius diligently applied himself to philosophy, and made rapid advances in its attainment. This too, was the case with Sabinillus, and above all with the senator Rogatianus^[55]. So deeply enamoured was this last nobleman with the charms of wisdom, and the discourses of Plotinus, and so attentive to the care of separating his soul from his corporeal life, that he neglected his wealth, and secular affairs, dismissed his servants, and rejected the dignities of the state. Hence, when he was chosen prætor, and the lictors waited for his appearance, he neither came into public, nor regarded the duties of his office, nor dwelt in the house allotted for his reception: but he supped and slept with certain of his friends and familiars, and gave himself to absolute retirement in the day. By this negligence and carelessness of life (says Porphyry), from being so vehemently afflicted with the gout, that he was obliged to be carried in a chair, he resumed his pristine strength and vigour. And from being so diseased in his hands, that he could not extend them when necessary, he so recovered their use by philosophic endurance, as to employ them with greater expedition than the manual mechanic. This great man, as we may suppose, possessed the principal place in the esteem of Plotinus, who was not sparing in his praise of so uncommon a character, and proposed him as an illustrious example to the pupils of philosophy. Happy Rogatianus! who could relinquish power for knowledge, and prefer the perpetual inheritance of wisdom to the gaudy splendors of title, and the fleeting honours of command. Alexandrinus Serapion too, was one of his associates, who was once a rhetorician, but afterwards, gave himself to philosophical disputations; though, shameful to relate, he was at the same time a slave to usury, and avarice. Besides all these (says Porphyry), he reckoned me a native of Tyre, among his most friendly adherents, whom he appointed to correct his writings.

The following particulars relative to composition are related by Porphyry of this extraordinary man. He could by no means endure to review twice what he had written, nor even to read his composition, through the badness of his sight. But while he was writing, he neither formed the letters with accuracy, nor exactly distinguished the syllables, nor bestowed any diligent attention on the orthography: but neglecting all these as trifles, he was alone intent to the intelligence of his wonderful mind; and, to the admiration of all his disciples, persevered in this custom to the end of his life. To a man of mere words, Plotinus will doubtless appear inexcusable for such *important* omissions: but to the sublime and contemplative genius, his negligence will be considered as the result of vehement conception, and profound cogitation. Such, indeed, was the power of his intellect, that when he had once conceived the whole disposition of his thoughts from the beginning to the end, and had afterwards committed them to writing, his composition was so connected, that he appeared to be merely transcribing from a book. Hence he would discuss his domestic affairs without departing from the actual intention of his mind; and at the same time transact the necessary negociations of friendship, and preserve a perpetual intelligence of his thoughts. In consequence of this uncommon power of intellection, when he returned to writing, after the departure of the person with whom he had been conversing, he did not review what he had written, owing, as we have observed, to the defect of his sight; and yet he so connected the preceding with the subsequent conceptions, as if his composition had never been interrupted. Hence he was, at the same time present with others, and with himself, so that, as Porphyry observes, the self-converted energy of his intellect was never remitted, except perhaps in sleep, which he very moderately indulged. And so vigorous and frequent was the conversion of his soul to intellect, that he would often abstain from bread, swallowed up, as it were, in the depths of contemplation.

Several women too, enamoured with the love of wisdom, were the auditors of Plotinus. The Platonic philosophy, indeed, as it necessarily combines truth with elegance, is naturally adapted to captivate and allure the female mind, in which the love of symmetry and gracefulness is generally pre-dominant. Hence, in every age, except the present, many illustrious females have adorned the Platonic schools, by the brilliancy of their genius, and as uncommon vigour and profundity of thought. This too, would doubtless be the case in our own country, if all the works of Plato and his disciples were but once faithfully and elegantly translated into English: but till the obstacle of Greek is removed, we may in vain expect thinking females^[56], and I had almost said Platonic philosophers among men. Porphyry adds, that many men and women of noble birth, when at the point of death, delivered up, and commended their children and all their substance to Plotinus as to a sacred and divine guardian. Hence, says he, you might see the house of Plotinus full, both of young men and virgins, among the number of

which was one Potamon, whom he educated with diligence and care. Nor was he wearied in hearing the procurators of his pupils, often rendering an account of their administration; nor did he disdain to pay attention to their expences, affirming, that as they did not yet philosophize, they ought to possess their own goods, and to receive, without detriment, an increase of their estate. Yet though he procured for so many pupils the chief necessities of life, the intellectual energy of his soul while he was awake, never suffered any interruption from externals, nor any remission of vigour. He was extremely mild, though not *meek*^[57], in his manners, and was easy of access to all his adherents and friends. Hence, so great was his philosophic urbanity, that though he resided at Rome twenty-six years, and had been the arbiter of many litigious causes, which he amicably dissolved, yet he had no enemy throughout that great and illustrious city. This last circumstance, indeed, reflects the highest honour on the philosophic character of Plotinus; but, at the same time, some merit is due to the age in which he fortunately lived. Had he been destined to make his appearance in the present times, unsupported by fortune, and with no other recommendation than an uncommon greatness of mind, and an unequalled depth of thought; from being despised, insulted, and distressed, he must surely have been indignant, though not morose, and severe though not agitated with wrath. He would have been scornful without pride, contemptuous without weakness, patient without servility, and solitary without affectation. He would have lived without notice, wrote with success, and died without regret. But born to a happier fate, his genius was not doomed to languish in the shades of obscurity, but attained to the blossom of perfection in the sun-shine of philosophy, and through the liberal pains of Grecian and Roman cultivation.

But though Plotinus was thus universally esteemed at Rome, and in general by all who had the happiness of his acquaintance, yet he had one vehement enemy in the person of Alexandrinus Olympius, who had been for a short time the disciple of Ammonius, who desired to arrogate to himself the chief place in philosophy, and endeavoured to render Plotinus the object of general contempt. So deadly, indeed, was his hatred of our philosopher, that he attempted to invade him, by drawing down the baneful influences of the stars. The attempt was, however, vain, and its effects noxious to their author. For the sidereal deflections, instead of being hurtful to Plotinus, were reflected on Olympius. Hence he exclaimed to his companions, “that the soul of Plotinus possessed such a mighty power, that it immediately repelled malignant influences directed upon his person, on the authors of the evil.” But Plotinus, when Olympius first machinated his sidereal enchantments, was conscious of his design, and said to his friends; “Now the body of Olympius is contracted like a purse, and all his members are bruised together.” After Olympius, therefore, had often found to his own detriment, that the baneful influences intended for Plotinus were repelled on himself, he desisted from such base and fruitless undertakings. Indeed, says Porphyry, Plotinus naturally possessed something greater than the rest of mankind, which the following extraordinary relation abundantly evinces. A certain Egyptian priest, who at that time visited Rome, and who became suddenly known to Plotinus by one of his friends (perhaps Porphyry himself), desirous to exhibit his wisdom in that illustrious city, persuaded our philosopher to attend him, for the purpose of beholding, through his invocations, his familiar *dæmon*; to which request he readily consented. But the invocation was performed in the temple of Isis; this being the only pure place in Rome the Egyptian priest was able to find. However, instead of a *dæmon*, as was expected, a god approached, who was not (says Porphyry) in the genus of *dæmons*. The Egyptian astonished at the unexpected event, exclaimed, “Happy Plotinus, who hast a god for a *dæmon*, and whose familiar attendant does not rank among the inferior-kind!” But this extraordinary and delightful vision was of short duration: for the priest affirmed it was not then lawful to interrogate any thing, nor any longer to enjoy the vision, because a certain common contemplative friend, who was present at the spectacle, suffocated some birds which he held in his hands for the sake of safety, either impelled by envy, or terrified with fear. As Plotinus, therefore, was allotted a *dæmon* belonging to the diviner orders, the divine eye of his soul was perpetually elevated to this guardian deity. On this account he composed a book *concerning every man’s familiar dæmon*, in which he diligently endeavours to assign the causes of the diversity subsisting among these attendants on mankind. As a still farther proof of his uncommon greatness of mind, Porphyry adds, that when Amelius who was an observer of sacred rites, in which he officiated, according to the Roman calends, once requested Plotinus to attend him in the discharge of these religious ceremonies, he replied, “It becomes them to approach to me, and not me to them.” But from what conceptions (says Porphyry), he spoke in such an exalted manner of himself, we were unable to conceive, and afraid to ask. We may, however, presume, that Plotinus meant to insinuate the high degree of purity and perfection of his intellectual part, which rendered him so superior to the use of corporeal sacrifices, and the cultivation of material deities, and *dæmons*, that he ought rather to be propitiated by others, than to propitiate himself. For a soul like his, was, indeed, to use his own expression, ὑερος θεός, *a posterior god*, ready winged for flight, and scarcely detained by the fetters of body. This I know will pass for great

arrogance and presumption among the philosophers of the present day, who consider *meekness* and *humility* as the highest ornaments of their nature, and the truest characteristics of genuine worth. But surely a sublime and godlike soul can never think *meanly* of its nature, or be willing to suppress and extinguish the inevitable consciousness of its own dignity and elevation. Humiliating conceptions flourish no where but in the breasts of the servile, or the base; and are the ornaments of no characters, but those of the impotent and the mean. Their influence is baneful to the advancement of science, and destructive of all genuine excellence and worth. They damp the glowing ardour of true theology, curb the celestial flight of philosophy, and blast the vigorous blossoms of genius. Let it, however, be remembered, that while we banish meekness, we are by no means the advocates of arrogance and conceit; but are alone desirous of vindicating the proper dignity of the worthy soul, and of rescuing its generous and ardent confidence from the frigid embraces of humiliating opinion. It is one thing to be *modest*, and another to be *meek*: for the former is the shadow attendant on genius, inseparable from its progress, and the symbol of its reality; but the latter is the dæmon of traffic, the inspirer of its projects, the support of its credit, and the harbinger of its appearance. It flies from the face of genius like the shadows of night before the beams of the morning, and, terrified at the approach of the elevated mind, hides itself in the dark retreats of trembling pusillanimity. But to return from this digression: Plotinus appears to have possessed an unequalled skill in physiognomy, as the following circumstance eminently evinces. A lady, named Chion, who, together with her daughters resided in his house, and there happily passed a chaste widowhood, was fraudulently deprived of a very valuable necklace. In consequence of this, all the servants and domestics were summoned into the presence of Plotinus, who regarded their several countenances, selected one, and accused him of the theft. The man was immediately chastised, and for a long time denied the fact, but at length confessed his guilt, and restored the necklace. In a similar manner (says Porphyry) he wonderfully predicted the destiny of the young men of his acquaintance; as of one Polemo, he foretold, that he would be very much addicted to love, and not arrive to the maturity of his age, which happened according to his prediction. But the last instance of his sagacity, related by Porphyry, excels all the rest, both in the singular skill displayed in its execution, and the happy consequences it produced. Porphyry, as we are informed by Eunapius, in his life, on his first acquaintance with Plotinus, bade a final farewell to all his preceptors, and totally applied himself to the friendship and confidence of this wonderful man. Here he filled his mind with science, and drew abundantly, without satiety, from the perennial fountain, seated in the sanctuary of the soul of Plotinus. But afterwards being conquered, as it were, by the magnitude of his doctrines, he conceived a hatred of body, and human nature, and could no longer endure the fetters of mortality. "Hence (says Porphyry) I formed an intention of destroying myself, which Plotinus wonderfully perceived, and as I was walking home, stood before me, and said, *Your present design, O Porphyry, is not the dictate of a sound intellect, but rather of a soul raging with an atrabilious fury*. In consequence of this, he ordered me to depart from Rome, and accordingly I went into Sicily, particularly when I heard that a certain worthy and elegant man dwelt at that time about Lilybæum. And by this means, indeed, I was liberated from this perturbation of soul, but was in the mean time hindered from being with Plotinus till his death."

But the great reputation of this divine man was not confined to the senate and people of Rome, for the emperor Galienus, and his wife Salonina, honoured his person, and revered his doctrine. Indeed, so highly was he esteemed by the emperor, that, relying on his benevolence, he requested that a city in Campania, which had been formerly destroyed, might be restored, and rendered a fit habitation for philosophers; and besides this, that it might be governed by the laws of Plato, and called Platonopolis. Had this design succeeded, Plotinus intended to have dwelt there with all his disciples, and to have realized the beautiful republic, conceived by the godlike genius of Plato. The emperor, indeed, assented to his wishes, and the philosopher would have easily accomplished his intentions, if some of the emperor's familiars, impelled by envy or indignation, or some other unjust and selfish cause, had not warmly opposed its execution.

This extraordinary man (as we are informed by Porphyry) was strenuous in discourse, sagacious in ^{invention} ~~intention~~, and prompt in the most opportune perceptions: but he was frequently incorrect in his speech, as well as in writing; and this most probably owing to the vehemence and vigour of his conceptions. Besides this, while he was engaged in discourse, his intellect beamed through his corporeal frame, and diffused over his countenance its intimate light. He was, indeed, of a most beautiful aspect, but when he disputed (says Porphyry) he seemed far more lovely to the view. Then a placid gentleness appeared in receiving questions; and a vigour uncommonly robust was demonstrated in their dissolution. When Porphyry once had interrogated him for three days, *by what means principally the soul was united with the body*, he persevered in demonstrating the manner of its conjunction. And when a certain person, named Thaumasius, entered his school, for the purpose of discussing

common questions in philosophy, and premised that he wished to hear the explanatory sentences of Plotinus, but that the questions and answers of Porphyry were by no means adapted to a disputation of this kind, Plotinus replied: *unless we dissolve the doubts arising from the interrogations of Porphyry, we shall not be able to comment any thing, in an uninterrupted series of discourse.* But he wrote with a most intense acuteness of thought, and an abundant intellect. His writings are remarkably sententious, and he abounds, every where, more with profundity of sense than copiousness of words. “He poured forth (says Porphyry) many things agitated by the impulse of inspiring deity; and was often wonderfully affected with the object of his investigation.” With me indeed every page of his works is a *volume*, and every sentence an *oracle*. The latent dogmata of the Stoics, and Peripatetics, are inserted in his books; and more particularly the Sentences of Aristotle posterior to his Physics. He was ignorant of nothing pertaining to geometry, arithmetic, optics, and music, though he had never reduced these sciences to practice. The commentaries of the Platonic philosophers, Cronius, Numenius, Gaius, Atticus; as also of the Peripatetics, Aspasius, Alexander, Adrastus, &c. were read in his schools: but nothing was repeated from these in an uninterrupted series. For his conceptions were entirely his own; and his contemplations were different from theirs. In interpretation, and the discussion of questions, he bore the intellect of Ammonius. As soon as he was sufficiently imbued with reading, and had given, in a short discourse, sentences full of profound contemplation he arose, and left the school. Having once read the book of Longinus concerning principles, he said, *that Longinus was indeed a philologist, but by no means a philosopher*; and this indeed, as it appears to me, by a necessary consequence: for the knowledge of *words* is entirely foreign from the study of *things*. When Origen (not the Christian father of that name) once came into his school, Plotinus whose cheeks were covered with blushes, wished to rise, and being solicited by Origen to continue his discourse, he replied “that discourse ought to cease, when he who speaks perceives he addresses himself to those who are well acquainted with his doctrine.” And thus after a short dissertation he arose from thence.

When in the celebration of Plato’s nativity, Porphyry recited a poem *on the sacred marriage*^[58], and a certain person who was present objected that Porphyry was mad, because many things were said in the poem mystically, and inspired by a divine fury, Plotinus openly exclaimed, “You have shewn yourself at the same time both a philosopher and a priest.” On a certain time too an orator, named Diophanes, read an apology for the intoxicated Alcibiades in the banquet of Plato, endeavouring to prove that it was proper for the sake of learning virtue that the lover should expose himself to the object of his attachment, and not even refuse venereal congress. But while he was reading this licentious defence, Plotinus often rose from his seat, as if he would suddenly leave the assembly: but he restrained himself till it was finished. However, when he left the company, he commanded Porphyry to confute the oration. But when Porphyry desired the orator to lend him his discourse, for this purpose, and was refused, he answered him from recollection, and delivered his answer in the presence of the same auditors as had attended Diophanes. On this occasion Plotinus was so much rejoiced, that he often repeated in the assembly

“Thus write, and you’ll illuminate mankind^[59].”

Our philosopher too, applied himself to the rules of astronomy, though (says Porphyry) not according to a very mathematical mode. That is, as we may presume, he very little regarded the calculation of eclipses, or measuring the distance of the sun and moon from the earth, or determining the magnitudes and velocities of the planets: for he doubtless considered employments of this kind as more the province of the mathematician than of the profound and intellectual philosopher. The mathematical sciences are indeed the proper *means* of acquiring wisdom, but they ought never to be considered as its *end*. They are the bridge as it were between sense and intellect, by which we may safely pass through the night of oblivion over the dark and stormy ocean of matter, to the lucid regions of the intelligible world: and he who is desirous of returning to his true country will speedily pass over this bridge, without making any needless delays in his passage. Plotinus also diligently applied himself to the judgments of the astrologers; but when he found that their predictions were not worthy of belief, he often confuted their presages in his writings.

At that time there were many Christians, and likewise some heathens, who forsaking the ancient philosophers, became the followers of Adelphius and Aquilinus. These men circulated a variety of books of Alexander, Libycus, Philocomus, Demostratus, Lydus; and openly exhibited certain revelations of Zoroaster, Zostrianus, Nicotheus, Allogenes, Mesus, with others of a similar kind. By this means they deceived many, and were themselves deceived, asserting that Plato had by no means penetrated the depth of an intelligible essence. On this occasion, Plotinus urged many arguments in his disputations against these impostors, and composed a book in confutation

of their tenets, inscribed, *against the Gnostics*, leaving a farther discussion of their errors to the labours of his disciples. Hence Amelius composed forty books against the book of Zostrianus; and Porphyry shewed by a variety of arguments, that the writings which they attributed to Zoroaster, were adulterated and recent, and were composed by the propagators of the heresy, that their institutions might pass for the genuine doctrines of the ancient Zoroaster.

Many of the Greeks (says Porphyry) falsely accused Plotinus of privately usurping the doctrines of Numenius, which calumny Tryphon, a Stoic and Platonist, told to Amelius. On this occasion Amelius composed a book, inscribed by Porphyry; *Concerning the difference between the dogmata of Plotinus and Numenius*, which he dedicated to Porphyry. Every one of the books, indeed, of this great man bear such evident marks of original thought, and singular depth, the execution in each is so similar, and the conceptions so uncommonly abstruse, that no one can understand his meaning, and believe him indebted to the labours of others. Porphyry adds, that he was likewise considered by many as a mere trifler, and treated with contempt, because says he they could by no means comprehend his sayings. Besides the manners of Plotinus contributed to produce and increase this disdain, for he was foreign (says Porphyry) from all sophistical ostentation, and pride; and conducted himself in the company of disputants, with the same freedom and ease as in his familiar discourses. With the superficial and the vain, a haughty carriage and severe aspect are considered as the badges of wisdom: but nothing in reality is more foreign from its possession. For true wisdom when it is deeply possessed gives affability and modesty to the manners, illumines the countenance with a divine serenity, and diffuses over the whole external form an air of dignity and ease. Add to this, that Plotinus did not hastily disclose to every one, the syllogistic necessities, which were latent in his discourse. “The same thing (says Porphyry) happened to me, when I first heard Plotinus. On which account I endeavoured to provoke him, by writing against him, endeavouring to shew *that intellections are not external to intellect.*” But after the writings of Porphyry on this subject were read to Plotinus, he said, smiling; “It must be your employment Amelius, to dissolve these doubts, occasioned by his ignorance of our opinion.” After Amelius, therefore, had composed no small book against the objections of Porphyry, and Porphyry had again contradicted his writings, and was once more answered by Amelius: “At length (says Porphyry) having scarcely after all these attempts fathomed the depth of Plotinus, I changed my opinion, wrote a recantation of my error, which I recited in a general assembly, considered the books of Plotinus ever after as most worthy of belief, and provoked my master by every possible means to disclose his opinion in a more particular and copious manner.” This relation is a most egregious instance of the unequalled profundity, as well as excellence of Plotinus’s writings: for who will presume to question the merit of composition, which was at first so difficult to be comprehended, and afterwards so greatly admired by such a genius as Porphyry? By a genius equally accurate, elegant and profound, who knew how to combine the Graces with Philosophy and Science, and to adorn the majestic brows of truth with the flowers of enchanting elocution.

But the testimony of the celebrated Longinus concerning our philosopher, sufficiently evinces his uncommon excellence and worth; and in the present age will probably be more esteemed than the eulogium of Porphyry. In a letter, therefore, which he wrote to Porphyry desiring him to come from Sicily into Phœnicia where he resided, and to bring with him the books of Plotinus, he writes among other things as follows: “These books (meaning those of Plotinus) are not moderately faulty, so that I have no means of using them, though I desire above measure to consider what Plotinus has written concerning the *soul*, and on *being.*” And again: “Do not send these books but bring them with you, and not these alone, but any others which may have escaped the notice of Amelius. *For why should I not enquire, with the greatest diligence, after the writings of this man, which deserve the highest honour and veneration?* This indeed I have always signified to you, both when present and absent, and when you resided at Tyre, that I could not understand many of the hypotheses of Plotinus’s books, *but that I immoderately loved and revered the manner of his writing, the density of his conceptions, and the very philosophic disposition of his questions. And indeed I judge that the investigators of truth ought only to compare the books of Plotinus with the most excellent works.*”

This testimony of Longinus is the more remarkable, as, prior to this, he had for a long time despised our philosopher, through the ignorant aspersions of others. The wonderful genius of Plotinus, was indeed so concealed under the garb of modesty, that before fame had announced his worth, it was only visible to a penetrating and sagacious few. But Longinus (says Porphyry) thought the works of Plotinus which he had received from Amelius incorrect, through the fault of the transcribers, because he was unacquainted with his usual elocution: for if any, the books in the possession of Amelius were correct, because they were transcribed from the manuscripts of Plotinus. Porphyry has likewise preserved the preface of a book composed by Longinus,

inscribed, *concerning the end*, and dedicated to Plotinus and Amelius, in the course of which he thus speaks of our philosopher. “*Plotinus and Gentilianus Amelius are replete with a copiousness of propositions, which they studiously discuss, and have seriously chosen the employment of writing, using a mode of contemplation peculiar, and their own. And Plotinus indeed, as it seems, has more certainly explained the Pythagoric and Platonic principles than his predecessors. For the writings of Numenius, Cronius, Moderatus, and Thrasyllus, are not to be compared, for accuracy in any part, with the books of Plotinus on the same subjects.*”

If such then is the decision of Longinus concerning the abilities and writings of this extraordinary man: of Longinus who is celebrated by one of our first poets, *as inspired by all the Nine*; and whose literary reputation is universal; what judgment must we form of the philosophic taste of the present age, when we find that the very name of Plotinus is known but to a few, and his works scarcely to any? The inference is obvious: let the reader draw it, and lament. But (says Porphyry), if it be requisite to employ the testimony of the wise, who is wiser than a god? Than a god, who truly said of himself:

[60] The number of the sands is known to me,
And the broad measure of the mighty sea.
I know the thoughts within the dumb conceal'd,
And words I hear, by language unreveal'd.

And this is no other than Apollo, who when Amelius enquired, where the soul of Plotinus had emigrated, answered in divine numbers, as follows:

To strains immortal full of heav'nly fire,
 My harp I tune well strung with vocal wire;
 Dear to divinity a friend I praise,
 Who claims those notes a god alone can raise.
 For him a god in verse mellifluous sings,
 And beats with golden rod the trembling strings.
 Be present Muses and with general voice
 And all the powers of harmony rejoice;
 Let all the measures of your art be try'd,
 In rapt'rous sounds, as when Achilles dy'd:
 When Homer's melody the band inspir'd,
 And god like furies every bosom fir'd.
 And lo! the sacred choir of Muses join,
 And in one general hymn their notes combine.
 I Phœbus in the midst to whom belong
 The sacred pow'rs of verse, begin the song.
 Genius sublime! once bound in mortal ties,
 A dæmon now and more than mortals wise;
 Freed from those members that with deadly weight
 And stormy whirl inchain'd thy soul of late:
 O'er lifes rough ocean thou hast gain'd that shore,
 Where storms molest, and change impairs no more;
 And struggling through its deeps with vig'rous mind,
 Pass'd the dark stream, and left base souls behind.
 Plac'd where no darkness ever can obscure,
 Where nothing enters sensual and impure;
 Where shines eternal minds unclouded ray,
 And gilds the realms of intellectual day.
 Oft merg'd in matter by strong leaps you try'd,
 To bound aloft, and cast its folds aside;
 To shun the bitter stream of sanguine life,
 Its whirls of sorrow, and its storms of strife.
 While in the middle of its boist'rous waves,
 Thy soul robust, the deeps deaf tumult braves;
 Oft beaming from the gods thy piercing sight,
 Beheld in paths oblique a heav'nly light:
 Whence rapt from sense with energy divine,
 Before thy eyes immortal splendors shine;
 Whose plenteous rays in darkness most profound,
 Thy steps directed and illumin'd round.
 Nor was the vision like the dreams of sleep,
 But seen while vigilant you brave the deep;
 While from your eyes you shake the gloom of night,
 The glorious prospects burst upon your sight:
 Prospects, beheld but rarely by the wise,
 Tho' men divine, and fav'rites of the skies.
 But now set free from the lethargic folds,
 By which th' indignant soul dark matter holds;
 The natal bonds deserted, now you soar,
 And rank with Dæmon forms a man no more.
 In that blest realm where love and friendship reign,
 And pleasures ever dwell unmixt with pain;
 Where streams ambrosial in immortal course
 Irriguous flow, from Deity their source.
 No dark'ning clouds those happy skies assail,
 And the calm æther knows no stormy gale.
 Supremely blest thy lofty soul abides,
 Where Minos and his brother judge presides;
 Just Æacus, and Plato the divine,
 And fair Pythag'ras there exalted shine;
 With other souls who form the general choir,
 Of love immortal, and of pure desire;
 And who one common station are assign'd,
 With Genii of the most exalted kind.
 Thrice happy thou! who life's long labours past,
 With holy Dæmons dost reside at last:
 From body loosen'd, and from cares at rest,
 Thy life perpetual, and divine thy feast.

[234]

[235]

Now ev'ry Muse who for Plotinus sings,
Here cease with me to tune the warbling strings;
For thus my golden harp with art divine,
Has told, Plotinus! endless bliss is thine.

“According to this oracle then (says Porphyry) Plotinus was worthy, and mild, gentle and endearing, and such as we found him to be by invariable experience. And again, it asserts that he was vigilant, endued with a purified soul, and always elevated to divinity, which he ardently loved. Likewise that he endeavoured by exerting all his powers to emerge from the bitter waters of this sanguine life. Hence when by the assistance of this blessed light, he had often raised himself by intellectual conceptions, to that first god who is superior to intellect, and had ascended according to all the gradations in the banquet of Plato to an union with his ineffable nature, this supreme principle suddenly appeared to him, neither possessing any form, nor any idea, but established above intellect, and every intelligible essence. And to this supreme god I Porphyry once approached, and was united with his nature, when I was sixty-eight years of age. The end of life therefore appeared to Plotinus: for the end and scope of existence to him, was a conjunction with that deity who is every where present. But he four times obtained this end, while I resided with him, not in capacity, but by an ineffable energy. Besides the oracle adds, that the gods often surrounding Plotinus with divine splendors, directed him in the right path, while they benignantly extended to his eyes abundant rays of celestial light: so that he may be said to have composed his books from the contemplation, and intuition of divinity. But from internal and external vigilance, he is said by the oracle, to have seen many and most beautiful spectacles, which no other philosopher has easily beheld. For human contemplation may indeed have various degrees of excellence, but when compared with divine knowledge, cannot fathom a depth, such as is penetrated by the gods. Hitherto the oracle shews, what were the energies of Plotinus, and what he obtained, while surrounded with body. But after his solution from body, it declares that he arrived at the blessed society, where friendship, sweet desire, joy, and love united with the deity, perpetually reign. Besides this, how the sons of the divinity, Minos, Rhadamanthus, and Æacus, are appointed the judges of souls; and that Plotinus departed to these, not for the purpose of receiving their decisions of his conduct, but to enjoy their conversation, with whom also other gods of the most exalted order converse. Where Plato and Pythagoras reside, and other sublime souls, who compose the choir of immortal love; and where the most blessed dæmons have fixed their abode. And lastly, that the life of the inhabitants, in these celestial regions is ever flourishing and full of joy, and perseveres in perpetuity of bliss, from the benignant communications of divinity.” And thus much for the life of Plotinus, who was a philosopher unequalled for the strength and profundity of his intellect, and the purity and elevation of his life. He was a being, wise without the usual mixture of human darkness, and great without the general combination of human weakness and imperfection. He seems to have left *the orb of light*, solely for the benefit of mankind; that he might teach them how to repair the ruin contracted by their exile from good, and how to return to their true country, and their proper kindred and allies. I do not mean that he descended into mortality for the purpose of enlightening the vulgar part of mankind: for this would have been a vain and ridiculous attempt. The splendour of truth cannot be apprehended by eyes totally fixed in the dark night of oblivion; but previous to this, punishment must be inflicted, and purgation employed; the labours of Hercules must be accomplished, and the sufferings of Ulysses endured. But he came as a guide to the liberal few, who are struggling to gain the lost region of light, but know not how to break the fetters by which they are detained; and who are impatient to leave the obscure cavern of sense, where all is delusion and shadow, and to ascend to the realms of intellect, where all is substance and reality.

Let us now consider what were the principal tenets of the Platonic theology, which this extraordinary man restored, and illustrated in his writings. And, in the first place, he every where profoundly and copiously proves the super-essential nature of *the one*, or the supreme principle of things, which is one of the principal doctrines in the *Parmenides*, and is more plainly asserted in the *Republic* ^[61]. However prior to Plotinus ^[62], the interpreters of Plato ascended no higher than to intellect and being, and by this means placed a compound, and not a perfectly simple nature at the summit of the universe. This doctrine, which is called by Cudworth, *high-flown*, *phantastical*, and *unsafe*, will, I am sure, be deemed no better than *jargon* and *reverie*, by modern philosophers, who, so far from being able to conceive a cause superior to being, scarcely possess a thought which is not the progeny of body and sense. It is, however, sufficient for our present purpose to prove that this is the opinion of Plato, and to shew how it is defended by Plotinus. In the sixth book of the *Republic* then, Plato most beautifully evinces the super-essential nature of the first cause, whom he calls the good, by his analogy to the sun, as follows. “Have you not observed, with respect to the author of the senses, in how perfect a manner he has formed the power of sight, and of being visible? I have not entirely perceived it, replied he. But consider it in this manner. Do hearing and

sound require any other species, in order that the one may hear, and the other be heard, which third nature when absent, the one shall not hear, nor the other be heard? There is nothing, said he. It appears to me, indeed, that many others (that I may not say none), require no such third species. Or are you able to find any such power? By no means. Have you not perceived that sight, and the object of sight, require such a nature? How? When sight is present with the eyes, and they are directed to vision, when colour also is present, unless a third species is present, naturally formed for the purpose, the sight will be without vision, and the colours will be invisible. But what do you call this? What you call light. You speak the truth. Indeed, the sense of seeing, and the power of being seen, are joined together by a bond the most honourable of all other conjunctions, and by no trifling form, if light be not dishonourable. Whom then of the celestial gods can you assign as the cause of this, that light causes our sight to see in the best manner, and that objects are perceived by the eyes? The same as you and others assign; for you interrogate concerning the sun. But the sight is thus affected with reference to this god. How? Neither is the sight the sun; nor is the eye in which vision resides the sun. It is not. But of all the organs of sense, the eye participates most of the sun. Greatly so. Does it not preserve the power which it possesses, as infused from the sun? Entirely. Besides, the sun is not sight, but its cause, and is on this account beheld by sight. It is plainly so. This is what I called the son of the good, which the good generated analogous to itself: that as *this* in the intelligible place, is to intellect and the objects of intelligence, so is *that* in the visible place to sight. How is this? Explain yourself more largely. You know that the eyes as often as they are not turned towards objects whose colours the splendour of day irradiates and discloses, but to such as are faintly illuminated by nocturnal rays, grow dim by the vision, and appear almost blind, as if perfect sight was not resident in their nature. So it happens. But as often as they are turned to objects which the sun illustrates, they perspicuously perceive, and in the very same eyes, sight appears to be contained. It is so. Thus think also concerning the soul. For when it adheres to that in which truth, and being itself shines forth to view, then it understands and knows this, and appears to possess intelligence. But when it is carried to that which is mingled with darkness, which is generated and destroyed, the sharpness of its sight is blunted, it is conversant with various opinions, and it seems to be destitute of intellect. So it appears. Hence, that which affords truth to objects of intelligence, and extends the power of intellection to him who understands, you may call the idea of the good, the cause of science, and truth perceived by intellect. But since these two are so beautiful, I mean knowledge and truth, if you esteem the good itself, as something different from these, and far more beautiful, you will think in a proper manner. And, as it is proper to believe, that light and sight possess a certain form of the sun, but are by no means the sun itself: so here it is proper to judge, that knowledge and truth possess a certain form of the good itself, but are by no means the good; for its majesty is far more venerable and august.” And a little after he adds: “You may say therefore, that the good, not only affords to objects of knowledge the power of being known, but likewise distributes their being and essence, while in the mean time *the good itself is not essence, but above essence*, excelling it both in dignity and power.”

It is plain, therefore, from the words of Plato himself, that he considered the supreme principle of things superior to being; and consequently this doctrine was not devised by the latter Platonists, contrary to the opinion of their divine master. But this is likewise evident from the testimony of Speusippus, the immediate successor of Plato, who, as we are informed by Proclus^[63] confirmed this doctrine from the most ancient authority, and asserted, “that the ancients considered *the one*, as better than being, and that the principle of being was free from all proportion to the subsequent order of things, as the good itself is separated from every condition of any particular good.” To this most respectable evidence we may also add, that of the philosopher Moderatus, who, as we are informed by Simplicius^[64], declares, “that, according to the Pythagoreans (from whom Plato, it must be observed, received the greater part of his philosophy), the first one is above all essence.”

This sublime theory was supported by Plotinus, with all that truly philosophic accuracy and depth, ^{fd239} which his writings are every where so remarkable. Indeed, it appears to have been his favourite topic; for he has employed considerable parts of many of his books in its illustration and defence. Nor can we wonder at his partiality for this exalted speculation, if we consider that a union with this ineffable nature, was the great aim of all his desires, and the only end of all his studies and pursuits. This was the divinely solitary light to which his intellectual eye was ever directed, and which so abundantly illumined the most secret recesses of his soul. Here he discovered the true fountain of good, and drank deep of its perennial streams. And lastly, here he derived those inestimable stores of knowledge, which he so fortunately transmitted to future generations. That the English reader, therefore, may have a specimen of his inimitable writings on this abstruse subject, and may see some of the deepest mysteries of the Greek theology disclosed, I shall present him with a paraphrased translation of two books of Plotinus: the first of which is inscribed, *That Intelligibles are not external to Intellect, and concerning*

the Good, and the other, *Concerning intelligible Beauty*. I have particularly chosen these, not only because they admirably unfold the depths of the Platonic philosophy and theology; but because the first relates to the vision of the *supreme*, explaining the wonderful manner in which it is accomplished; and the second describes the method of becoming united with the intelligible world. The Platonic reader will find in these books (if I have done justice to their divine author), instances of sublimity beyond all comparison with any other writings; and specimens of a profundity of thought unequalled by any other philosopher. I am sensible that the great labour I have employed in the translation, will be, most probably, lost on the present generation: but though I write with no views nor desires of popular renown, yet I flatter myself with the approbation of more equitable posterity. The fifth book, therefore, of the fifth Ennead of Plotinus is as follows:

“Is it possible any one can think that true intellect, possessing true being, can at any time be deceived⁽²⁴⁾, and believe in things which have no real existence? Certainly no one. For how could it be intellect, if it is ever liable to deception? It is requisite, therefore, that it should always understand, and that nothing should ever be concealed from it, like those natures that are subject to oblivion. But it is likewise necessary that knowledge should reside in its essence, not like one imagining or doubting, or deriving information from another. Nor yet again, like knowledge collected from demonstration. For though it is granted that some things are collected by demonstration, it cannot likewise be denied that something is of itself known to intellect, at the same time that reason dictates, that all knowledge is essential to its nature. But it is now necessary to enquire after what manner we must distinguish the essential knowledge of intellect, and that which it obtains by investigation. Likewise from whence the certainty is derived to intellect, of its essential knowledge? From whence its faith is derived, that it is in such a condition? Because about things offered to the senses, the belief of which appears more certain, it is usual to doubt whether they possess their apparent nature in the subject things, or in certain passions only; where certainly the judgment of intellect, or, at least, of thought, is required. For though it should, perhaps, be granted, that the natures of sensible objects are contained in their subject bodies, yet what is known by sense, is nothing more than an image of the object; for sense cannot apprehend the thing itself, since it abides external to its perception. But intellect when it understands and apprehends intelligibles, if it knows these as something different from itself, after what manner is it connected with them? For it may happen that it shall not meet with them, and consequently that it may not understand: or perhaps then at last when it meets with them it will immediately understand, and thus it will not always possess intellection. And if it should be said, that intelligibles are conjoined with intellect, it remains to enquire what such a conjunction means. Besides the intellections themselves will be certain figures: and, if this is the case, they will be adventitious, and nothing more than certain pulsations. But after what manner will intellect be figured; and what will be the form of intelligibles? Lastly, from this hypothesis intelligence will be like sense, a perception of externals. After what manner then do these disagree among themselves? Shall we say in this, that one of them comprehends lesser concerns? Also, how can intellect know that it perceives something in reality? Or how will it be able to judge that this is good, or beautiful, or just? For every one of these will be different from intellect, nor will it contain the principles of judging by which it believes, but these also will be external to its essence; and in the same manner truth. Again, intelligibles themselves, are either destitute of sense, life and intellect, or they possess intellect. If they possess intellect, they will equally contain both, and this will be the true and first intellect. But of this also we enquire how it contains truth, intelligible itself, and intellect. Whether subsisting in the same and together, or in some other manner? But if intelligibles themselves are destitute of intellect and life, we must enquire what they are. For they are neither certain propositions, nor axioms, nor dictions. For if this were the case they would affirm something of other things, but would not be things themselves: as if they should say, that what is just is beautiful, when at the same time justice itself is different from the beautiful itself. But if they should consider as simple essences, the just itself, and the beautiful itself, apart from each other: in the first place, intelligible itself will not be a certain one; but every intelligible will be separate from others. In which case we must enquire where they are, and in what places they are separately disposed: afterwards in what manner intellect every where running round in a discursive procession, is able to find these: also how it abides: and again, how it abides or perseveres in the same; and what form or figure it is endued with. Unless, perhaps, intelligibles are situated like certain images formed from gold, or from some other matter by a statuary or painter. But if this be the case, intellect in its perceptions will be the same as sense. Besides in what respect among these, is *this* intelligible, justice, but *that*, something else. Lastly, this is the most powerful objection of all: viz. if any one should entirely admit, that these are extrinsical, and that intellect speculates them as having an external position, it necessarily follows that intellect does not possess the truth of these, but is deceived in the contemplation of each. For the object of its contemplation will be truly external: it will therefore behold them deprived of their intimate possession, and

containing only their images in a knowledge of this kind. Since, therefore, it does not possess truth itself, but only contains certain images of truth, it will possess what is false, and have nothing of truth. If then it knows that it contains only what is false, it must undoubtedly confess itself to be destitute of truth: but if it is ignorant of this, and thinks that it participates of truth, when at the same time it is destitute of its possession, it is deceived by a two-fold fallacy, and is very far distant from truth. For it is on this account, as I think, that truth is not to be found in sensible objects but opinion alone: because opinion is conversant in receiving, from whence its name is derived. On this account it receives something different from itself, since that also is different from which it possesses what it receives. If then truth is not resident in intellect, such an intellect cannot be truth, nor a true intellect, nor intellect at all: nor indeed will truth be resident in any other place.

Hence it is not proper either to investigate intelligibles separate from intellect, or to confess that the figures of things only are contained in intellect, or to deprive it of truth, while we admit it is ignorant of intelligibles, and that the objects of its intellection have no existence in the order of things. But it is necessary to attribute all things to true intellect, if it is requisite to induce knowledge and truth; to preserve beings themselves; and that knowledge by which the essence of every thing is known; and no longer to acquiesce in the resemblances and images of things, as when we alone understand the particular mode of existence, and not the real essence of a thing; in this case neither possessing the object itself, nor dwelling with it, nor conspiring into one with its nature. For intellect indeed truly knows, nor is any thing concealed from its essential intelligence, nor is it liable to oblivion, nor does it wander by investigation, but it contains truth, and the seat of things in its essence, and is ever vital and intelligent. All which properties, indeed, ought to reside in the most blessed nature, or where can any thing honourable and venerable be found?

Hence it neither requires demonstration, nor the faith of persuasion, that intellect is thus essentially intelligent: for it is entirely manifest to itself, and there is nothing more worthy of faith than its own essence. So that it contains real truth, not consonant to any other but to itself, nor does it pronounce and exist any thing besides itself: and that which it is, it pronounces. Who then can confute it? And from whence can he bring his confutation? For the argument which is adduced must revolve into the same with the former. And although it is employed as different, it is nevertheless referred to the thing proposed by the first argumentator, and is with it entirely one and the same. For nothing can be found more true than truth.

This one nature intellect therefore is all beings: it is truth: it is a great deity: or rather it is not any particular god: but is deservedly every deity. And such is the nature of this second divinity, appearing to beholders, before they survey that superior God, who is seated in sublimer majesty on the illustrious throne of intellect, depending from his ineffable nature. For it is highly proper that he should not subsist in an inanimate seat, nor again immediately occur to us moving in the circular chariot of soul, but that an inestimable beauty should wonderfully shine before his appearance, as before the presence of a mighty king. For to such as advance to his intuition it is ordained that lesser things should first occur and afterwards that such as are greater should gradually present themselves to the view; and that such as surround the king should be more royal, and the rest in a degree proportionate to their distance from his ineffable glory. But after all these, the mighty king himself, suddenly shines forth to the view: while the rest venerate the king, in a suppliant manner; such I mean as do not depart from thence till they have proceeded to the last spectacle of all, like those who are satisfied with the splendor of the attendants on majesty. Another king, therefore, reigns in this intelligible world, and his attendants are different from his nature. But this supernal king does not rule over foreign subjects, but he possesses a just and natural government, and a true kingdom: since he is himself the king of truth, and is naturally the lord of his offspring the universe, and of the divine company of immortal gods. Hence he is the king of a king, and of kings, and is called by a juster name, the father of the gods. Whom indeed Jupiter in this respect imitates, since he does not acquiesce in the contemplation of his Father, but proceeds beyond this to his grand-sire, as to an energy in the very subsistence (ὑπόστασις) of his essence.

But let us now ascend to the one itself, which is indeed truly one, not like other things which at the same time that they are many, are also one through the participation of unity. For we must now receive one itself, which is not one by participation, like such things as are not more truly one than many. We must likewise assert that the intelligible world is more one than other things, and that nothing is nearer than this to unity itself: at the same time that it is not purely one.

But for the present we desire to contemplate, if possible, the nature of the pure and true one, which is one from another, but from itself alone. It is therefore here requisite, to transfer ourselves on all sides to one itself,

without adding any thing to its nature, and to acquiesce entirely in its contemplation; being careful lest we should wander from him in the least, and fall from one into two. But if we are less cautious we shall contemplate two, nor in the two possess the one itself; for they are both posterior to unity. And one will not suffer itself to be numerated with another, nor indeed to be numbered at all: for it is a measure free from all mensuration. Nor is it equal to any others, so as to agree with them in any particular, or it would inherit something in common with its connumerated natures; and thus this common something, would be superior to one though this is utterly impossible. Hence neither essential number, nor number posterior to this, which properly pertains to quantity can be predicated of one: not essential number whose essence always consists in intellection; nor that which regards quantity, since it embraces unity, together with other things different from one. For the nature pertaining to number which is inherent in quantity, imitating the nature essential to prior numbers, and looking back upon true unity, procures its own essence, neither dispersing nor dividing unity, but while it becomes the duad, the one remains prior to the duad, and is different from both the unities comprehended by the duad, and from each apart. For why should the duad be unity itself? Or one unity of the duad rather than another, be one itself? If then neither both together, nor each apart is unity itself, certainly unity which is the origin of all number, is different from all these; and while it truly abides, seems after a manner not to abide. But how are those unities different from the one? And how is the duad in a certain respect one? And again, is it the same one, which is preserved in the comprehension of each unity? Perhaps it must be said that both unities, participate of the first unity, but are different from that which they participate: and that the duad so far as it is a certain one participates of one itself, yet not every where after the same manner: for an army, and a house are not similarly one; since these when compared with continued quantity, are not one, either with respect to essence, or quantity. Are then the unities in the pentad, differently related to one, from those in the decad? But is the one contained in the pentad, the same with the one in the decad? Perhaps also if the whole of a small ship is compared with the whole of a large one, a city to a city, and an army to an army, there will be in these the same one. But if not in the first instance, neither in these. However, if any farther doubts remain, we must leave them to a subsequent discussion.

But let us return to unity itself, asserting that it always remains the same, though all things flow from it as their inexhaustible fountain. In numbers, indeed, while unity abides in the simplicity of its essence, number producing another is generated according to this abiding one. But the one which is above beings, much more abides in ineffable station. But while it abides, another does not produce beings, according to the nature of one: for it is sufficient of itself to the generation of beings. But as in numbers the form of the first monad is preserved in all numbers, in the first and second degree while each of the following numbers do not equally participate of unity; so in the order of things, every nature subordinate to the first, contains something of the first, as it were his vestige or form in its essence. And in numbers, indeed, the participation of unity produces their quantity. But here the vestige of one gives essence to all the series of divine numbers, so that being itself, is as were the footstep of ineffable unity. Hence he who asserts that τὸ εἶναι, which is a denomination declarative of essence, is derived from τὸ ἓν, that is, *one*, will not perhaps deviate from the truth. But that which is called τὸ ὄν, that is, *being*, first of all shining forth from the depths of unity, and as it were not far proceeding from thence, is unwilling to advance beyond its original, but abides converted to its most interior retreats, where it becomes essence, and the essence of all things, and that which pronounces these; containing itself as it were in its labouring with sound; and declaring by its speech that it flows from one: and indeed τὸ ὄν thus pronounced, signifies its origin as much as possible. So that what becomes οὐσία, that is *essence*, and εἶναι or *to be*, imitate to the utmost their author, from whose unwearied power they perpetually flow. But intellect perceiving this, and being moved by the spectacle, and imitating what it knows, suddenly produces with an energetic voice the words ὄν, εἶναι, οὐσία, ἔστιν. For these sounds endeavour to express the substance of that which is generated, (the pronouncing nature labouring with the expression;) and imitate as much as possible the origin of being itself.

But this must be left to every one's particular determination. But since generated essence is form, (and that which is produced from thence can have no other appellation,) it is not a particular form, but universal, so that nothing else than this general form remains to species; and therefore it is necessary that one itself should be destitute of form. But since it is foreign from species neither can it be essence: since it is requisite that essence should be something determinate. But it is not lawful to consider unity itself as any thing particular and bounded, otherwise it would not be the principle, but that alone which you denominate something singular. If then all things are contained in that nature which is generated from the first, we must truly say that the author of all things, is not any one of these, and that he can alone be called that which is above all. But the natures produced from thence are beings, and being itself; and hence the one itself is superior to being. And that which is above

being, does not say I am this, nor does it determine any thing concerning its nature, nor does it tell its name, but it alone pronounces, *I am not this*, i.e. *I am nothing comprehensible and definite*. But it is impossible by this means, to comprehend its nature; since it is ridiculous to attempt to comprehend immensity itself. So that whoever attempts it, removes himself far from the least vestige of this nature. For as he who desires to know intelligible essence, then only perceives what is above sense, when he possesses no image of a sensible object: so he who desires to contemplate a nature superior to intelligible essence, will enjoy the ineffable vision, if he neglects every thing intelligible, while merged in the most profound and delightful of all contemplations; learning from hence, that he is, but neglecting the enquiry into what he is, as impossible to investigate. For this which is called *such*, signifies when applied to him, not such: since the appellation of such cannot belong to a nature, to whom the predication *what*, is not applied. But we labouring as it were with our difficulty of conception, are ignorant what denomination is proper to his nature, and desiring as much as possible to signify something to ourselves give a name to that which is ineffable. But perhaps this name which is called one derives its appellation from a certain negation of many. On which account the Pythagoreans denominated him Apollo, according to a more secret signification, which also implies a negation of many. But if any one establishes this name one, and affirms something according to its signification, both the name and the thing named will be more obscure than if its appellation had been entirely neglected. For perhaps the name was expressed that the investigator beginning from something signifying the greatest simplicity of all might arrive at that perfection of contemplation, as even to deny him the appellation of one; convinced that the best name indeed had been assigned him, but that it was unworthy to express the superlative excellence of his nature. For *this* cannot be reached by the hearing, nor be understood by any hearer: but if it is manifest to any one, it must be to the profound beholder. But if he who perceives, endeavours to behold form, he will lose the intuition of this ineffable nature.

Again, the energy of vision is two-fold, as it happens with respect to the eye. For one thing, indeed, is a spectacle to the eye, that is, the form of the sensible object, but another, that by which it perceives the form, and which though itself sensible, is different from the sensible form. Hence it is the cause by which form is beheld, is inherent in form, and is perceived connected with its nature: though on this account it is not clearly perceived, since the eye more intently directs itself to the illuminated object than to the illuminating cause. But when there is nothing besides itself, it is beheld with a sudden and universal vision, though it should then be perceived adhering to some other object: for if it was entirely separate and alone, it could not be subject to sensible inspection; since the light of the sun flourishing in the sun itself, would perhaps escape our sense, unless its more solid orb was the subject of its splendor. But if it should be said that the whole sun is light, it is perhaps only asserted for the sake of explanation: for light is in no form of other visible objects, and is perhaps nothing else than that which is visible^[65], while other things are visible, but not light alone; since their natures are various and composite. In like manner the eye of intellect, sees from another light things illuminated by that first nature, and in them it truly sees their illuminating source. But when it too earnestly converts itself to the nature of the illuminated objects, it perceives less their splendid original. And if at any time it should dismiss the visible objects, and attentively survey the light by which it perceives, it will then view light itself, and the principle of light. But because it is requisite that intellect should behold a light of this kind, not as any thing external; let us return again to the example of the corporeal eye, which on a time does not perceive external and foreign light, but previous to this beholds a light more peculiarly its own, and by far more lucid, shining in a certain inviolate and pure seat; either when it perceives before itself a ray darting from its transparent receptacle, through the darkness of night; or when not disposed to behold other objects, it confines itself under the covering of the eye-lids, and in the mean time produces from itself a purer light within; or lastly, when some one by pressing the corners of his eye-lids, views the inward light of the eye. For then, indeed, by not seeing he sees, and then sees in the most exalted degree; for he views light itself: while other things which were the objects of this vision before, were indeed lucid, without being light. In like manner intellect concealing and separating itself from all other concerns, and confining itself in its most inward retreats, and perceiving nothing, will immediately behold light, not subsisting in another, but by itself alone, perfectly pure, and suddenly shining from itself, with a splendor ineffably sacred and divine.

But in this case it will be doubtful from whence such a light shines; whether from something external, or rather from an internal source: and again when it departs we may happen to say, this was something intimate, and again not intimate. But, indeed, it is not lawful to enquire from whence it originated, for it neither approached hither, nor again departs from hence to some other place, but it either appears to us, or does not appear. So that we ought not to pursue it, as if with a view of discovering its latent original, but to abide in quiet, till it suddenly

shines upon us; preparing ourselves for this blessed spectacle, like the eye waiting patiently for the rising of the sun, until appearing above the horizon, and emerging, as the poets say, from the bosom of the ocean, he presents himself to the sight. But from whence does this light which the sun imitates supernally shine? And what is the nature which it transcends, when it perspicuously presents itself to our view? Indeed it illuminates intellect, intently surveying its lustre. So that intellect stops itself in beholding, as having now arrived at the desired end of its vision, looking upon nothing else than the beautiful itself; converting itself wholly to its contemplation, and dedicating itself entirely to its enjoyment. Hence abiding in this delightful state, and as it were replete with divine vigour, it beholds itself in the first place now become more beautiful and refulgent, as being nearer to that which is highest and best. But he will not approach in the manner some may expect; since he will come as if not coming. For he will be present before and above all things, even before intellect approaches to the vision. But it is intellect which properly approaches and departs; which departs indeed when it is ignorant where it should abide, and where this divine principle abides: because indeed it truly abides in no being. And if intellect could be no where (I do not mean with respect to place only since this also is free from the affections of place) but entirely no where, it would doubtless always behold his divinely solitary nature, although it would become united with him, not as perceiving, but as abiding in his nature; and this not as if intellect and this highest principle were two. But now because it is intellect, it thus sees when it sees, by that which it contains different from intellect, and which is the very summit and flower of its essence. And, indeed, it is wonderful in what manner this first god is present without approaching, and how while he is no where, he is at the same time every where. This indeed is wonderful from its very condition, but to him who profoundly knows the thing itself, it would rather be admirable if the contrary should be affirmed. Or rather, indeed, it cannot exist otherwise than as the object of vehement admiration. For such is the nature of the supreme.

Whatsoever is produced by another, is either contained in its author, or in some other nature, if anything besides its author remains: for since it is produced by another, and requires something different from itself to its generation, it every where requires another nature for its support, and consequently reposes in another, from the necessary indigence of its being. And thus it is appointed by nature, that such things as are last, should be established in such as are immediately above them: and again things prior to these, in such as are similarly prior, and always one thing in another up to the first principle of all. But the highest principle, because he has nothing prior to his nature, cannot subsist in any other. And hence because he is not in another, but others subsist in their superiors, on this account he comprehends all things in the immensity of his nature. But while he embraces them, he is not dissipated into their essence, since he contains them without being contained; yet in this case, there is nothing exists, with which he is not present: for unless he was present he could not contain: and again, if he did not contain, he could not be present. So that he is present, and yet not present: for, because he is not comprehended by any thing, he is by no means present; but because he is free from all circumscription, he is not hindered from being present every where: for if he were restrained, he would certainly be defined by some particular being, and subsequent natures, would be left destitute of his presence; and, thus far the first deity would reign, nor would any thing farther subsist in his nature, nor would he abide in himself, but become subservient to others. Whatever, therefore, subsists in any thing different from itself, is properly there, where it subsists. But such as are not any where, are on this account present every where. For whatever is excluded from some particular place, is comprehended in some other, so that it is false to affirm of such a nature that it is not contained some where. If then it is true that the supreme principle is not in any particular place, and false, that he is somewhere (lest he should be contained in another), he is on this account absent from no being or place. But if he is no where absent, certainly because he is not somewhere, he will be every where present in himself: for one part of him will not be here, and another there, nor yet the whole of him in one particular place only, so that he will be every where totally present; since no one being contains him, nor yet in another sense does not contain him, since he is so contained, that he may rather be said to contain. But in order to illustrate the present subject, let us consider our visible universe, for if there were no other world superior to this, it would neither be contained in the world, nor yet in place. For what place could there be prior to the existence of the world? But the parts of the world are reduced to the universe, and are placed in its comprehensive bound. And soul is not in the world, but rather the world is in soul: for neither is body the place of soul, but soul is in intellect, and body in soul. Lastly, intellect abides in another, which is no longer dependent on any thing superior, and in which it is compelled to repose: so that the highest principle is properly contained in no other, and is on this account said to be no where. Where then do other things subsist? Doubtless in that which is first. Hence he is neither absent from others, nor is contained in them, while at the same time he contains all things in the immensity of his nature. Hence too, on this account he is considered as the good of the universe; because all things subsist by him, and are

referred to him as their divine original. But they are so referred to him, that some are more excellent than others, because some are more proximate than others to his ineffable nature.

But let me intreat you, not to endeavour to perceive him through the medium of other natures, for otherwise you will not discover the highest principle himself, but only a vestige of his divinity. But consider with yourself what that is, which can alone be perceived abiding in itself, perfectly pure and unmixed, and which is of such a kind that all things participate yet none contain its nature; so that nothing else can be such as he is, and yet it is necessary that such a nature should subsist. What being then can at once apprehend the whole of his power? For if any one apprehends the whole, in what respect does he differ from his nature? Must he be received then according to a part? But you who are intent on beholding him, should survey him with a universal vision, and at the same time be cautious not to tell yourself the whole of your perception, or you will become intellect, intelligent: but he will immediately fly from your intuition, or rather you will retire from him. But when you behold, behold him totally; and when you energize with intellect concerning him, whatever you retain in your memory of his nature, be careful to understand it as the good. For he is the cause of a wise and intellectual life; since he is that power itself, from which life and intellect is produced; and he is the author of essence and being, because he is the one itself. And he is perfectly simple, and the first, because he is the principle of all. For all things flow from him as their original source; motion first proceeded from him, yet is not contained in his nature; station likewise originates from him, because he is superior to want: for he is neither moved, nor at rest, since he contains nothing in which he can either repose or revolve. For about what, or to what, or in what can he either be moved, or repose, since he is the first? But neither can he be defined, for what can bound his nature? Nor yet again, is he infinite, like an immense bulk. And where can he be said to advance, as if he were indigent, who is in want of nothing? But his power contains infinity itself. Nor is he ever deficient, since beings who are superior to defect derive this perfection from the inexhaustible plenitude of his nature.

But this infinite is so called, because it is not more than one, and because it does not contain any thing, by which any part as it were of its nature can be bounded. Indeed, from its being one, it is neither measured, nor proceeds into number; and therefore is neither terminated by another, nor by itself: for if this were the case it would become two. Nor again has it any figure, because it has no parts, nor form. Do not, therefore, seek after its ineffable vision with mortal eyes; nor attempt to perceive by any corporeal means, that which reason proves to be so remote from the comprehension of sense. Do not, I say, think it can be known in the manner they imagine, who consider all things as sensibles; and thus entirely subvert that which *Is*, in the most exalted degree. For those things, which some consider as having the most real being, have the most unreal. And that which is great in quantity is least in being: but that which is first in the principle of being, and something more excellent than essence; so that our opinion must become the very opposite to this, or we shall be destitute of the union with this most exalted deity. Just as those who in solemn festivals, through a shameful gluttony, fill themselves with food which it is unlawful for those to touch who intend an entrance to the gods; esteeming the aliment of the belly more certain than the contemplation of the god whose rites are to be celebrated, and on this account they depart destitute of the sacred visions. For in such holy rites when the god is not beheld, his existence is denied by those who consider as alone certain that which is tasted and perceived by the flesh. Just as if any one should be lost in sleep through the whole of life, and should therefore believe in the visions of sleep, as alone certain and real. But if any one happens to rouse him, as one who does not believe in objects beheld with open eyes, should suddenly return again to sleep, and the delusions of dreams.

Again, it is requisite for the purpose of perceiving, to assume that organ by which each particular ought to be beheld. The eyes for some, the ears for others, and so of the rest. And it is necessary to believe, that other things are the peculiar objects of intellect, and that to understand is not the same as to hear and to see; for this would be as absurd as if any one should command the ears to perceive, and should on this account deny the existence of voices, because they are not the objects of sight. Hence, we must consider such as these ignorant of that which from the beginning to the present day they desire and affect: for all things desire that which is first from a necessity of nature, prophesying, as it were, that they cannot subsist without the incomprehensible energies of his nature. Besides the knowledge of beauty, happens to such souls as are roused and knowing; and is attended with a stupor, and the excitation of love. But good, because present from the beginning to our innate appetite, abides with us even when asleep, and never seizes its spectators with astonishment, because it is always present, and requires no peculiar reminiscence to convince us of its presence. But the love of beauty, when it first offers itself to the view, produces molestation, because it is requisite to seek after beauty by knowledge: but a love of this kind since it is the second, and belonging to those who are intelligent, plainly indicates that beauty is itself the

second; and the desire of good, since it is more ancient, and does not require the assistance of the senses, testifies that good itself is more ancient than beauty, and is superior to its nature. Add to this, that all beings think they shall be sufficient to themselves, if they obtain good; as if secretly convinced they shall then at length arrive at the desired end: but all do not think the possession of beauty, will be sufficient to the completion of their wishes. Besides some judge that what is beautiful, is beautiful to itself, but not to them, as is the case with this our apparent beauty. For they judge that its possessor is beautiful; and consider it sufficient to appear beautiful though deprived of its real possession: but they do not desire to possess good in opinion, but in reality. For all things especially strive to procure for themselves that which is first; and contend with beauty, as it were with a desire of victory, as if conscious it was generated, as well as themselves. Just as if some one posterior to a king, should study to equal in dignity another who immediately follows the king, and is the next to him in royal pre-eminence; because he depends on one and the same principle as his rival, being ignorant, indeed, that he himself depends on the king, but that the other precedes him in priority and perfection of nature. But the cause of the error is their both participating of the same; and one itself being prior to both. Besides it appears that good itself is by to means indigent of the beautiful, but the beautiful cannot subsist without the good. Hence good is gentle, mild, placid, delicate, and such as every one wishes it to occur. But beauty either renders the soul stupid, or mingles the excited pleasure with grief. Lastly, it often causes incautious souls to deviate from good, as the beloved object often separates the lover from his parent. For beauty is of a junior nature, but good is more ancient, not indeed in time, but in truth, because it possesses a prior power: for it possesses universal power. But that which is subordinate to the good, does not receive all power, but such only as it is requisite for a nature posterior to the first, and originating from him to receive. So that he is the lord of this posterior power, and is in no respect indigent of his offspring, the beautiful, since he existed such as he is prior to its generation; and would have suffered no loss in the perfection of his nature, if this had not been generated. And if some other could be produced from his nature, he would not envy it the possession of being. But now nothing farther can be generated: for nothing remains, which has not been already produced, since the universe is complete. But this highest principle is not all things, for in this case he would be indigent of all: but surpassing all things, he is able to produce and permit all things to themselves; while, at the same time, he is eminently exalted above all by the incomprehensible dignity of his nature.

But since the supreme principle is good itself and not merely good, it is requisite he should contain nothing in himself, since he does not even contain good. For if he possessed any thing, he would either possess good, or that which is not good: but in that which is properly the first good, non-good, can have no subsistence; nor yet can good itself contain good. If then it neither possesses non-good, nor good, it contains nothing; and if it contains nothing it is alone, dwelling in solitary unity, retired from the universality of things. If then other natures are either good (yet not good itself); or, perhaps, such as are non-good, but he contains neither of these, certainly by possessing nothing he is good itself. If then any one adds to his nature either essence, or intellect, or beauty, by such an addition he deprives him of being the good itself. As on the other hand by taking away all things, and affirming nothing concerning his nature, nor deceiving in any respect, as if something was present with his nature, we shall permit him to be what he is; testifying concerning him none of these properties of being, which are not present with a cause so sublimely remote from essence itself. In which respect those for the most part err, who, when they are ignorant how any one ought to be praised, detract from the glory of the subject of their praise, while they add such things to his nature, as are beneath its dignity; not knowing how to accommodate true praise to its proper object. On this account we ought also, in the first place, to beware, lest we add any thing posterior, and unworthy to the divine object of our praise; and to observe that he who surpasses all these, is, indeed, their proper cause without possessing any of their properties and affections. For the nature of good does not consist in being either all things, or some one particular of all. Since, if he was some one particular of all, he would be contained under one and the same nature together with all. But if he is under one and the same nature together with others, he will vary from others, only by a certain proper difference and addition. Hence, in this case, he will be two and not one; one part of which two, I mean that which is common to it with the rest, will be non-good: and the other will be good. He will, therefore, be mixed from good and non-good, and consequently will not be the pure and first good. But that will be the first good, of which this participating becomes good beyond the common condition. This then will be good by a certain participation: but that of which this participates, will be none of the universality of things; and such, therefore, must be the condition of the good itself. But if this too contains good as a part, for it is difference by which this is a composite good; it is necessary that this should depend on another, which is entirely simple, and alone good. And hence this which is various depends upon that which is good alone. So that it appears, that what is first, and the good itself, is above all beings, is good alone, and contains nothing in

its nature, but is perfectly free from all mixture; and that it is above all, and is the divinely solitary cause of all. For neither does beauty nor being originate from evil, nor yet from such things as are indifferent: for the efficient is better than the effect; since it is more perfect and divine.” And this much for the first book of Plotinus, which we proposed to insert; the other on intelligible beauty is as follows:

“Since we must confess that the soul which contemplates the intelligible world, and beholds the beauty of it, the true intellect, may also perceive the father of this divine world, who is superior to intellect: let us now endeavour to the utmost of our ability to behold, and to express to ourselves (as much as such things can be expressed) how we may in the best manner survey the beauty of intellect, and the world which it contains. Suppose then, two stony masses placed near each other, one of which is incomposite, and destitute of artificial form: but the other is fashioned by art into some divine, or human statue. And if divine, let it be the statue of some Grace or a Muse: but if human, not that of any particular man, but rather of some one which art has collected together from all beautiful forms. The stone then which is disposed by art into the beauty of form, will immediately appear beautiful, but not because it is a stone; or other mass would be similarly beautiful; it is therefore beautiful because it possesses the form which art applies. Matter, therefore, had not this form, but it existed in the thinking artist before it came into the stone. But it was in the artificer, not on account of his possessing eyes and hands, but because he was endued with art. This beauty, therefore, existed in art in a much more excellent manner. For the form itself which abides in art does not proceed into the stone, but this abides in indivisible union, while an inferior form proceeds from this, which neither remains in itself pure, nor is such as the artist wishes, but such as the subject matter is capable of receiving. But if art operates according to what it is, and to what it possesses, but it fashions beautiful forms, according to the reason by which it acts: hence reason is a much greater and truer beauty, since it contains the beauty of art; and is greater and more excellent than every thing which proceeds into external form. For so far as form proceeding into matter is extended, so far it becomes more debile than that which abides in one. Since whatever suffers distance in itself, departs from itself, and the integrity of its nature; whether it is strength diffused into some participant; or heat, or power, or beauty extended to some subject, and divided about the fluctuating receptacle of matter. Again, every efficient according to itself, ought to be more excellent than its effect: for that which is unharmonious does not form a musician, but this is the work of harmony; and that music which is above sense, produces the harmony in sensible sound. But if any one despised the arts, because they operate imitating nature, in the first place, it must be confessed, that natures also imitate other things: and in the next place, that arts do not simply imitate that which is perceived by the eyes, but recur to those reasons from which the energy of nature consists. Besides this, they produce many things from themselves, and add something where any thing is wanting to the perfection of the whole; because they contain beauty in themselves. Lastly, Phidias himself fashioned his Jupiter, not by imitating any spectacle proper to the senses; but conceiving the god such as he would appear, if he should be willing to exhibit himself to our eyes.

But for the present let us neglect the arts, and consider those beautiful natural effects, which art is said to imitate, i.e. all rational and irrational animals; but especially whatever amongst these are more exactly finished: I mean where the Demiurgus ruling over matter, invests it with the form he desires it should participate. What then is beauty in these? For it is not blood and menstrea, but colour and figure different from these; or it is nothing; or something destitute of figure; or it is that which, as it were, contains something simple like matter. From whence arose the beauty of Helen, for which so great a contest ensued? From whence shines the beauty of other forms similar to Venus? And from whence did the form of Venus herself arise? Or that of any man entirely beautiful, or of some god, whether they are among the number of things subject to our sight, or among those which are not subject, and yet have in themselves a conspicuous beauty. Is not this every where form, descending into that which is produced by the artificer, in the same manner, as it was said that the beauty of artificial figures, proceeded from the arts. What then? Are works beautiful indeed, and reason existing in matter? But is reason separate from matter, which exists in the soul of the agent, and which is first in dignity and rank, not beautiful, but is reduced into one with its subject matter? But if bulk is beautiful, so far as bulk, it follows that active reason, because it is not bulk, is not beautiful: though if form, whether contained in a small or in a large mass, moves and affects in a similar manner the mind of the beholder, certainly beauty is not to be attributed to the magnitude of bulk. Hence, so long as form is external to the soul, we do not perceive, and are not moved by its power: but when it is well conceived in the soul then it affects us with delight. Again, the form of things alone, flows through the eyes, otherwise the most ample figures could not penetrate through such narrow receptacles. But magnitude is contracted, not from its being great in bulk, but rather because great in species or form. Besides it is necessary that the cause itself of a beautiful effect, should be either deformed, or indifferent, or beautiful. If it

is deformed, it cannot produce the contrary to deformity. If it is indifferent, why should it rather produce any thing beautiful, than deformed. But, indeed, it is necessary that nature the artificer of things so beautiful, should possess a beauty more primary and exalted. But with regard to us, when we behold nothing inward, and are entirely ignorant of internal beauty, we follow what is external, unconscious in the mean time that the cause of motion is profoundly latent in the depths of the soul; just like one, who on perceiving his own image, and being ignorant from whence it came, should follow its shadowy and unreal progression. But that there is something else which allures followers to itself, and that beauty does not consist in magnitude is sufficiently testified, by the beauty inherent in disciplines, offices, and the soul: where certainly a more true beauty flourishes; which is then manifest, when we contemplate the wisdom in a worthy mind, and are delighted with the contemplation, and in love with its beauty; not then surveying the corporeal face, which perhaps is not beautiful, but neglecting the whole form of the body and pursuing inward beauty to its most sacred and profound retreats. But if such a soul does not yet incite you to denominate it beautiful, neither on surveying yourself inwardly, will you be delighted with yourself as with something beautiful. Hence while so affected, you will vainly investigate true and intimate beauty: for you will seek after the purity of beauty, not with something pure, but with that which is base; and hence too, a discourse on things of this kind is not to be addressed to all men. Because if you behold yourself beautiful, you may obtain a reminiscence of beauty itself.

The reason therefore of the beauty contained in nature is the exemplar of the beauty appearing in bodies; but the exemplar of natural beauty, is a more beautiful reason contained in soul, from which the beauty of nature flows. But this shines brighter in a worthy soul, already advanced in beauty, than in nature herself: since it adorns such a soul, and affords a light, derived from one much greater; and which is no other than the first beauty. Thus abiding in the soul, it leads it to consider, what that superior reason of beauty may be, which is no longer generated nor placed in another, but abides perpetually in itself. Hence it is not reason, but the author of that reason which is first: since indeed the first reason is a certain beauty subsisting in soul as in matter. But its author is intellect, which is always the same, and not sometimes intellect; because intelligence does not happen extrinsical to this true and original intellect. But what image are we able to receive of such an intellect? For whatever is enquired after externally, is doubtless sought for from something worse than intellect. An image therefore of intellect must be obtained from intellect itself; so that we must not speak of it through the medium of an image; but we must receive a certain portion of gold, as a representative of universal gold. And unless this received gold is pure, we must purify it either in reality, or at least in our discourse; demonstrating that this which is received by us, is not universal, but only a particular portion of gold. Thus then let us ascend higher from our intellect now purified, to intellect itself; and let us begin with the gods themselves, contemplating the intellect which they possess. For all the gods are venerable and beautiful, and endued with an inestimable gracefulness. But what is the cause of such beauty? It is intellect, energising in the most exalted manner, which produces their divinely beautiful appearance. For it is not because their bodies are beautiful that they are gods, but from the possession of intellect, since the participation of body, is not essential to divinity. For they are not at one time wise, and at another time the contrary; but they are perpetually wise, with a tranquil, stable, and pure intellect, understanding all things, and knowing not human concerns properly, but their own, that is such as are divine, and such as intellect itself perceives. But the gods who inhabit this visible heaven, for they abound in divine leisure, assiduously contemplate, as if it were above them, what the primary and intelligible heaven contains. But those who are stationed in this higher world, contemplate its inhabitants possessing the whole of this diviner heaven. For all things there are heaven. There the sea, animals, plants, and men are heaven. Lastly every portion of this heaven is celestial. But the gods who reside there, do not disdain men, nor any other of its inhabitants, because every thing there is divine; and they comprehend the whole of this intelligible region attended with the most perfect repose.

Hence the life of these divinities is easy, and truth is their generator and nurse, their essence and nutrient: hence they perceive all things, not such indeed as are subject to generation, but such as abide in essence: they likewise perceive themselves in others. For all things are there perfectly perspicuous. Nothing there is dark, nothing opposing, but every thing is conspicuous to all, intrinsically and universally. For light every where meets with light. Each thing contains in itself all, and all things are again beheld in another. So that all things are every where, and all is all. There every thing is all. There an immense splendour shines. There every thing is great, since even what is small is there great. There the sun is all the stars; and every star is a sun, and at the same time all the stars. But one thing excels in each, while in the mean time all things are beheld in each. There motion is perfectly pure: for the proceeding motion is not confounded by a mover foreign from the motion. Station also

there is disturbed by no mutation: for it is not mingled with an unstable nature. Besides beauty there is beauty itself, because it does not subsist in beauty. But every thing abides there not as if placed in some foreign land; for the being of each is its own stable foundation: nor is its essence different from its seat; for its subject is intellect, and itself is intellect. Just as if any one should conceive this sensible heaven, which is manifest and lucid to the eyes, germinating into stars by its light. In corporeal natures indeed, one part is not every where produced from another, but each part is distinct from the rest. But there each thing is every where produced from the whole; and is at the same time particular, and the whole. It appears indeed as a part: but by him who acutely perceives, it will be beheld as a whole: by him I mean, who is endued with a sight similar to that of the lynx, the rays of whose eyes are reported to penetrate the depths of the earth. For it appears to me that this fable, occultly signifies the perspicuousness of supernal eyes. Besides the vision of these blessed inhabitants is never wearied, and never ceases through a satiety of perceiving. For there is no vacuity in any perceiver, which when afterwards filled up, can bring intuition to an end. Nor can pleasure ever fail through the variety of objects; or through any discord between the perceiver and the thing perceived. Besides every thing there is endued with an untamed and unwearied power. And that which can never be filled, is so called, because its plenitude never spurns at its replenishing object. For by intuition it more assiduously perceives. And beholding itself infinite, and the objects of its perception, it follows its own nature as its guide in unwearied contemplation. Again, no life there is laborious, since it is pure life: for why should that labour, which lives in the best manner? But the life there is wisdom, a wisdom not obtained by arguments like ours, because it is always total, nor is in any part deficient, from which it might require investigation. But it is the first wisdom, not depending on any other; and essence itself is there wisdom; yet not in such a manner that essence is first, and then wisdom succeeds as secondary and an adjunct. Hence, no wisdom is greater than this, but there science itself is the associate of intellect, because they both germinate, and beam with divine splendors together: in the same manner as by a certain imitation they report that justice resides with Jupiter. For every thing of this kind exists there like a lucid resemblance perspicuous from itself, so as to become the spectacle of transcendently happy spectators.

The magnitude and power therefore of wisdom itself, is sufficiently evident from its containing within itself, and producing beings: for all things which are true pursue wisdom, depend on it for their being, originate together with it, and have one and the same essence: and lastly essence there is no other than wisdom itself. But we do not yet approach to this exalted knowledge, because we consider sciences, as certain speculations, and rules, and a conflux of propositions, which indeed ought not with propriety to be attributed to the sciences we possess. But if any one doubts concerning our sciences, we must neglect the discussion for the present, at the same time assuming an occasion from hence let us dispute concerning that science, which Plato beholding in the intelligible world says, that science there is not one thing in another. And this investigation will be proper to us, if we profess ourselves worthy an appellation of this kind.

Whatsoever is made by nature or art, is produced by a certain wisdom, and every where wisdom is the leader of action. But wheresoever a certain wisdom fabricates, there are indeed arts of this kind. But the artificer himself is again referred into natural wisdom, according to which art produces every work; not by being collected from speculations, but as one certain whole; nor as composed from many into one, but rather as resolving itself from one into many. If any one therefore places this wisdom as the first in intelligible dignity, it will be sufficient, since it does not originate from another, and does not subsist in any other essence. But if he should say that reason is placed in nature, and that the principle of this is nature, we must enquire from whence nature possesses reason. Because if it is said to possess it from another, we again enquire of that other; and if it possesses it from itself, our investigation is finished. But if they fly to intellect, there again we must enquire, whether intellect generates wisdom. And if they confess it does, we ask from whence? But if it conceives wisdom from itself, it could not accomplish this, unless intellect is wisdom itself. True wisdom therefore is essence, and true essence is wisdom; and the dignity of essence is derived from wisdom. For it appears that true essence originates from wisdom. Hence whatever things are destitute of the wisdom of essence; so far indeed as they are made by a certain wisdom, they are essences; but because they do not contain in themselves any wisdom, they are not true essences. No one therefore ought to think that in the intelligible world, either the gods themselves or any of its transcendently happy inhabitants, contemplate certain rules of propositions; but that each of the objects there contained, offers itself to the beholders, like a beautiful spectacle, such as may be imagined to exist in the soul of a man divinely wise. Not indeed like painted resemblances, but true beings shining with intellectual splendors: on which account the ancients called ideas, beings and essences.

But the wise men of the Egyptians whether from a certain accurate science, or from natural instinct, when they determined to signify to us the mysteries of wisdom, appear to me not to have used figures significant of letters, discourses, and propositions, nor things imitating voices and axioms; but rather by describing and painting the particular images of particular things in their sacred concerns, to have occultly signified the discursive energy of the thing itself. For indeed every image is a certain science and wisdom; it is likewise a subject; and is a spectacle collected into one; and is neither cogitation, nor counsel. But afterwards from this image, or wisdom collected into one, an evolved resemblance is produced in something else, speaking in a discursive transition, and finding out the causes why things are thus instituted: while the thing thus beautifully disposed, excites admiration. Hence it is said that he will admire wisdom, who considers how without containing the causes of her essence, she affords to others which are fashioned according to her nature, their particular mode of existence. This beautiful disposition of things then, which is scarcely manifest from enquiry, if any one should discover, he must own it requisite that in the intelligible world, things should subsist previous to all argument and enquiry, as in one great nature which harmonizes the whole.

Can we think that this universe, which we confess to be derived and to exist in this manner, from another, was so composed by its artificer, that he thought within himself concerning the earth; and considered that it ought to rest in the middle? And that afterwards he reasoned concerning the connection of water with earth, and the orderly disposition of things as far as to the heavens? But in the next place concerning all animals, and such, and so many forms of particular vital beings, as they are at present; and the disposition as well of the inward as of the external parts and members? And lastly that he began to produce things in energy, as they were disposed in himself? But such a consideration could not subsist with the artificer of the universe. For how could it take place in him, who had not as yet seen such things in existence? Nor is it possible that he could fabricate, by receiving external assistance, after the manner of human artificers, who operate with hands and instruments: for hands and feet were posterior to his energy. It remains therefore that all things must subsist in their divine cause, and since no medium intervenes, that by the propinquity of being itself, to another, its image and similitude should as it were on a sudden shine forth, whether from itself alone, or through the ministry of soul. For it is of no consequence at present whether or not the world was fabricated properly through a certain soul, if it is but admitted that all things emanated from thence, and subsist there in greater beauty and perfection. For here they are mixt, but there they are pure. But this universe proceeding, from thence, is comprehended by forms from beginning to end. In the first place matter is the receptacle of the elementary forms, and of others in continual succession; so that it is difficult to find matter, thus concealed under a multitude of forms. But since it possesses a certain ultimate form, it easily becomes the subject of every form. Hence since the exemplar of the universe is form, he produced all forms; and this without any difficulty or violence, because the artificer there is a divine universe, and essence, and form. Hence too his fabrication was easy, and without labour: for there was no impediment; and on this account he now rules over his work with absolute dominion. And although some particulars are every where in opposition to others, yet they cannot now oppose the universal fabric, for it abides as the whole. Indeed I think if we were the first exemplars of things, and at the same time essence, and forms, and if the form which operates here was our essence, that our fabrication would rule without labour, though man as at present should fabricate a form different from himself. For becoming man he ceases to be the universe: but when he ceases to be man as Plato says, he raises himself on high, and governs the world. *For being made of the whole, he also makes the whole.* But that we may return to our design, you may indeed produce a reason, why the earth is placed in the middle, and why it is round, or why the zodiac is situated in a certain place: but in the intelligible world it was not deliberated so to be, because it was requisite; but rather because it is as it exists, on this account it is constituted as it ought: just as if previous to a syllogistic energy through causes, the conclusion itself should remain indubitably certain, without any propositions. For nothing there depends on consequences, nothing becomes certain from consideration: but it subsists prior to consequence, and all consideration. For all these are posterior, *reason, demonstration, faith.* Since on account of the principle all these exist, and are thus disposed. But it is rightly said that the causes of the principle are not to be sought after; especially of a perfect principle, which is the same with the end: for that which is both principle and end, is at the same time the whole, and perfect in every part.

Intellect itself, therefore, is the first beauty; it is total, and is every where total, without suffering a defect of beauty in any part. What then is the beautiful itself to be called? Certainly, not any thing which is not the whole itself, but either possesses a part only, or is entirely destitute of its participation. Indeed unless this is the beautiful itself, what else can merit this appellation? For that which is prior to intellect, does not will itself to be beautiful,

but is something ineffably more excellent. Hence that which first presents itself to our view, because it is form, and a spectacle of intellect, is by this means lovely, and pleasant to the sight. On this account Plato wishing to intimate to us this truth, represents the demiurgus of the universe, approving his own perfect work; willing from hence to exhibit, by something more manifest to our apprehension, the beauty of the exemplar, and of his great idea, as perfectly lovely. For as often as any one admires a work, fabricated according to an exemplar, he must particularly admire the exemplar itself. Nor ought it to seem wonderful if in the mean time such a one, is ignorant of what he suffers: since terrene lovers, and those who admire corporeal beauty, are ignorant that they are thus affected, on account of supernal beauty. But that Plato refers the demiurgus of the universe loving his work, to the divine exemplar, is evident from hence: for he says, that he was delighted with his work, and wished to render it still more similar to its exemplar: evincing from this the beauty of the exemplar, for says he its work is beautiful, because it is the image of its artificer. For indeed unless that was inestimably beautiful, what would be more beautiful than this universe, which is subject to our corporeal sight? On which account they do not perceive rightly, who detract from the beauty of this sensible world; unless in detracting they perceive that this universe is not the intelligible world.

Let us then receive by cogitation this our sensible world, so disposed that every part may remain indeed what it is, but that one thing may mutually reside in another. Let us suppose that all things are collected as much as possible into one, so that each particular object may first present itself to the eyes; as if a sphere should be the exterior boundary, the spectacle of the sun immediately succeeding, and an image of the other stars, and the earth, the sea and all animals should appear within, as in a diaphanous globe: and lastly let us conceive that it is possible to behold all things in each. Let there be then in the soul a lucid imagination of a sphere, containing all things in its transparent receptacle; whether they are agitated, or at rest; or partly mutable, and partly stable. Now preserving this sphere receive another in your soul, removing from this last the extension into bulk, take away likewise place, and banish far from yourself all imagination of matter: at the same time being careful not to conceive this second sphere, as something less than the first in bulk, for this must be void of all dimension. After this invoke that divinity who is the author of the universe, imaged in your phantasy, and earnestly intreat him to approach. Then will he suddenly come, bearing with him his own divine world, with all the gods it contains. Then will he come, being at the same time one and all, and bringing with him all things concurring in one. There indeed all the gods, are various amongst themselves in gradations of power, yet by that one abundant power they are all but one, or rather one is all: for the divinity never fails, by which they are all produced. But all the gods abide together, and each is again separate from the other in a certain state unattended with distance, and bearing no form subject to sensible inspection: or one would be situated differently from the other, nor each be in itself all. Nor again does any one of these possess parts different from others, and from itself: nor is every whole there a divided power, and of a magnitude equal to its measured parts; but it is indeed a universe, and a universal power, proceeding to infinity in a power, which is the parent of energy. But this divine world is so truly great, that its parts become infinite. For where can any thing be said to exist, with which it is not extended? This sensible world too is great, and all powers are contained in its ample bosom: but it would be much greater, and that in a manner perfectly ineffable, if it was free from the diminutive power of body. And if it should be said that the power of fire and of other bodies is great, it must be remembered that true powers are infinite, and that it is only from an ignorance of these, that corporeal natures appear to have being, and to operate by corrupting, separating, and ministering to the generation of animals. But these indeed corrupt, because they are themselves corrupted, and they generate because they are generated. But the power which flourishes there, possesses being alone, and is alone beautiful, without any external and adventitious qualities, which only derogate from the dignity of essence. For where can there be any thing beautiful, deprived of being? And where again can essence abide, if it wants the presence of beauty? For while beauty is taken away, essence is destroyed. On this account being itself is desirable, because being, and beauty are the same: and the beautiful is lovely; because it is being. But it is not proper to enquire which is the cause of the other, since the nature of each is one and the same. The false essences indeed of bodies, require a certain image of beauty, extrinsically acceding, both that they may appear beautiful, and that they may inherit in obscure portion of being. For they so far partake of essence as they participate of beauty, consisting in form: and by how much the more they receive of this kind of beauty, so much the more of perfection do they inherit: for by this means a beautiful essence, and beauty itself is more peculiar to their nature.

On this account Jupiter himself, who is the most ancient of the other gods which he leads, proceeds first to the contemplation of the intelligible world. But afterwards the subordinate gods, demons, and souls follow him, who are able to perceive such transcendently lucid objects. And this divine world shines upon them, from a certain

occult place, which is no other than the abode of ineffable unity. But it illustrates all the divinities with its light: and excites to itself superior souls who are afterwards converted to its splendid vision, which before they were incapable of perceiving; and which like the sun dazzles the eye unaccustomed to intellectual light. And while some with elevated eyes, easily bear its intuition, others who are more distant from its nature are disturbed with the vision. But since each of these blessed inhabitants, perceives according to his ability, all of them indeed behold this intelligible world, with its various contents, yet they do not all retain the same spectacle, but while they are lost in attentive vision, one beholds the lucid fountain and nature of the just itself, while another abundantly perceives temperance itself, but not such as that which resides with men, when they enjoy its possession. For this our temperance imitates the supreme: but that diffusing itself in all things, as if about all the magnitude of its nature, is finally perceived by those, who have already beheld many perspicuous spectacles. On this account the gods behold every thing separate, and at the same time all things together: they perceive too divine souls there, whose vision is universal; and their nature becomes such from unbounded perception, that they contain all things from the beginning to the end.

These divine objects therefore, Jupiter himself and those of us who together with Jupiter love this intelligible world, happily contemplate, together with that universal beauty shining from all, and whatever participates of the beauty, which there abides. For every thing there glitters, and illuminates the spectators with its light, so that they become beautiful by its lustre: just as it happens to those who ascend the highest mountains, where the earth is yellow: for they are immediately infected with the colour, and become similar to the earth, to which they ascend. But the colour which flourishes in the divine world is beauty itself; or rather every thing there is wholly colour, and profound beauty. For beauty there, is not like that which flourishes in the superficies of bodies: but among those who do not perceive the whole, that alone which is resplendent in the superficies is considered as beauty. But those who are totally filled with the intoxicating nectar of divine contemplation, since beauty diffuses itself through every part of their souls do not become spectators alone. For in this case the spectator is no longer external to the spectacle: but he who acutely perceives, contains the object of his perception in the depths of his own essence; though while possessing, he is often ignorant that he possesses. For he who beholds any thing as external, beholds it as something visible, and because he wishes to perceive it attended with distance. But whatever is beheld as *perceptible*, is beheld externally: but it is requisite we should transfer the divine spectacle into ourselves, and, behold it as one, and as the same with our essence: just as if any one hurried away by the vigorous impulse of some god, whether Apollo or one of the Muses, should procure in himself the intuition of the god; since in the secret recesses of his own essence, he will behold the divinity himself. But if any one of us who is not able to perceive himself entirely comprehended by this divinity, should produce a spectacle into his view, for the purpose of assisting his vision, he should produce himself; and he will then perceive an image of the intelligible world, now become more beautiful and divine. But afterwards neglecting the image although beautiful, and conspiring with himself into one, and no longer separating his essence, he will become *one all* together with that deity, who silently flows into his soul; and he will be present with him as far as he is able, and as much as he desires. But if he should return from this divine union into two, and is in the mean time pure, he will nevertheless dwell proximate to its essence; so that by conversion, he may again be present and become united with his divinity. But the gain of the soul will consist in this ineffable conversion. Indeed, when it first attempts this union, it perceives itself, as long as it is different from the god: but when it has penetrated into its most intimate recesses, it will then find itself in possession of the intelligible universe; and casting sense behind, fearing lest it should become different, it will be one with this divine world. And if it desires to perceive as something different, it will place itself external to its object. But it is requisite that the soul which is about to perceive a divinity of this kind, should possess a certain figure of his nature, and assiduously persevere, while it endeavours perspicuously to know him; and thus well understanding the importance of its pursuit, and trusting it is about to enter on the most blessed vision, should profoundly merge itself in contemplation, till instead of a spectator, it may become another specimen of the object of its intuition; such as it came from thence, abundantly shining with intellectual conceptions. But how can any one reside in the beautiful itself, unless he perceives it? Indeed, if he perceives it as something different, he will not as yet abide in beauty. But becoming beautiful, he will thus especially exist in beauty. If then vision is directed to something external, it is not proper that vision should be there, or if it is it should become one with the object of perception. But a doubt of this kind is like a certain consciousness of some one fearing, lest if he wished to perceive more vehemently, he should depart from himself. For thus disease more vehemently impels and excites our sensation: but health dwelling with us more quietly, exhibits a truer knowledge of itself, since it is present with silence and tranquillity, as something familiar and allied to us; and conspires into one with our composition. On the contrary disease possesses nothing

domestic, but is entirely foreign from our nature; and hence its presence is more manifest on account of its diversity: but such things as are peculiarly our own, are present with us, without any manifest sensation. So that when we are in this condition, we are then most of all known to ourselves; since our science in this case is one and the same with our essence. Hence, in the divine world, when we are most knowing according to intellect, we appear to be ignorant, expecting the passion of sense, which says it does not perceive; nor indeed does it see; nor can it ever attain to the intuition of such exalted objects. That which distrusts its vision then is sense: but it is something else which perceives. And if this too should doubt, it is no longer its true self. For neither can this last when it places itself externally, behold that which is intelligible, as if it were sensible, and to be seen with corporeal eyes.

But it has been shewn how the soul may be able to accomplish this as different from its object, and how when the same. But what will the perceiver relate whether abiding as different, or the same? He will tell that he saw this god, who is the same with the intelligible world, generating a beautiful son, and producing all things in his essence without any labour and fatigue. For this deity being delighted with his work, and loving his progeny, continues and connects all things with himself, pleased both with himself, and with the splendors his offspring exhibit. But since all these are beautiful, and those which remain within are still more beautiful, Jupiter the son of intellect alone shines forth externally, proceeding from the splendid retreats of his father. From which last son, we may behold as in an image, the greatness of his fire, and of his brethren those divine ideas, who abide in occult union with their father. But this ultimate progeny does not affirm in vain, that he proceeds from his parent intellect: for he is another world, proceeding from this first, and becoming beautiful, like an image of beauty. For it is not lawful that the image of beauty and of essence, should not be beautiful. Hence, he in every respect imitates his exemplars. For he possesses life, and the gift of essence as a certain imitation of stable essence, and life ever vigilant: he possesses also beauty, so far as he proceeds from thence; and perpetual duration, as a moving image of the eternity of intellect abiding in one: for if this is not admitted, he would at one time exhibit his image and not at another. But he is not an image fabricated by art; and every image formed by nature, lasts as long as its exemplar endures. Hence they do not conceive rightly, who think this world may be destroyed, that which is divine remaining in the full perfection of its essence, and thus imagine the world generated, and that its author on a certain time consulted concerning its production. Such as these indeed neither wish to understand, nor are at all acquainted with the mode of its formation, and are ignorant that so long as the splendors of that divine world endure, so long will this visible universe beam from thence, and will never be destroyed, since the original of each is the same. But the intelligible world always was, and always will be: appellations of this kind being adopted from necessity, for the purpose of conveying the conceptions of our minds.

Saturn, therefore, who according to poetical fable is feigned bound, because he always perseveres in the same divine energies of his nature: who is also reported to have delivered the government of this universe to his son Jupiter (for it was not proper that he having dismissed his government, should follow a nature junior and posterior to himself, since he comprehends in himself the plentitude of all beauty.) Saturn, I say, omitting all subordinate natures established in himself his father Cælum, and raised himself on high as far as to this ineffable principle. He likewise established succeeding natures originated posterior to him, from his son. And thus he possesses a middle situation between both, through a diversity of section from that which is above him, and from his abstaining from inferior concerns, while he is fabled by a subordinate care to be bound in chains; thus obtaining a middle situation between his greater father, and his inferior son. But since his father Cælum, is something greater than beauty, hence Saturn or intellect is the first beauty, though soul is likewise beautiful: yet intellect is more beautiful than soul, because soul is only its vestige; and is naturally beautiful through this, though it is far more beautiful when it beholds the perfect nature of intellect. If then the soul of the universe (that we may use words more generally known), and Venus herself is beautiful, what must be the beauty of intellect? For if soul and Venus possess this from themselves, how great must be the splendor of intellect? But if from another, from whom does soul possess the beauty as well acceding, as natural to her essence? Indeed, whenever we are beautiful, we become so from the possession of our own nature alone: but we are base, when we are precipitated into an inferior nature. So that we are beautiful when we know, but base when we are ignorant of ourselves. Beauty, therefore, shines in Saturn or intellect, with primary splendors. But are these considerations sufficient to a knowledge of the divine world the intelligible place? Or must we proceed another way in its investigation?"

And thus much for the doctrine of Plotinus, as delivered by him in the two preceding inestimable books. I shall only add the following observations concerning the Platonic triad of principles, as conceived and illustrated

by this extraordinary man, and some reflections concerning the Christian trinity, with which I shall take my leave of Plotinus.

According to Plotinus then, as the divinely solitary principle of things is perfectly simple, it necessarily follows that he must be perfectly sufficient, and perpetually exuberant. Hence, he must be a producing cause; and that which he first produces, must be the most similar of all things to himself. And this is no other than intellect, or the intelligible world, the nature of which has been so divinely explained by this philosopher, in the preceding book. Now this intelligible world on account of its perfect similitude to the one, contains all multitude in occult and indistant union: for it is requisite that multitude should exist occultly, before it is scattered abroad and diffused into separate forms; and that it should be concealed in the profound recesses of intellect, before it emerges into the diffused splendors of multitude perfectly divided and discreet. Just as the duad is posterior to unity, and contains number, without being perfect number itself. But as it is necessary that this occult multitude, should be perfectly diffused, in order to the actual diversity of things, and the existence of the sensible world, hence a third procession originates, in which multitude no longer subsists in indivisible union, but proceeds from the sanctuary of intellect into absolute diversity and separation. And this third principle is no other than soul, which expands the impartibility of intellect, and unfolds all that was involved in the unity of intellectual perception. Now, besides these, there can be no other principles: for after the cause by which multitude is perfectly evolved, nothing but the gradation and diversities of multitude can subsist. Hence, as Plotinus justly observes^[66], “we ought not to entertain any other principles, but having established the *simple good* as first, we should place the supreme intellect as the next, and then the universal soul as the third in descent. For this is the proper order according to nature, neither to make more, nor less intelligibles than these three. For he who contracts the number of these, must of necessity either suppose soul, and intellect to be the same, or else intellect and the first good. But that all these three are different from each other we have often asserted and proved.”

It must here, however, be observed, as Dr. Cudworth justly remarks^[67], that this third hypostasis or principle, is not the immediate soul of the world (according to Plotinus, and the best of the Platonists) but ψυχὴ ὑπερκόσμιος, a super-mundane soul. For thus Proclus plainly asserts^[68], not only of Amelius, but also of Porphyry, who followed Plotinus in this particular. “After Amelius, Porphyry, thinking to agree with Plotinus, calls the super-mundane soul, the demiurgus of the world, and that intellect to which it is converted not the demiurgus, but the paradigm of the world.” Indeed, this super-mundane soul must be too nearly allied to the supreme intellect, to become the immediate animating principle of the world; and as the gradation of things throughout the universe, subsists by the most gentle and easy declension, the descent would be precipitate, to make the highest soul connected with the mundane body. Besides as multitude subsists retired and concealed in the supreme intellect, such an intellect cannot be the artificer of the world, since all forms reside there in stable and indivisible union: but a procession and extension of these forms is requisite to the production of the visible universe. And as every cause is superior to its effect, and as the mundane soul must be connate with the world, hence the demiurgus of the world, must be superior to the mundane soul.

Such then is the Platonic triad, composed from three distinct, and different principles; and having^[69] no similitude except in name, to the trinity of the Christian faith, as established by *law*. The Platonic philosophers indeed took a bold flight, for they soared to the principle of things, and drew abundantly from the ineffable and eternal fountain of good: but they never rose so high as to discover that the three persons of their triad were identically one. As men merely assisted by the illuminations of intellect, they saw the necessity of three principles, to the existence of the universe, but they had not yet penetrated the awful veil of the most mystic theology, and beheld the triple deity, seated on the tremendous throne of *unintelligible* faith. They were capable of demonstrating that the principle of all was perfectly *simple* and *one*: but their eyes were not acute enough to survey the *triplicity* of the *one*. Had they but been blest with the *light* of the moderns, what wonders would have opened to their view! They would then have understood the trinity in unity, and the godhead in the manhood, absurdity involved in mystery, and mystery in absurdity. In short, they would have discovered, that the supreme so far from being separate from multitude, and superior to essence itself, as they fondly imagined, took upon himself the actual form of a man, that he might enlighten the vilest and most obscure of mankind, and that by suffering an ignominious death, he might appease his own wrath, and satisfy the vengeance of his injured deity. However, an impartial reader must confess, that considering their ignorance of these sublime particulars, their discoveries were admirable and profound; and a sagacious modern will doubtless rejoice to find that they

believed in a god, who was the principle of things, though at the same time they were so blind, as not to perceive that like *Cerberus* he was triple!

SECTION II.

Porphyry the favourite disciple of Plotinus next demands our attention: but unfortunately we have scarcely any other particulars of his life, worthy our attention, than those we have already delivered in the history of Plotinus. I shall, therefore, only add, that he was born at Tyre, in the twelfth year of the emperor Alexander Severus, and of Christ 233; and that he died at Rome more than seventy years old, in the latter part of the emperor Dioclesian's reign. Of his great abilities we have already given ample testimony: it now remains that we shew how much he contributed to the restoration and perfection of the Platonic theology. We are informed by Proclus^[69], that it was usual with Plato, and his most genuine associates, to call all beings, by the appellation of intellect. "Hence (says he), in many places they establish *the good intellect, and soul*, as the three principles of things, calling intellect *every being*." Now this was eminently the case with Plotinus, who in intellect or the intelligible world, comprehends all the intelligible gods, all true beings, and the multiform variety of ideas. Hence, he was more anxiously employed in profoundly investigating the nature of this divine world, than in scientifically unfolding the order of the beings it contains. Indeed, his genius on every subject was more adapted to an intimate perception of the occult essence of a thing, than to explaining its gradual evolution, and describing the mode of its participations. However, though he did not prosecute the more particular processions of divinity himself, yet he took care to insert the principles of this sublime investigation, in his writings; and to lay the foundation of that admirable and beautiful system, which was gradually revealed by succeeding Platonists, and at last received its ultimate perfection, by the subtle and elegant genius of Proclus.

Porphyry, however, appears to have been the first who wrote any thing explicitly on this interesting^[360] subject. "For he composed (says Proclus^[70]) a treatise concerning principles, in which he demonstrates by many and beautiful reasons, that intellect is indeed eternal, but that it contains in itself something more ancient than intellect, which is conjoined with the one." Now this something which is more ancient than intellect, but inferior to the one itself, can be nothing else than an Henad, or posterior monad; and if so there must be an order of Henades prior to that of intellects, which is most beautifully and copiously proved by Proclus, in his books on Plato's Theology, and is demonstrated in the following theological institutions. But that this was likewise the doctrine of Plotinus is plain from his own words^[71]. "It is necessary (says he) that the principle and cause of intellect, and the deity himself, should be present with the soul; and that this supreme principle should be present, not in any divisible manner, but perfectly stable, not indeed in place, but in himself. And, in the mean time, he must be considered in each of the natures capable of receiving him, as something different from their essence, just as the centre also abides in itself. But every line in the circle terminates its point in the centre, to which it tends. And by something analogous to this, which our essence contains, we also may touch, coalesce, and be conjoined with the supreme." Hence it appears how egregiously Dr. Cudworth^[72] is mistaken, in asserting that the doctrine of the Henades, was a figment of the Platonists posterior to Plotinus: for if they are the summits of our souls, and intellects, they must likewise be the very flowers, as it were, of universal souls and universal intellects.

As the most valuable of Porphyry's works are unfortunately lost, and this most probably through the malice of sectaries, and the unjust interposition of regal authority, we can scarcely give any other proof of his great contributions to the advancement of the theology, than by enumerating his writings on this occasion. And in the first place, next to the two books on principles which we have already mentioned, his treatise, *Concerning the Philosophy from Oracles*, merits our attention. In this curious and valuable work (as we are informed by Eusebius^[73]) he collects, a variety of the answers, as well of Apollo, as of other deities, and beneficent dæmons: proving by this means the virtue of theology, and promoting *theosophy*, or the study of divine wisdom.

2. In the next place, we may deservedly rank his *Explanation of many of Plotinus's Books* (according to the testimony of Eunapius.) For surely nothing could more contribute to the restoration of this theology, than a commentary on the writings of him who first uncovered its veil, and happily penetrated into the mysteries it contains.

3. To these succeed his *Commentaries on the Timæus of Plato*, which are every where cited by Proclus, with the praise they doubtless deserved. Indeed, if we consider the sanctity of Porphyry's genius, his great abilities, and universal attainments, we cannot sufficiently regret the loss of his writings on a dialogue which has ever been esteemed uncommonly abstruse, and replete with the most mystical theology.

4. *Concerning the divine Names.* This work is mentioned by Suidas, and promises by its title, to have been an inestimable treasury of recondite wisdom.

5. *Concerning the Allegories of the Grecian and Egyptian Theology.* How much Porphyry, excelled in works of this kind, is sufficiently evident from his explanation of the Cave of the Nymphs in Homer, which is fortunately preserved; and the translation of which in the following pages, will both adorn the present history, and, I doubt not, be acceptable to the reader.

6. *On the Regress or Re-ascent of the Soul.* The design of this excellent work, as we are informed by St. Augustin, who often cites it, in his City of God^[74], was to admonish us, that we should fly from all body, that the soul may abide in felicity with the deity. According to the same Augustin too, in this book there was a copious discourse concerning the purgation of the soul by the *theurgic* art, which we cannot wonder this *reverend father* should both ridicule and disclaim: for how can the faith of a *bishop* agree with the dogmata of a *philosopher*? But it is here necessary to inform the reader that the theurgical art, explained and recommended by Porphyry in this treatise, consists in purifying the imaginative spirit or vehicle of the soul, which according to its various condition either elevates the soul to a superior state, or draws it down into the darkest recesses of the earth. Lucas Holstenius in his account of the life and writings of Porphyry, is very diffuse in relating and defending the opinions of Augustin on this particular, but takes no notice of the Platonic Synesius^[75], who in his admirable book on dreams, speaks so appositely and fully on this most interesting subject, that I cannot refrain from presenting the English reader with a translation of his doctrine concerning this phantastic spirit, or as he calls it spiritual soul. “In what the disease (says he) of this spirit consists, by what means it languishes and is dulled, and how it becomes purified and defecated, and restored to its natural simplicity and perfection, must be learned from the arcana of philosophy; from which being purified by the lustrations of mysteries it passes into a divine condition of being. But it is requisite to banish all influxions externally, before the phantastic spirit can superinduce the divinity. And whoever preserves it pure by a life according to nature, will render it prompt for this most exalted employment. For this spirit understands the affection of the soul, and is not destitute of sympathy towards it, like its testaceous vestment the body, which has a condition opposite to the more excellent affections of the soul. But the primary and proper vehicle of this phantastic spirit, when the soul is in a flourishing condition, is attenuated and ethereal: but when the soul is badly affected, then this vehicle is dulled, and becomes terrene. For this phantastic spirit is situated in the confines of the rational and brutal nature, is of an incorporeal and corporeal degree; and is the common boundary of both, and the medium which conjoins divine natures with the lowest of all. On this account it is difficult to comprehend its nature by philosophy: for it collects that which accords with itself, as it were from neighbouring natures, and from the extremes of each; and comprehends in one essence things separated by so great an interval from its own. But nature extends the latitude of a phantastic essence, through many conditions of things: for it descends even to animals to whom intellect is not present. In this case, however, it is no longer the vehicle of a diviner soul, but presides over its subject powers, becomes the reason of the animal with which it is connected, and is the occasion of its acting with much wisdom and propriety.

“But this phantastic spirit may be even purified in brutes, so that something better may be induced; and all the genera of dæmons derive their essence from a life of this kind, for their whole essence is composed from the phantasy, and from inward imaginations. But many of the energies of the human nature consist from this alone, or if from something else, yet this prevails the most: for we are not accustomed to cogitate without imagination, unless some one should perhaps for a moment be able to pass into contact with an immaterial form. But to transcend the phantasy in rational energies, is not less difficult than blessed. Hence (says Plato) the possession of intellect and wisdom in old age is desirable above all things, signifying by this, intelligence shining without imagination; because intelligence when conversant with a common life, belongs to the phantasy, or at least to an intellect energizing through the medium of the phantasy. Hence too, this animal spirit which divine men have denominated the spiritual soul, becomes a god, and an omniform dæmon, and an image, in which the soul suffers the punishment of its guilt. And in conformity with this the oracles also compare the life of the soul in this animal spirit to the imaginations of dreams. Philosophy too, agrees in asserting, that preceding lives are certain preparations to those in a subsequent order, while the possession of the best habit in souls renders this spirit more adapted to elevation, and wipes away the profound stains of a baser affection. Hence by natural allurements, this spirit is either elevated on high, on account of its heat and dryness, which Plato signifies by the wings of the soul, and Heraclitus when he says, *that a dry soul is the wisest*: or becoming bulky and humid, it merges itself in the recesses of the earth by a natural gravity; and is thus concealed in darkness, and hurled into a subterranean

region^[26]. For a place of this kind is peculiarly adapted to humid spirits; and the life there is unhappy, and obnoxious to punishment. It is however possible by labour and time, and a transition into other lives, for the imaginative soul when purified, to emerge from this dark abode: for it passes its course through lives of a two-fold nature; and alternately approaches to superior and subordinate conditions of being.

“But the soul in its first descent, derives this spirit from the planetary spheres, and entering this ~~a~~²⁷²boat associates itself with the corporeal world, earnestly contending that it may either at the same time draw this spirit after it, in its flight, or that they may not abide in conjunction. Indeed it is rarely though possible to be accomplished, that the one deserts the other in descending to the earth: for it is unlawful not to believe in mysteries of known credibility and truth. But the souls’ regression will be base, if she neglects to restore, that which is foreign from her nature, and leaves about the earth, what she had received from on high. And this indeed one or two may obtain as a gift of divinity and initiation. For it is instituted by nature, that the soul, once seated in this phantastic spirit, should either follow, or draw, or be drawn, yet so as to remain copulated with this spirit, till it again ascends from whence it came. Hence when on account of its depravity this spirit grows heavy, at the same time, it draws down the soul, which had yielded to its gravitation. And the dread of this is what the oracles announce to our intellectual conceptions, when they advise: *Nor decline beneath, into the obscure world, whose depth is always an unfaithful bottom, and an infernal darkness, squalid, rejoicing in shadows, and full of stupidity and folly*. For how can a stupid and foolish life be expedient to intellect? But the inferior region, accords with the image, or spiritual soul, on account of an affection of spirit corresponding with such a place: for like rejoices in like.

“On this account, if by conjunction, one is produced from the two, intellect also will be merged in pleasure. But the extremity of all evils consists in not perceiving the present evil: for this belongs to such as have no desire to emerge, but like those whose skin is hardened by disease, as they are no longer tormented with pain, so neither are they anxious to be cured. Hence penitence possesses a peculiar power of re-elevating the soul. For he who endures his present state with sorrow and remorse, will meditate his flight: *and the will is the greatest part of purgation*. Indeed through the means of this both our deeds and discourses extend their hands to assist us in our ascent: but this being taken away the soul is deprived of every purifying machine, because destitute of assent, which is the greatest pledge of reconciliation. Hence both here and elsewhere, punishments bring with them the greatest utility to the order of things, while they oppose molestation to delight, and banish stupid pleasure from the soul. Misfortunes too, which are said to happen contrary to our deserts, are of the greatest advantage in extirpating the affections by which we are captivated with externals: and thus the doctrine of a providence is confirmed to the intelligent, from the very circumstances which produce diffidence in the ignorant. For no place would be left for the soul to take her flight from the dominion of matter, if in the present state she lived free from the incursions of evil: and hence it is proper to believe, that the præfects of the infernal regions have invented vulgar prosperities, as the snares of the soul. It may therefore be said that souls emigrating from hence drink of oblivion: but the cup of oblivion is extended to souls entering into the present life, by pleasure and delight. For when the soul descends spontaneously to its former life, with mercenary views, it receives servitude as the reward of its mercenary labours. But this is the design of descent, that the soul may accomplish a certain servitude to the nature of the universe, prescribed by the laws of Adrastia, or inevitable fate. Hence when the soul is fascinated with material endowments, she is similarly affected to those, who though free born, are for a certain time hired by wages to employment, and in this condition captivated with the beauty of some female servant, determine to act in a menial capacity under the master of their beloved object. Thus in a similar manner, when we are profoundly delighted, with external, and corporeal goods, we confess that the nature of matter is beautiful, who marks our assent in her secret book: and if considering ourselves as free we at any time determine to depart, she proclaims us deserters, endeavours to bring us back, and openly presenting her mystic volume to the view, apprehends us as fugitives from our mistress. Then indeed the soul particularly requires fortitude, and divine assistance, as it is no trifling contest, to abrogate the confession and compact which she made. Besides in this case, force will be employed: for the material inflictors of punishments will then be roused to revenge, by the decrees of fate, against the rebels to her laws. And this is what the *sacred discourses* (ἱεροὶ λόγοι)^[27] testify by the labours of Hercules, and the dangers which Hercules was required to endure; and which every one must experience who bravely contends for liberty, till the phantastic spirit becomes superior to the dominion of nature, and is placed beyond the reach of her hands.

“But if this leap from matter should happen within the boundaries of nature, the soul will be depressed, and require more weighty contests: for matter now fully convinced that we are fugitives, will not be sparing of

punishment; and though we may despair of our ascent, she will chastize us for the endeavour, and no longer propose our choice of living from two urns, which Homer occultly intimates are two portions of matter. And Jupiter himself in this place, according to the same divine poet, is the moderator of matter, distributing a two-fold condition of fate; from which good is never found sincere, and without a mixture of evil, though it is possible that some unfortunate being may participate of the worse condition without any portion of good.

“In short all lives are conversant with the fluctuations of error, when the soul does not speedily return,^[77] after its first descent. But consider with how great an interval, this spirit energizes in our nature: for when the soul is inclined downwards, *the spirit also (according to the sacred discourse) grows heavy, and sinks, till it falls into a region profoundly dark: but when the soul rises from this obscurity, the phantastic spirit also attends it, as far as it is able to follow.* And it will attend, till the soul arrives at a condition of being the most opposite from its nature. Hear too, what the oracles declare on this occasion. *Nor should you leave the most abject part^[78] in the precipice of matter: for there is a place for the image in the region every where resplendent with light.* But this place is opposed to the region totally dark; and to him who acutely perceives, a still deeper meaning will be found in the words. For the oracle not only seems to recal into the spheres, the nature which had proceeded from thence, but also intimates that whatever of sublime fire or air, the soul descending from on high, had attracted into a phantastic essence, before she was invested with this terrene bark, must be elevated together with the more exalted part. For the dregs of matter, or the most abject part, cannot signify a divine body. But reason dictates that things which communicate, and conspire in unity, cannot be destitute of mutual relation, and connection with each other, particularly when the places of their residence have a kindred position: as fire is proximate to an orbicular body, and does not like the earth possess the extremity of things.

“Again, if better natures yielding to the subordinate, should at any time unite in conjunction with these, they would produce in matter an indissoluble body, from their superior dominion: and in this case perhaps, the baser nature, no longer opposing the energy of the soul, but becoming gentle, obedient, and obsequious, and exhibiting the middle nature of the phantastic spirit without dissipation, may become ethereal together with the dominion of the rational soul; may be the attendant of its elevation; and may ascend if not to the summit of all, at least to the extremity of the elements, and arrive at the region in every part lucid, and divine. For it possesses says the Oracle a certain place in this region; i.e. it is received into a certain order of an orbicular body. And thus much may suffice, concerning the parts and condition of the elements, which the reader may either believe, or reject as he pleases.

“But it cannot be denied that the corporeal essence of the phantastic spirit, when arrived at this place, ^[79] at the same time elevated with the returning soul, and adapted to the spheres; or in other words it is brought back to its proper nature and pristine condition. These two regions, therefore, are situated in perfect opposition to each other: the one profoundly obscure, but the other every way lucid, obtaining the extremities^[79] of felicity and misery. But how many middle regions do you think are situated in the concave space of this mundane orb, partly lucid, and partly dark, in all which the soul lives, with this phantastic spirit, and alternately changes its forms, and manners, and life? When, therefore, it returns to its proper nobility, it becomes the storehouse of truth: for it is then pure and pellucid, and perfectly immaculate; and has power, if willing, to become a god and a prophet. But when it falls from this elevation, it becomes dark, erratic, and fictitious: for the obscurity of the spirit, cannot perceive the perspicuity of true beings. Lastly, when it possesses a middle situation, it partly wanders, and partly pursues the truth. You may also by this means, explore a demoniacal nature, and its order: for to pursue truth entirely, or to wander but a little from its contemplation, is divine, or nearly divine. But a condition of being, erroneous in predictions, necessarily belongs to such as are assiduously inclined to nature, who are obnoxious to passion, and perfectly ambitious: for by this means such a condition becomes subterranean^[80], and forsakes divinity, and its more ancient dæmon; though by a contrary mode of proceeding it may resume its pristine associations, and occupy the place prepared for a more excellent nature.

“And from hence we may apprehend the state of the soul while connected with the present body: for he^[81] whose phantastic spirit, is pure and composed, and who, both waking, and sleeping, receives true resemblances of things, he indeed, possesses a token that the figure of his soul will pass into a better condition of being. Nor is the judgment trifling which we may form respecting the affection of the animal spirit, from the imaginations which it principally produces, and in which it is employed, when free from external pulsation; philosophy supplying us with judgment and admonition, respecting its nutriment, and the diligent care we should employ to prevent its deviation from the right. But its best education, consists in always energizing according to an intuitive and

perceptive power, and in taking care that the principal energy of the soul is always intellectual; and that as much as possible we always pre-occupy the absurd and rash impetuosities of the phantasy. But this is no other than a conversion of the soul to that which is best, and forsaking all communion with an inferior nature, except what the strongest necessity compels us to adopt. But an intellectual perception above all things separates, whatever is contrary to the true purity of the phantastic spirit: for it attenuates this spirit in an occult and ineffable manner, and extends it to divinity. And when it becomes adapted to this exalted energy, it draws by a certain affinity of nature, *a divine spirit*, into conjunction with the soul: as on the contrary when it is so contracted and diminished by condensation, that it cannot fill the ventricles of the brain, which are the seats assigned to it by providence, then, nature not enduring a vacuum, *an evil spirit* is insinuated in the place of one divine. And what will not the soul suffer, when assiduously pressed by such an execrable evil? For such is the constitution of things, that the regions of the phantastic spirit must either be filled with a superior or subordinate nature: but the latter is the punishment of the impious, who defile the divine part of their essence; and the former is either the end of piety, or proximate to the end.” Thus far the excellent Synesius, who, I doubt not, was greatly indebted to Porphyry’s book on the regress of the soul, for this admirable discourse; as it is evidently pregnant with the most recondite theology. But let us consider this interesting subject more minutely.

Though the theurgical art is unfortunately lost, by means of which we might obtain the best method of purifying the phantastic spirit; yet we must not suppose that it is utterly impossible to accomplish this desirable end, without its assistance. Synesius in the preceding beautiful quotation informs us, “that an intellectual perception attenuates this spirit, in an occult and ineffable manner, and extends it to divinity.” Indeed, nothing can so effectually contribute to separate the phantasy from this terrene body, as a continual intellectual illumination. Now this can only be acquired by long habits of meditation, accompanied with a vehement thirst after truth, which gradually withdraw the soul from sensible perturbations, produce the contemplative virtues, and dispel the darkness of corporeal imaginations. Science, indeed, is the first requisite in acquiring this purification of the phantasy, I mean the mathematical science; by whose assistance, we first recognize the glimmerings of truth, and discover the dawning beams of intellect emerging, as it were, from the night of oblivion. When the liberal soul first discovers this light, though but feeble and transient, she rejoices at the happy event, and is anxious to procure its continuance and increase. She now despises outward corporeal form, and becomes deeply enamoured with those purer forms in the phantasy, which she has found to be the receptacles of truth. And this is the first degree of purification. But after this if by a fortunate event, from contemplating universals in imaginative figures, she should rise to speculate their subsistence in cogitation, and in the rational soul, she will then discover a much brighter light; though even this will not be constant and serene: for it will be present only when she is deeply engaged in such *middle contemplations*. Indeed, as cogitation is the medium between sense and intellect, so the light attending its energies, has a middle subsistence between the obscurity of the former, and the invariable splendors of the latter. This light, however, will so purify the phantastic spirit, that all its images will possess a considerable degree of perspicuity and lustre. There now remains only the third step, in order to produce the perfection of purity, and to conjoin the phantasy with divinity: and this is no other than an intimate conversion of the soul to the energies of intellect. For by a long and vigorous exercise of this kind, a constant and ineffable light will continually illuminate the phantasy, so as to render all its images pure and pellucid, and perfectly abolish the obscurity of sensible impressions. We may add too, as a symbol of this exalted purgation, that a perpetual serenity, unceasing delight, and occasional rapture will be produced in the soul. The *will*, now entirely free will be intimately converted to that which is best; the desires will breathe nothing but the ardour of intellectual energy; and the passions will no longer be at variance with reason. In this delightful state, the vehicle of the phantastic spirit will become so attenuated and ethereal, that all sensible harmony will waken the soul to an immediate recollection of ideal harmony; all external figures will recall to her memory ideal form; and all lucid bodies will represent with advantage to her inward eye the brighter light reflected in the mirror of imagination. Indeed, sensible light, will be found to possess a remarkable sympathy with this purer light of the soul. For when this intellectual splendor is firmly introduced, and illuminates every part of the phantasy, the smallest spark, and the most glimmering ray of external light, will call forth into energy that sacred light, which is now perfectly seated in the sanctuary of the soul. Such too will be the temperament of the soul in this case, that she will spontaneously utter musical sounds, as indications of the harmony within; and as echoes of the perpetual felicity she enjoys. And such are the methods of acquiring, and such the tokens of possessing purity of imagination, which he who obtains will understand; but which will appear incomprehensible, and ridiculous to him, who is not advancing in its acquisition.

And here it may not be improper to observe, that the phantasy in this purified state, affords indubitable tokens of the possession of truth; and serves as an instrument by which we may discover false opinions from such as are true. For the images attending the perceptions of reality, will always be lucid; and this in proportion to the certainty they contain. Hence, whenever the soul is full, and as it were, pregnant with true conceptions, certain bright phantasms, as the progeny of her rational energies, will drop into the mirror of imagination, and appear like images clothed with light. For the phantasy will now no longer be similar to the dark and irriguous cavern of Calypso (which appears to be the emblem of imagination in an unpurified state), illuminated by sense as by an artificial fire; but it will be totally diaphanous and full of light. It will, indeed, in every respect resemble the palace of Ithaca, when enlightened by the golden lamp of Minerva, during the removal of the arms by Ulysses and Telemachus. Of which we may say with the greatest propriety:

Not such the sickly beams which unsincere,
Gild the gross vapour of this nether sphere^[81].

And he who knows the truth of what I assert, may exclaim with rapture, like Telemachus:

[278]

What miracle thus dazzles with surprise!
Distinct in rows the radiant columns rise:
The walls where-e'er my wond'ring sight I turn,
And roofs amidst a blaze of glory burn!
Some visitant of pure ethereal race,
With his bright presence deigns the dome to grace.

7. In the last place, we may deservedly rank among the theological writings of Porphyry, his treatise *Concerning the Cave of the Nymphs*, in the 13th book of the *Odyssey*. This admirable work is fortunately preserved: and as it contains some deep arcana of the natural and symbolical theology of the ancients, together with some beautiful observations respecting the allegory of Ulysses, I persuade myself the following paraphrased translation of this work, will be acceptable to the lovers of ancient learning and philosophy.

“^[82]What are we to understand by the Cave, in the island of Ithaca, which Homer describes in the following verses?

High at the head a branching olive grows,
And crowns the pointed cliffs with shady boughs.
A cavern pleasant, though involv'd in night,
Beneath it lies, the Naiades delight.
Where bowls and urns, of workmanship divine,
And massy beams in native marble shine;
On which the Nymphs amazing webs display,
Of purple hue, and exquisite array.
The busy bees, within the urns secure
Honey delicious, and like nectar pure.
Perpetual waters thro' the grotto glide,
A lofty gate unfolds on either side;
That to the north is pervious by mankind:
The sacred south t' immortals is consign'd.

i.e. “an olive with spreading branches stands at the head of the Ithacensian port; and near it is a cave both pleasant and obscure, which is sacred to the nymphs who are called Naiades. Within the cavern, bowls and capacious amphora are formed from stone, in which the bees deposit their delicious honey. There are likewise within the cave long stony beams, on which the nymphs weave purple webs wonderful to the sight. Perpetual waters flow within the grotto. But there are two gates: one towards the north gives entrance to mortals descending; but the other towards the south which is more divine, is impervious to mankind; and alone affords a passage to ascending immortals.”

That the poet does not describe this cave according to truth, is evident from hence, says Cronius, that^[80] one of those who have handed down to us the situation of that island make any mention of such a cave. This likewise, says he, is manifest, that it would be very absurd for a mortal man, such as Homer, to expect, that in describing a cave fabricated merely by poetical licence, and thus arbitrarily opening by a new art a path to gods and men in the region of Ithaca, he should gain the belief of mankind. It is equally as absurd to suppose, that nature herself

should point out in this place, one path for the descent of all mankind, and again another path for all the gods. For indeed the whole world is full of gods and men; but it is impossible to be persuaded that in the Ithacensian cave men descend, and gods ascend. Cronius, having promised thus much, affirms that it was evident not only to the wise, but also to the vulgar and unlearned, that the poet under the veil of allegory, concealed some mysterious signification. But the investigation of the particular meaning of these gates, and of the cave of the nymphs, he leaves to others more disposed to such curious enquiries, as likewise why it is both pleasant and obscure; since darkness is by no means delightful, but is rather productive of aversion and horror. Also why it is not simply sacred to nymphs, but it is accurately added, which are called Naiads. Why likewise the cave is represented as containing bowls and urns, when no mention is made of their receiving any liquor, but the bees are said to deposit their honey in these vessels, as in hives? Then again, why are oblong beams placed here for the nymphs; and these not formed out of wood, or any other ductile matter, but from stone, as well as the bowls and urns? Which last circumstance is indeed less obscure; but that on these stony beams, the nymphs should weave purple garments, is wonderful not only to the sight, but to the auditory sense. For who would believe that goddesses weave garments in a cave involved in darkness, and on stony beams; especially while he hears the poet affirming that the purple webs of the goddesses were exposed to human inspection. Besides this too is wonderful, that the cave should have a double entrance; one prepared for the descent of men, the other for the ascent of gods. And again, that the gate pervious by men should look to the north, but the portal of the gods to the south. Since the reason of this distribution affords just matter for surprize and enquiry: and why an eastern and western situation was not preferred. For almost all temples have their entrance and statues towards the east: but those who enter them look towards the west, when standing with their faces turned to the statues they honour and worship the gods. Hence since this narration abounds with obscurities, it follows that it is neither a fable, rashly devised for the purpose of procuring delight, nor contains a true and certain description of a place: but that something is signified by the poet, under its obscure disguise; who likewise places, with a mystic intent, an olive at the entrance of the cave. All which particulars the ancients thought very laborious to investigate and explain, and we who succeed them are of the same opinion, while endeavouring from our own inventions to unfold the concealed meaning of the allegory. Hence those men appear to have written very negligently concerning the situation of the place, who believe both the cave and its contents, to be a mere poetical figment. But the best and most accurate writers of geography, and among these Artemidorus the Ephesian, in the fifth book of his work, which consists of eleven books, thus writes: “The island of Ithaca, containing an extent of 85 stadii^[83], is distant from Panormus, a port of Cephalenia, about 12 stadii^[84]. It has a port named Phorcys; in which there is a shore, and on that shore a cave sacred to the nymphs, in which the Phæacians are reported to have placed Ulysses.”

By no means therefore is this cave a mere Homerial figment. But whether the poet describes it according to its real nature, or adds something of his own invention, yet the same questions remain to be solved; whether you are disposed to investigate the intention of the poet, or of those who consecrated the cave. Since neither did the ancients consecrate temples without fabulous symbols; nor is it usual with Homer to relate any thing rashly concerning their peculiarities. For indeed, by how much the more any one endeavours to shew, that this description of the cave is not an Homeric fiction, but was consecrated to the gods, before Homer’s time; by so much the more he evinces, that this sacred cave is filled with ancient wisdom. On which account it is highly worthy our investigation, and necessary that its symbolical consecration and obscure mysteries should be rendered evident by the light of philosophical enquiry.

Antiquity then with great propriety consecrated caves and dens to the world, whether taken collectively^[284] the universe, or separately according to its parts. Hence they considered earth as the symbol of that matter from which the world is composed; so that, according to the opinion of some, matter and earth are the same: by the symbol of a cave, signifying the formation of the world from matter. For indeed caves are most commonly spontaneous productions, congenial with the earth herself, and comprehended by one uniform stone; whose interior part is concave, and whose exterior parts are extended over an immense space of earth. But the world being self-born, (i.e. produced by no external cause but from a principle within,) and in perfect symphony with itself, is allied to matter which they call, according to a secret signification; a stone and a rock. For like these hard bodies it is sluggish and inert, and receives the impression of ornamenting form: at the same time they considered it as infinite on account of its formless nature. But since it is continually flowing, and of itself destitute of the supervening investments of species by which it is formed and becomes visible, the flowing waters, darkness, or, as the poet says, obscurity of the cavern exhibit apt symbols of what the world contains on account of that matter with which it is connected. Hence through the dark union of matter, the world is obscure and dark, but from the

presence and supervening ornaments of form (from which it derives its name) it is beautiful and pleasant. The world therefore may with great propriety be called a cave; agreeable indeed, at its first entrance, on account of its participation of form, but involved in the deepest obscurity to the intellectual eye which endeavours to discern its dark foundation. So that its exterior and superficial parts are pleasant, but its interior and profound parts obscure: and its very bottom is darkness itself. After the same manner the Persians mystically signifying the descent of the soul into an inferior nature, and its ascent into the intelligible world, initiate the priest or mystic in a place which they denominate a cave. For according to Eubulus, Zoroaster first of all among the neighbouring mountains of Persia, consecrated a natural cave, florid and watered with fountains, in honour of Mithras the father of all things: a cave in the opinion of Zoroaster bearing a resemblance of the world fabricated by Mithras. But the things contained in the cavern, being disposed by certain intervals, according to symmetry and order, were symbols of the elements and climates of the world. We find too that after Zoroaster it was usual with others to perform initiatory rites in caves and dens, whether natural or artificial. For as they consecrated temples, groves, and altars to the celestial gods; but to the terrestrial gods and to heroes altars alone, and to the subterranean divinities vaults and cells; so to the world they dedicated caves and dens; as likewise to nymphs, on account of the waters trickling, and dispersed through caverns, in which the nymphs called Naiads, as we shall shortly observe, preside. But the ancients not only considered a cave as the symbol of this generated and sensible world, but as the representative of every invisible power: because as a cave is obscure and dark, so the essence of these powers is unknown. Hence Saturn fabricated a cave in the ocean itself, and concealed his children in its dark retreats. Thus Ceres educated Proserpine with her nymphs in a cave; and many other particulars of this kind may be found by any one who peruses the writings of Theologists. But that caves are attributed to nymphs, and especially to Naiads, who dwell near fountains, and are called Naiads from the waters over whose flowing streams they preside, the hymn to Apollo indicates in these words:

[85]“The nymphs residing in caves shall deduce fountains of intellectual waters to thee, (according to the divine voice of the Muses,) which are the progeny of a terrene spirit. Hence waters bursting through every river, shall exhibit to mankind perpetual effusions of sweet streams.” From hence as it appears to me the Pythagoreans, and after them Plato took occasion to call the world a cave and a den. For the powers which are the leaders and guides of souls thus speak in a verse of Empedocles.

“We will enter into this cave covered with rocks.”

And Plato in the seventh book of his Republic, speaking of the condition of mankind in this sensible world, says, “Behold men as if dwelling in a subterranean cavern, whose entrance opens through the whole cave to the admission of the light.” But when the other person in the dialogue says, you relate an absurd similitude, he subjoins: “It is requisite, friend Glaucus, to apply this similitude to all that has been previously said: assimilating this terrene habitation which is the object of corporeal sight, to the dark residence of a prison: but accommodating the fire shining in the recesses of the cavern to the solar light.” And thus it is sufficiently evident, that theologians have considered a cave as a symbol of the world, and of the powers it contains. But we observed that they likewise considered a cave as the symbol of an intelligible essence; led to this opinion by reasons very different from the former. For they placed it as a symbol of the sensible world, because caverns are dark, stony and humid; resembling in all these respects the world on account of the obscure nature of that matter from which it is composed, the continual impression of forms to which it is obnoxious, and the constant flowing of all its parts. But a cave resembles intelligible essence, both because invisible to the eyes and sense, and because its substance is solid, firm, and durable. And in the same manner particular virtues or powers are inconspicuous, especially such as are united with matter. For they did not consider a cave as the symbol of a material and immaterial nature on account of its figure as some have suspected: (since every cave is not circular as appears from this Homeric cavern with a double entrance) but from surveying the natural condition of caves, involved in the depths of obscurity and night, and formed from the union of a hard and stony substance. Again, since a cave has a two-fold similitude, it must agree in some particulars with sensible substance, but in others with an intelligible essence. Thus the present cave since it contains perpetual waters, in this respect resembles a substance united with matter, and not that which is immaterial and intelligible. On this account the cave is not sacred to mountain divinities, to those who dwell on hills, or to other deities of this kind, but to Naiads so called by the Greeks from νάματα, fountains; because they preside over waters: and this term is commonly applied to all souls passing into the humid and flowing condition of a generative nature. These souls they considered as incumbent on the water, which is nourished by a divine spirit as Numenius affirms: and hence a prophet said, *that the spirit of God moved*

on the waters. The Egyptians likewise on this account place all dæmons, not connected with any thing solid or stable, but raised on a sailing vessel; and it is known that humor invades the sun itself, and all animals descending into generation. Hence Heraclitus observes “that it appears delightful, and not mortal to souls, when they are born connected with humidity.” And he says in another place, speaking of unembodied souls, “we live their death, and we die their life.” Hence the poet calls men existing in generation διεπούς, i.e. humid, because their souls are drenched in moisture. On this account too, such souls delight in blood and humid seed: but water administers nutriment to the souls of plants. Besides, according to the opinions of some men aerial and celestial bodies, are nourished by the vapours of fountains and rivers and other exhalations. Thus the Stoics assert that the sun is nourished by the exhalation of the sea; the moon from the effluvia of fountains and rivers; but the stars from the exhalation of the earth. Hence according to them the sun is a certain intellectual composition formed from the sea; the moon from river waters; and the stars from terrene exhalations. It is necessary therefore that souls, whether they are corporeal or incorporeal, while they attract bodies, must verge to humidity, and be incorporated with humid natures; especially such souls, as from their material inclinations ought to be united with blood, and confined in humid bodies as in a watery tegument. Hence the souls of the dead are evoked by the effusion of bile and blood: and souls ensnared by corporeal love, and attracting to their nature a humid spirit, condense this watery vehicle like a cloud; for a cloud is nothing more than humour condensed in the air. But the pneumatic part thus condensed, through too great an abundance of humour becomes the object of corporeal sight. And among the number of these we must reckon those apparitions of images, which from a spirit coloured by the influence of imagination, present themselves to mankind. But pure souls are averse from generation; on which account the same Heraclitus observes “a dry soul is the wisest.” But souls thus desiring to be mingled with body, and attracting a humid vapour, by their propensity to generation render their pneumatic part moist and wet, and by thus verging to the ever-flowing waters of generation, are deservedly called Naiads. Hence it is customary with the Greeks to call nymphs γαμουμένας, or married, as those who are copulated to generation; and to wash in a bath whose waters are derived from fountains or perpetual rills. This world then is sacred and pleasant to nymphs, i.e. to souls proceeding into a material nature, and to genii participating of generation, although it is naturally dark and opaque; on which account some are of opinion that souls are composed from a certain aerial opacity. Hence a cave is a habitation peculiarly adapted to such souls; since it is both pleasant and obscure, like this material region, in which souls reside. A cave likewise through which perpetual waters flow is well adapted to nymphs, the divinities of waters. The present cave therefore must be allowed sacred to souls, and to those more particular powers denominated nymphs, who from their being præfects of rivers and fountains are called πρυγαιαί and ναϊδές, i.e. fountain and river divinities. What then are the different symbols, some of which correspond to souls, and others to the divinities of waters, by which it may be manifest that this cave is at the same time dedicated and consecrated to both? We reply that the stony bowls and urns are symbols of the aquatic nymphs. For vessels of the same form are symbols of Bacchus; but their composition is testaceous, that is, from baked earth. And indeed such as these are correspondent to the gift of this god; since the fruit of the vine is brought to a proper maturity by the celestial fire of the sun. But the stony bowls and urns, are most admirably accommodated to nymphs presiding over waters which flow from rocks. And what symbol is more proper to souls descending into generation, and the tenacious vestment of body, than as the poet says, “Nymphs weaving on stony beams purple garments wonderful to behold?” For the flesh is generated in and about the bones, which in the bodies of animals may be compared to stones. On which account these textorial instruments, are fabricated from stones alone. But the purple garments plainly appear to be the flesh with which we are invested; and which is woven as it were and grows by the connecting and vivifying power of the blood, diffused through every part. Besides, purple garments are tinged with the blood of animals; and flesh is produced and subsists from blood. Add too that the body is a garment with which the soul is invested; a circumstance indeed wonderful to the sight, whether we regard its composition, or consider the connecting band by which it is knit to the soul. Thus according to Orpheus, Proserpine who presides over every thing generated from seed, is represented weaving a web; and the ancients called heaven by the name of πέπλος, which is as it were the veil or tegument of the celestial gods. But why are the amphora represented filled with honey-combs, and not with water? For in these as he says the bees deposit their honey. But the word τιθαίωσσειν, signifies nothing more than τιθέναι τὴν βόσιν, i.e. to deposit aliment. And honey is the nutriment of bees.

Indeed, theologists have made honey subservient to many and various symbols, because it is induced with a variety of powers: for it possesses a purging and preserving quality. Hence bodies are kept from putrefaction by its use, and ulcers of long standing are purified: besides it is sweet to the taste, and bees produced from putrid oxen, collect it by a wonderful art from flowers. On this account when in the sacred rites called λεοντικά, those

who are initiated, pour honey instead of water on their hands, it is signified by this practice, that their hands should be pure from every sorrowful, noxious, and abominable concern. Thus, others purify the initiated by a purgatorial rite from fire, but are averse from water as the enemy of fire. Besides they purify the tongue from all the defilement of evil with honey. But when the Persians offer honey to the guardian of fruits, they regard its preserving power as a symbol of its similitude to divine nature. In like manner when the poet pours nectar and ambrosia into the nostrils of the slain, for the purpose of preserving the body from putrefaction, some have interpreted honey as the aliment of the gods. For Homer in a certain place calls nectar yellow; which is also the colour of honey. But whether or not honey is to be taken for nectar, we shall hereafter more accurately examine. Again, we find in Orpheus that Jupiter employs stratagems against Saturn from honey. For Saturn full of honey is intoxicated, his senses are darkened as if from the effects of wine, and he sleeps: just as Porus, according to Plato, is distended with nectar; for wine (says he) was not yet known. But night admonishes Jupiter to employ the stratagem of honey, according to Orpheus, in these words,

“As soon as you behold him spread under the lofty oaks, intoxicated with the sweet honey produced by the bees, bind him in chains.”

Saturn, therefore, intoxicated with honey is bound by Jupiter; and castrated in the same manner as Cælum. But the theological poet intimates by this fable that the divine essences are, as it were, bound, and drawn down by delight into the fluctuating empire of generation; and that when resolved in pleasure, they produce certain powers by their seminal virtue. Thus Saturn castrates Cælum, who, by his desire of coition descends to earth. But the intoxication of honey, signifies among theologists nothing more than the desire of coition; by the ensnaring power of which Saturn is castrated. For Saturn and his orb is the first of the celestial spheres, which moves contrary to the course of Cælum or the heavens. But certain virtues descend as well from the heavens as from the wandering stars, and the influences of the heavens are received by Saturn, and those of Saturn by Jupiter. Hence, since honey is assumed in purgations, and as an antidote to putrefaction, and aptly represents the pleasure and delight of descending into the fascinating realms of generation, it is accounted a symbol well adapted to nymphs the divinities of waters; signifying the nature of the waters over which they preside free from putrefaction: intimating likewise the purgative quality of the waters and their co-operating in the business of generation. For water promotes generation. The poet, therefore, very properly represents the bees as depositing their honey in bowls and urns: since bowls signify fountains; and on this account a bowl or cup is placed next to Mythras instead of a fountain. But we draw the waters of fountains in Amphora; and fountains and rivers are proper to aquatic nymphs, and especially to the nymphs called by the ancients souls, which antiquity likewise peculiarly denominated μελίσσας, i.e. artificers of sweetness or bees: for souls are, indeed, the authors of all the pleasure peculiar to our nature.

Hence Sophocles does not speak improperly, when he says,

“The swarm of the dead utters a buzzing noise.”

But the priestesses of Ceres, as ministers to the terrene goddess were formerly called bees; and her daughter Proserpine μελιττώδη, or delicious, alluding to the sweetness of honey. Besides the moon who is the queen of generation was denominated by the ancients a bee, and likewise a bull: for the exaltation of the Moon is Taurus; and bees are generated from oxen; on which account they are called βουγενεῖς, which name is likewise attributed to souls proceeding into generation. Also the god Mercury is esteemed a stealer of oxen, who is secretly conscious of generation. Besides honey is considered as a symbol of death, in the same manner as gall is of life; whether they indicated by such similitudes that the life of the soul dies by the noxious embraces of pleasure, but enjoys life from bitterness, which by its disgusting sensation prevents the soul from sinking into that drowsy oblivion produced by corporeal delight (on which account they sacrificed gall to the gods); or whether the symbol originated from considering that death is the end of evils, but that the present life is laborious and bitter. But it is here necessary to observe that they did not promiscuously call all souls descending into the whirl of generation bees; but only those who, while residing in this fluctuating region, acted justly; and who, after being in a manner acceptable to the divinities returned to their pristine felicity. For the bee is an animal, accustomed to return to its former place; and is studious of justice and sobriety, on which account libations with honey are called νηφάλιοι, or sober. The ancients likewise refrained from sitting on beans, which they considered as a symbol of generation proceeding in a regular series without being intercepted; because this leguminous vegetable is almost the only one, amongst other fruits, whose stalk is perforated throughout without any intervening knots. We must,

therefore, admit that honey-combs and bees are symbols, as well peculiar as common to nymphs the divinities of waters; and at the same time to souls wedded to the humid and fluctuating nature of generation.

But let us now return to the cave and consider its double entrance. The most ancient of mankind then, before temples were raised to divinity, consecrated caves and dens to the gods. Hence the Curetes in Crete dedicated a cave to Jupiter; in Arcadia a cave was sacred to the Moon, in Lyceum to Pan, and in the island Naxos to Bacchus. The worship of Mithras too, wherever this god was known was performed in caves. But with respect to this cave of the nymphs in Ithaca, Homer was not alone content with saying that it had two gates, but he adds that the one looks to the north, and the other, more divine, to the south; concerning which he does not mention whether it is pervious to the descent of either immortals or mankind, as is the case with the northern entrance, but he only says,

“The other of these tends to the south, which is not pervious to men, but is alone open to immortals.”

It remains, therefore, to investigate either the secret meaning of those who first instituted this cave, according to the poet's description; or what occult signification Homer himself intended to convey, if it is nothing more than a fiction of his own inventing. Since then, the present cave in an eminent degree is a symbol and image of the world, as Numenius and his familiar Cronius affirm, it is necessary, in order to elucidate the reason of the position of the gates, to observe that there are two extremities in the heavens; viz. the winter-solstice, than which no part of heaven is nearer to the south; and the summer-solstice which is situated next to the north. But the summer tropic, that is, the solstitial circle is in Cancer, and the winter tropic in Capricorn. And since Cancer is the nearest to the earth, it is deservedly attributed to the moon, which is itself proximate to the earth. But since the southern pole by its great distance is inconspicuous to us, Capricorn is ascribed to Saturn, who is the highest and most remote of all the planets. Again, the signs from Cancer to Capricorn are situated in the following order; the first is Leo called by astrologers the house of the sun; afterwards Virgo, or the house of Mercury; Libra of Venus; Scorpius of Mars; Sagittarius of Jupiter; and Capricornus or the house of Saturn. But from Capricorn in an inverse order, Sagittarius is attributed to Saturn; Pisces to Jupiter; Aries to Mars; Taurus to Venus; Gemini to Mercury; and last of all Cancer to the Moon. From among the number of these theologists consider Cancer and Capricorn as two ports; Plato calls them two gates. Of these, they affirm that Cancer is the gate through which souls descend, but Capricorn that through which they ascend, and exchange a material for a divine condition of being^[86]. Cancer is, indeed, northern and adapted to descent: but Capricorn, is southern, and accommodated to ascent. And, indeed, the gates of the cave which look to the north, are with great propriety said to be pervious to the descent of men: but the southern gates, are not the avenues of the gods, but of souls ascending to the gods. On this account the poet does not say it is the passage of the gods, but of immortals; which appellation is also common to our souls, whether in their whole essence or from some particular and most excellent part only, they are denominated immortal. It is reported that Parmenides mentions these two ports in his book, concerning the nature of things: as likewise that they were not unknown to the Egyptians and Romans. For the Romans celebrate their Saturnalia when the sun is in Capricorn, and during this festivity the servants wear the shoes of those who are free, and all things are distributed among them in common; the legislator intimating by this ceremony, that those who are servants at present, by the condition of their birth, will be hereafter liberated by the Saturnalian feast, and by the house attributed to Saturn, i.e. Capricorn; when reviving in that sign, and being divested of the material garments of generation, they return to their pristine felicity, and to the fountain of life. But since the path beginning from Capricorn is retrograde, and pertains to descent; hence the origin of the word Januarius or January from Janua a gate, which is the space of time measured by the sun while returning from Capricorn towards the east, he directs his course to the northern parts. But with the Egyptians the beginning of the year is not Aquarius, as among the Romans, but Cancer. For the star, so this borders on Cancer, which star the Greeks denominate Κύων, or the Dog. When this star rises they celebrate the calends of the month, which begins their year; because this is the place of the heavens where generation commences, by which the world subsists. On this account the doors of the Homeric cavern, are not dedicated to the east and west, nor to the equinoctial signs, Aries and Libra, but to the north and south, and particularly to those ports or celestial signs which are the nearest of all to these quarters of the world: and this because the present cave is sacred to souls, and to nymphs the divinities of waters. But these places are particularly adapted either to souls descending into generation, or to such as are separating from it. On this account they assigned a place congruous to Mithras, near the equinoctial; and hence he bears the sword of Aries, because this animal is martial, and is the sign of Mars: he is likewise carried in the Bull the sign of Venus; because the Bull as well as Venus is the ruler of generation. But Mithras is placed near the equinoctial circle, comprehending the northern parts on his right, and the southern on his left hand. Likewise to

the southern hemisphere they added the south, because it is hot, and to the northern hemisphere, the north, on account of the coldness of the wind in that quarter. Again, it was not without reason that they connected winds with souls sinking to generation, and again separating themselves from its stormy whirl: because, according to the opinion of some, souls attract a spirit, and obtain a pneumatic substance. Indeed, Boreas is proper to souls passing into generation: for the northern blasts recreate those who are on the verge of death; and refresh the soul reluctantly detained in the body. On the contrary, the southern gales dissolve life. For the north, from its superior coldness, collects into one, detains and strengthens the soul in the moist and frigid embraces of terrene generation: but the south dissolves the humid bands, and by its superior heat, having freed the soul from the dark and cold tenement of the body, draws it upward to the incorporeal light and heat of divinity. But since our habitable orb verges mostly to the north, it is proper that souls born in this turbulent region should be conversant with the north wind; and those who depart from hence with the south. It is, indeed, on this account that wind blowing from the north, is immediately on its commencement vehement; but the south, on the contrary, is more vehement towards the end. For the former hangs directly over the inhabitants of the north pole, but the latter is more distant, and the blast from places very remote, is more tardy than from such as are near; but when it is gradually collected it blows abundantly and with vigour. Hence, because souls enter into generation, through the northern gate, they have feigned this wind to be amatorial; and hence the poet:

[87]“Boreas changed into the form of a horse mingled himself with the mares of Ericthonius; and they big with young produced twice six foal.” And they report that he committed a rape on Orithyia, from whom he begot Zetis and Calais. But attributing the south to the gods, when the sun is at his meridian, they draw the curtains before the statues of the Gods in temples; and conceal them from the view, observing the Homeric precept, that it is not lawful for men to enter temples when the sun is inclined to the south: “for this path is open to immortals alone.”

[88]Hence when the god is at his meridian they place a symbol of mid-day and of the south in the gate of the temple. Besides, in other gates it was esteemed unlawful to speak at all times; because they considered gates as sacred. On this account too the Pythagoreans, and wise men among the Egyptians, forbade any person to speak while passing through gates or portals; for at that time the divinity who is the principle of the universe is to be worshipped in silence. But Homer was not ignorant that gates are sacred, because he represents Oeneus in the place of supplication knocking at the gate.

Before his gates the aged Oeneus came,
And suppliant shook their well-compacted frame [89].

Besides he knew that the gates of heaven were committed to the care of the hours, commencing in cloudy places; and which are opened and shut by the clouds: for he says,

“Whether they unfold, or close a dense cloud [90].”

Hence likewise they are said to resound because thunders roar through the clouds.

Heaven’s gates spontaneous open to the powers,
Heavens sounding gates kept by the winged hours [91].

Besides Homer elsewhere makes mention of the gates of the sun, signifying by these Cancer and Capricorn [92] for the sun proceeds as far as these signs, when he descends from the north to the south; and from thence ascends again to the northern parts. But Capricorn and Cancer are situated about the milky circle, Cancer occupying the northern extremity of this circle, and Capricorn the southern. Again, according to Pythagoras *the people of dreams* [92] are souls, which are reported to be collected in the milky way; the appellation of which is derived from souls, nourished with milk after their lapse into the whirls of generation. Hence those who desire to evocate departed souls, sacrifice to them with milk sweetened with honey: convinced that by the allurements of pleasure, these souls would desire to pass into generation, with the very beginning of which milk is generally produced.

Besides the southern regions produce small bodies, because being attenuated by the heat they are dried up [93] and dried up: and by a contrary reason all bodies generated in the north are large, as is evident in the Celtæ or Gauls, Thracians, and Scythians; and these regions are humid and abound with much pasture. For the word Boreas is derived from the Greek βορέα, which signifies aliment. Hence also the wind which blows from a land

abounding in nutriment is called βορρᾶς or nutritive. From these causes therefore the northern parts are properly adapted to the class of souls obnoxious to mortality and generation; but the southern quarter to immortals, exempt from the mutability inseparable from the flowing realms of generation: in the same manner as the east is attributed to the gods, and the west to demons. Hence since diversity is the origin of nature, the ancients considered every thing with a double entrance, as the symbol of nature. For the progression of things is either through an intelligible or a sensible nature. And if through a sensible nature, either through the sphere of the fixed stars, or through the orbs of the planets; and again either with an immortal or a mortal motion. Likewise one centre or hinge of the world is above the earth, but the other is subterranean; and one part of the heavens is eastern, and another western. In like manner some parts of the world have a dexter, and others a sinister position. Thus to night is opposed to day; and the harmony of the universe consists from the amicable junction of contrary and not similar natures. Plato also makes mention of two gates, one of which affords a passage to those ascending into the heavens, the other to those descending on the earth: and theologists place the sun and moon as the gates of souls, which ascend through the sun and descend through the moon. So, according to Homer,

“Two urns by Jove’s high throne have ever stood,
The source of evil one, and one of good^[93].”

But Plato, in his *Gorgias*, by vases understands souls, some of which are beneficent, and others malignant, and again some are rational and others irrational. But souls are denominated vases because they are capacious of certain energies and habits, after the manner of vessels. In *Hesiod* too we find one vase shut, but the other opened by pleasure, who diffuses its contents, and leaves nothing but hope behind. For in whatever concerns a depraved soul diffused about the dark and turbulent nature of matter, deserts the proper order of its essence; in all these, it is accustomed to nourish itself with the pleasing though delusive prospects of hope.

Since then every two-fold division is a symbol of nature, this Homeric cavern has with great propriety two gates, numerically different; the one peculiar to gods and pure souls; but the other to such as are mortal and depraved. Hence Plato took occasion to speak of bowls, and to substitute vases for Amphora, and two gates, as we have already observed, in the place of two ports. Also *Pherecydes Syrus*, mentions recesses, and dens, caves, gates, and ports, by which he insinuates the generation of souls, and their separation from a material nature. And thus much for an interpretation of Homer’s cave, which we appear to have sufficiently explained, without adducing any farther testimonies from ancient philosophers and theologists, which would give an unreasonable extent to our discourse.

One particular however remains to be explained, and that is the symbol of the olive at the top of the cavern; since Homer appears to insinuate something egregious by giving such a position: for he does not merely say that an olive grows in this place, but that it flourishes at the head or vertex of the cave.

“High at the head a branching olive grows,
Beneath a gloomy grotto’s cool recess, &c.”

But the growth of the olive in such a situation is not fortuitous as some may suspect, since it finishes and ^{completes} the ænigma of the cave. For as the world was not produced by the blind concurrence of chance, but is the work of divine wisdom and an intellectual nature, hence an olive the symbol of divine wisdom, flourishes near the present cavern, which is an emblem of the material world. For the olive is the plant of Minerva, and Minerva is wisdom. And since this goddess was produced from the head of Jupiter, the theological poet gives a proper position to the olive, consecrated at the head of the port: signifying by this symbol that the universe is the offspring of an intelligible nature, separated indeed by a diversity of essence, though not by distance of place from his work; and by unremitting and ever present energies, not remote from any part of the universe, but situated as it were on its very summit, that is governing the whole with perfect wisdom from the dignity and excellence of his nature. But since an olive always flourishes, it bears a similitude peculiar and convenient to the revolutions of souls in this material region. For in summer the white part of the leaves is upwards, but in winter it is bent downwards. On this account also in prayers and supplications they extend the branches of an olive, presaging from this omen that they shall exchange the sorrowful darkness of danger for the fair light of security and peace. But the olive is not only of an ever-flourishing nature, it likewise bears fruit, which is the reward of labour, is sacred to Minerva, supplies the victors in athletic labours with crowns, and affords a friendly branch to the suppliant petitioner. Thus too the world is governed by an intellectual nature, and a wisdom ever flourishing and vigilant, who also bestows on the conquerors in the athletic race of life, the crown of victory, as the reward of severe toil, and patient

perseverance: and the mighty builder who supports the universe by his divine energies, invigorates miserable and suppliant souls, contending for the most glorious of all prizes, the olympiad of the soul.

In this cave therefore, says Homer, all external possessions must be deposited; here, naked and assuming a suppliant habit, afflicted in body, and casting aside every thing superfluous, sense too being averse from needless possessions, it is requisite to sit at the foot of the olive, and consult with Minerva, by what means we may most effectually amputate and destroy that hostile rout of passions, which lurk in the secret recesses of the soul. Indeed as it appears to me it was not without foundation that Numenius thought the person of Ulysses in the Odyssey represented to us a man who passes in a regular manner over the dark and stormy sea of generation^[94]; and thus at length arrives at that region, where tempests and seas are unknown, and finds a nation

“Who ne’er knew salt, or heard the billows roar,”

Again, according to Plato, the deep, the sea, and a tempest are so many symbols of the constitution of matter^[96] and on this account, I think, the poet called that port by the name of the marine-god Phorcys.”

“But it is the port of the ancient marine Phorcys^[95]”.

Likewise his daughter Thoosa, is mentioned in the beginning of the Odyssey. But from Thoosa the Cyclops^[94] was born, whom Ulysses deprived of sight that he might by this means while sailing over the stormy ocean be reminded of his sins, till he was safely landed in his native country. On this account too, a seat under the olive is proper to Ulysses, as to one who supplicates divinity, and would please his natal dæmon with a suppliant branch. For indeed it will not be lawful for any one to depart from this sensible life in a regular way and in the shortest time, who blinds and irritates his material dæmon; but he who dares to do this, will be pursued by the anger of the marine and material gods, whom it is first requisite to appease, by sacrifices, labours, and patient endurance; at one time by contending with perturbations, at another time by employing stratagems of various kinds, by all which he transmutes himself into different forms; so that at length being stripped of the torn garments by which his true person was concealed, he may recover the ruined empire of his soul. Nor will he even then be freed from molestation, till he has entirely passed over the raging sea, and taken a long farewell of its storms; till though connected with a mortal nature, through deep attention to intelligible concerns, he becomes so ignorant of marine and material operations, as to mistake an oar for a corn-van.

Nor is it proper to believe that interpretations of this kind are forced, and are nothing more than^[98] the conjectures of ingenious men: but when we consider the great wisdom of antiquity; and how much Homer excelled in prudence and in every kind of virtue, we ought not to doubt, but that he has secretly represented the images of divine things under the concealments of fable. For it is not possible that this whole exposition could be devised, unless from certain established truths, an occasion of fiction had been given. But rejecting the discussion of this to another work, we shall here finish our proposed explication of the cave of the nymphs.

It is now requisite that we should direct our attention to Iamblichus, the celebrated disciple of Porphyry, ⁶¹⁰ on account of the sublimity of his genius, and his admirable proficiency in theological learning, was surnamed, *the divine*. This extraordinary man who appears to have been born for the advancement of theology, though zealously attached to the Platonic philosophy, yet explored the wisdom of other sects, particularly of the Pythagoreans, Egyptians, and Chaldeans; and formed one beautiful system of recondite knowledge, from their harmonious conjunction. There is a short life of this philosophic hero extant, by Eunapius, the substance of which is as follows; Iamblicus was descended of a family equally illustrious, fortunate, and rich. His country was Chalcis, a city of Syria, which they denominate Cælen. He associated with Anatolius, who was the second to Porphyry, but he far excelled him in his attainments, and ascended to the very summit of philosophy. But after he had been for some time connected with Anatolius, and most probably found him insufficient to satisfy the vast desires of his soul, he applied himself to Porphyry, to whom (says Eunapius) he was nothing inferior, except in the structure and power of composition. For his writings were not so elegant and graceful as those of Porphyry: they were neither agreeable, nor conspicuous; nor free from impurity of diction. And though they were not entirely involved in obscurity, and perfectly faulty; yet, as Plato formerly said of Zenocrates, he did not sacrifice to the *mercurial Graces*. Hence he is far from detaining the reader with delight, or inviting him to a perusal of his works; but he rather seems to avert and dull the attention, and frustrate the reader's expectation. However, though the surface of his conceptions is not covered with the flowers of elocution, yet his thoughts contain a most admirable depth, and his imagination is truly divine. He shared in an eminent degree the favour of divinity on account of his cultivation of justice; and obtained a multitude of associates and disciples, who came from all parts of the world, for the purpose of participating the streams of wisdom, which so plentifully flowed from the sacred fountain of his wonderful mind. Among these was Sopater^[96] the Syrian, of the greatest eloquence, both in composition and discourse; Eustathius the Cappadocian; and of the Greeks, Theodorus and Euphrasius. All these were excellent for their virtues and attainments, as well as many others of his disciples, who were not much inferior to the former in eloquence; so that it seems wonderful, how Iamblichus, could attend to them all, with such gentleness of manners and benignity of disposition.

He performed some few particulars relative to the veneration of divinity, by himself, without his associates and disciples; but was inseparable from his familiars in most of his operations. He imitated in his diet the frugal simplicity of the most ancient times; and during his repast exhilarated those who were present by his behaviour, and filled them as with nectar by the sweetness of his discourse. Some of these inflamed with an unwearied desire of hearing his wisdom, and incapable of being satiated with its pleasure, were his constant guests, and once addressed him as follows: "Why, O divine master, do you thus act alone, without communicating to us your most consummate wisdom? Yet it has been reported to us by your servants, that you have been seen, while engaged in prayer, elevated more than ten cubits from the ground, your body and garments at the same time being changed into a golden colour; and that when your prayers have been finished, your body has returned to its pristine form, and descending to the earth you have associated and discoursed with us as before." Upon this Iamblichus laughed (though he was not addicted to laughter) and replied: "He who invented this false relation, was not unpleasant; but in future, nothing shall be transacted without you."

The two following circumstances, relative to the theurgical powers of this wonderful man, are related by Eunapius, which the reader may credit or reject as he pleases. At that season of the year, when the sun rises in conjunction with the dog-star, Iamblichus went with his disciples to sacrifice, in one of the suburbs of the city; and after the sacrifice was performed they returned to town, gently walking along, and discoursing concerning the gods, as a subject very proper for the occasion. Then Iamblichus, who was perfectly lost in thought in the midst of the discourse, whose voice was fallen, and eyes immoveably fixed on the earth, turned to his companions and exclaimed: "Let us take another road, for not far from hence there is a funeral procession". Iamblichus accordingly chose a purer way, and was accompanied by some who were ashamed to forsake their master: but the greater part, among whom was Aedesius, obstinately persisted in the former road, ascribing the affair to the vanity and superstition of the man, and tracing the event with avidity and caution. In the mean time, those whose office consists in burying the dead approached, contrary to the expectation of his disciples; and upon enquiring whether they had taken that road from the first, they answered in the affirmative, and that no other path led to the place of their destination. But the second relation is far more wonderful than the present: for in the first (says Eunapius) perhaps the sight and smell of Iamblichus was more powerful than that of his disciples. His associates

therefore, not satisfied with this testimony of his extraordinary powers, were desirous to try him in a greater affair, and upon soliciting Iamblichus for this purpose, he replied that a proof of this kind was not dependant on his own will, but must be referred to a proper opportunity. In a short time after this, they all went to Gadara in Syria, a place so famous for baths, that, after Baïæ in Campania, it is the second in the Roman empire. Here a dispute concerning baths arising while they were bathing, Iamblichus smiling, said to them: "Though what I am about to disclose is not pious, yet for your sakes it shall be undertaken;" and at the same time he ordered his disciples to enquire of the natives, what appellations had been formerly given to two of the hot fountains, which were indeed less than the others, but more elegant and graceful. Upon enquiry, they found themselves unable to discover the cause of their nomination; but were informed that the one was called ἔρως, *eros*, or *love*, and the other ἀντέρως, *anteros*, or *the god who avenges the injuries of lovers*. Iamblichus immediately touching the water with his hand (for he sat perhaps on the margin of the fountain) and murmuring a few words, raised from the bottom of the fountain, a fair boy, of a moderate stature, whose hair seemed to be tinged with gold, and the upper part of whose breast was of a luminous appearance. His companions being astonished at the novelty of the affair, let us pass on says he, to the next fountain; and at the same time he arose, fixed in thought, and performing the same ceremonies as before, called forth the other love, who was in all respects similar to the former, except that his hair, scattered in his neck was blacker, and was like the sun in refulgence. At the same time both the boys, eagerly embraced Iamblichus, as if he had been their natural parent: but he immediately restored them to their proper seats, and when he had washed departed from the place. After this affair, the astonishment of his familiars and disciples was so great, that they submitted to the doctrines of Iamblichus with implicit assent. Eunapius observes that other extraordinary particulars were related of Iamblichus, but that they had too much the appearance of fables to be combined with historical veracity. He adds that he should fear the preceding relations, were delusive and fictitious, if they had not been confirmed by men who were eye-witnesses of their reality.

A celebrated philosopher named Alypius, lived at the same time as Iamblichus, who was deeply skilled in dialectic; but was of such a short stature, and so slender in body, that he exhibited the appearance of a pygmy. However his great abilities amply compensated for this trifling defect; and he might be said to have emigrated into soul alone, by which he was possessed as by some inspiring god. This Alypius had many followers, but his mode of philosophising, was confined to private conference and disputation, without committing any of his dogmata to writing. Hence his disciples gladly applied themselves to Iamblichus, desirous to draw abundantly from his copious mind, as from a perennial and overflowing fountain. The same therefore of each continually increasing, they once accidentally met like two refulgent stars, and were surrounded by so great a croud of auditors, that it represented some mighty musæum. While Iamblichus on this occasion waited rather to be interrogated than to propose a question himself, Alypius, contrary to the expectation of every one, relinquishing philosophical discussions, and seeing himself surrounded with a theatre of men, turned to Iamblichus, and said to him: *tell me O philosopher, is the rich man unjust, or the heir of the unjust? For in this case there is no medium.* But Iamblichus hating the acuteness of the question, replied, "This kind of disputation, O illustrious man, relative to external concerns, is foreign from our philosophical mode; since we alone propose as subjects of speculation, characters replete with philosophic virtue." After he had said this he departed, and at the same time all the surrounding multitude was immediately dispersed. But Iamblichus collecting himself when alone, and admiring the acuteness of the question, often privately resorted to Alypius, whom he vehemently extolled for the subtilty of his judgment, and the sagacity of his genius; and whose life he historically and copiously delineated. This Alypius was an Alexandrian by birth, and died in his own country, worn out with age: and after him Iamblichus^[97], leaving behind him many roots and fountains of philosophy; which, through the cultivation of succeeding Platonists, produced a fair variety of vigorous branches and copious streams.

The writings of this extraordinary man, though inestimably valuable, are not numerous; and the greater part are unfortunately lost. The only one which is preserved, relative to the Platonic theology, is the following:

On the mysteries of the Egyptians, Chaldæans, and Assyrians: or an answer to the epistle of Porphyry to the prophet Anebo. This admirable book contains many of the greatest arcana, of the ancient theology; respecting gods and dæmons, their cultivation and commerce, and the conjunction of the soul with divinity. It fully solves all the doubts concerning the impassivity of a divine nature; demonstrates its omnipresence, and never-failing energy; shews that we are continually surrounded with its light; and that all the divinities subsist in indivisible union, and indissoluble consent. There is an excellent Greek and Latin edition of this work, published, with copious notes, by the learned Gale: and it is greatly to be wished, though but little to be expected, that it was once

translated into English, accompanied with a philosophical comment, which might both disclose its beauties, and reveal the sacred mysteries it contains.

Among the lost writings of Iamblichus, respecting theology, we may reckon in the first place, *three books, concerning the physics, ethics, and theology of arithmetic*: or φυσικὰ ἠθικὰ καὶ Θεολογούμενα Ἀριθμητικῆς. These three books, form the fifth, sixth, and seventh, of a great work by Iamblichus, in ten books, entitled, *a collection of the Pythagoric dogmata*. And the seventh book, Fabricius thinks is still extant.

2. *Concerning the gods*. From this work the emperor Julian derived most of the dogmata contained in his elegant oration to the sun.

3. *Commentaries on the Parmenides, Timæus and Phædo of Plato*. The inestimable value of the first and second of these commentaries is sufficiently evident from the frequent mention made of them by Proclus, in his writings on these dialogues; and from the admirable passages contained in them, which he has fortunately preserved.

4. *Concerning the perfection of the Chaldaic philosophy*. The twenty-seventh book of this great work, is cited by Damascius, in his MS. treatise περὶ ἀρχῶν: and this whole discourse was studied with avidity by Proclus, and enabled him as we are informed by Marinus, to ascend to the very summit of theurgic virtue. And thus much for the works of Iamblichus relative to the Platonic theology.

But here we may observe with wonder how the deepest mysteries of this theology, became more and more explicitly unveiled, in proportion as the Roman empire was hastening to its dissolution; and Christianity to an universal establishment. Though the works of Plotinus and Porphyry contain all the arcana of theology, yet they contain them occultly and concisely. Their depth is in a great measure latent, and their fire condensed. But in Iamblichus we find greater copiousness and precision: theology is rendered more easy of access, and her light is more widely diffused. The profundity of barbarian theology is more accurately explored, and its consent with that of Pythagoras and Plato more abundantly and distinctly evinced. We find in his works, mystery united with bright evidence, religion with sublime philosophy, and science with divine illumination. Now this difference in the mode of unveiling the Platonic theology, is perfectly agreeable to the state of the Roman empire, and the new religion, at the periods when these modes were adopted. In the times of Plotinus and Porphyry, when Galienus, and Dioclesian swayed the sceptre of the world, Rome was in the middle of her course to destruction and Christianity had nearly accomplished one half of her journey to ecclesiastical empire. However as neither the fall of Rome, nor the establishment of Christianity, were then absolutely certain, these philosophers were cautious in disclosing all, that a baser period might require. This period Iamblichus was destined to see approach under the reign of the emperor Constantine; when the new religion was established, and the old treated with ridicule and contempt. Indeed the new religion had no sooner ascended the throne, and assumed the reins of arbitrary power, but she was surrounded with myriads of unphilosophic converts, and in her progress to despotism, drew after her the capital of Rome; and at once fixed the destruction of its ancient empire. And thus we may see that the writings of Iamblichus were perfectly correspondent to the depravity of the times.

The most celebrated disciple of Iamblichus, appears to have been one Œdesius a Cappadocian, who was of noble birth; but, as is generally the case with philosophers, possessed but a slender estate. According to Eunapius who wrote his life, he was not much inferior to Iamblichus, except in a divine afflatus, which seems to have been peculiar to that illustrious hero. To Œdesius we may add Maximus and Dexippus, both disciples of Iamblichus; and frequently cited by Simplicius in his elaborate commentary on the predicaments of Aristotle. But here we must regret, that none of the immediate successors of Iamblichus, contributed any thing to the advancement of the ancient theology. They revered indeed the arduous flights, and divine genius of their master; but never attempted even to imitate, what they could not equal, and were content to gravel without presuming to soar. The iniquitous times indeed of the emperor Constantine, may afford a reasonable apology for the decay of genius, and the langour of philosophy. The destructive rod of ecclesiastical empire was already extended; and its lethargic influence was already felt on the active spirit of liberal investigation. Religious faction had now started from the bosom of delusion; and holy persecution, was hastening from the infernal seats, to massacre the nations, and deluge Europe and Asia in blood. The peaceful and instructive disputes of philosophers, were now beginning to be exchanged for the jargon of orthodox and heterodox sectaries; and the calm voice of ancient theology, was silenced by the barbarous and tumultuous sounds, of Arian and Trinitarian clamours. This alarming change however, checked only for a short period, the generous ardour of the philosophic genius: for the æra was now at hand in which theology was destined to display the full blaze of her celestial light. *Sacred* zeal indeed presumed

to hurl the darts of *faith*, against her venerable person: but her arm was destitute of vigour, and her weapons fell innocent to the ground. The buckler of *true* theology was not to be transpierced, by such imbecil darts; and the attempt was like that of weak old Priam, against the strong and youthful Pyrrhus.

—— ——— telumque imbelle fine ictu
Conjecit; rauco quod protenus ære repulsum,
Et summo clypei necquicquam umbone pendit.

But the order of discourse now brings us to a survey of the last branch of the *theological tree*, in which we may discover amidst numerous ramifications, and elegant foliage, exhaustless vigour, and luxuriant fruit. The source of this illustrious branch was the great Athenian Plutarch, of whom such honourable mention has been made, in the preceding life of Proclus^[98]. To Plutarch succeeded Syrianus and Olympiodorus; and to these Hermeas and Proclus. It was by the labours of this last philosophical hero, that theology received the consummation of excellence, and exhibited diffused elegance, combined with majesty and strength. This will be evident from perusing his life, and studying his more abstruse writings, among which the following elements may be deservedly ranked. Though Marinus as we have observed in the life of Proclus, was his immediate successor, yet Asclepiodotus the master of Damascius, was his best disciple; and was most capable of receiving the exuberant streams of wisdom, which vigorously flowed from his philosophic mind. After Asclepiodotus an illustrious series of philosophers succeeded, who terminated the golden chain of Platonists, and were the last advocates for the dignity of ancient wisdom and theology. These great men were Zenodotus, Severianus, Ammonius Hermias, Hierius, Asclepius, Simplicius, Isidorus, Damascius, Diogenes, Eulalius and Priscian. But of all these, none except Damascius, appears to have contributed any thing to the perfection of theology: for the works of the rest consist for the most part in excellent commentaries on Aristotle; but Damascius, in his book, on principles, has preserved a most valuable store of recondite wisdom, and unfolded some of the sublimest mysteries of ancient theology. This inestimable work is however still in manuscript, and is not likely in the present age to emerge from its shameful concealment.

Seven of the preceding illustrious heroes, who were united by friendship as well as philosophy, Damascius the Syrian, Simplicius of Cilicia, Eulalius the Phrygian, Priscian the Lydian, Hermias and Diogenes of Phœnicia, and Isidorus of Gaza, disgusted with the religion of their sovereign Justinian, determined to seek from Chosroes the Persian king, that liberty of conduct which their native country denied. Chosroes though a barbarian, was deeply skilled in the philosophy of Plato and Aristotle; and was so imbued with the dogmata of Plato, that not one of his abstruse dialogues escaped his penetrating genius. The ill success however of these philosophers in their journey to Persia, gives us reason to suspect that the philosophic attainments of Chosroes, were influenced more by pride than the love of truth: and that he affected the name without possessing the requisites of a sage. The return of these philosophers was precipitate, and their disappointment extreme. They derived however a considerable advantage from their expedition; and the conduct of Chosroes in this particular will confer immortal honour on his character and name. He was the means of procuring for the seven sages, an exemption from the barbarous penal laws of Justinian against the Pagans; and thus enabled them to end their days in security and peace, and in the enjoyment of that liberty of conscience which no religion before the Christian, ever attempted to destroy.

The reign of Justinian, indeed, as it firmly established the Christian religion, terminated the glorious empire of philosophy, by suppressing the schools of Athens, and suspending the ecclesiastical sword over the heads of heathen theologists. But the fall of philosophy was naturally succeeded by the darkness of delusion and ignorance; by the spirit of wild fanaticism, and intolerant zeal; by the loss of courage and virtue; and by the final dissolution of the empire of the world. She was ruined indeed but not without revenge. War, pestilence, and famine, were the scourges of a prince who had presumed to demolish her schools, and intercept the diffusion of her sacred light: and his reign was disgraced by an irreparable decrease of mankind, in the most fertile regions of the earth. We may add too that his dominions were alarmed with the dreadful blaze of two mighty comets, whose malignant light foretold approaching calamities and war; and signified perhaps the establishment of religious anarchy, and the commencement of *barbarous impiety* and folly. And to complete this catalogue of prodigies and desolation, every year of his reign was marked with violent earthquakes of uncommon duration, and incredible extent. The whole surface of the Roman empire was agitated with horrid internal convulsions; and enormous chasms were formed by the earth's strong vibrations. Large bodies were discharged into the air, and the sea concurring in the general ruin, overflowed or deserted its natural bounds, by alternately advancing and retreating

with accumulated majesty and strength: and a mountain was torn from Libanus, and hurled into the waves, amidst the dreadful tossings of the deep. History after this period exhibits nothing but religious dissensions, despicable *councils*, and bigoted sects; the enmity of *saints*, and the discord of Nestorians and Jacobites, Maronites and Armenians, Copts and Abyssinians. *Religious* war, and *pious* rebellion succeeded to philosophical theory; and Nestor and Cyril led the confused and clamorous dance of ecclesiastical disputation.

It would neither be consistent with the design of this history, pleasing to the author, nor entertaining³¹⁷ the Platonic reader to trace the rapid increase of barbarism and ignorance, after the abolition of the Athenian and Alexandrian schools. It will be sufficient to observe that the jargon of innumerable sects, established a tyranny unknown to the Pagan world, the tyranny of religious despotism; and finally extirpated from the earth, the dominion of ancient wisdom and virtue. From the incredible multitude of different persuasions, Christianity lost all appearance of a *revelation*; and by the conduct of its professors, seemed rather calculated to confound than illuminate mankind. The same infatuated spirit has indeed marked its progress to the present day; and we find that in proportion as this baneful zeal prevails, knowledge retires, virtue droops, and magnanimity is destroyed: hypocrisy becomes the substitute for generosity; and whining cant succeeds the decent confidence, inspired by genuine dignity and worth. As the rapidity of a river is increased by the contraction of its channel; so its vigour is diminished by the multiplication of its streams. In a similar manner, the influence of any religion is lessened, when it is divided into various streams of opinion, by the discord of party, and the zeal of profession. The energy of the whole is lost by diffusion; and the river of the *Church* is weakened by the numerous and narrow rivulets of *Dissenters*. Experience unfortunately shews, that the professors of a national religion, are generally men of greater integrity, than those who compose the dissenting sects; and the fact may be supported by a rational theory. The trifling employments, groveling cares, and contemptible fame which are necessarily connected with religious dissension, unavoidably debilitate the mind, and contract the heart. The whole attention is engrossed in regarding the little concerns, and supporting the narrow opinions of a party; and that strength of understanding, and integrity of character, which are requisite to acquire eminence in science and virtue, are lost in imbecile exertions and hypocritical cant. It is on this account that I should prefer a dissenter in Scotland, and a papist in France, to a dissenter and catholic in England: for in those countries they cease to be sectaries, and may consequently in some degree become virtuous and wise.

It seems at first sight surprising that there should be no sects among the Grecian polytheists: they³¹⁸ were unanimous in their belief of a multitude of gods subordinate to one supreme: their mode of worship was uniformly the same; and they appear to have had no conception of religious innovation. Shall we say that a religion is false in proportion to its unity; that truth may be branched out into an endless variety of discordant streams; and that error alone resists the power of copious and confused division? Such a speculation is indeed curious, but not safe; and its result would perhaps be more logical than *orthodox*, and more informing than discreet! Let us therefore direct our attention, to a more important subject, and consider the excellence of the Christian religion with respect to the commercial interests of mankind. That Christianity is not favourable to philosophy (I mean that of the ancients) is evident from its causing the destruction of the ancient schools; which it has not yet restored, though more than a thousand years have elapsed since their dissolution. Indeed the wisdom of a *sage*, is not likely to coincide with the doctrine of a *fisherman*; and implicit faith, ill suits with liberal doubt, and severe investigation. However, the spirit of *meekness*, which Christianity so admirably inculcates, though opposite to the *dignity* of philosophy, promotes the humility of merchandize, and facilitates the emoluments of trade. It enables men to suppress their passions from considerations of interest; teaches them to refer every thing to private advantage; and to consider magnanimity as a dangerous and arrogant virtue. It is to this spirit, that we must ascribe the great extent of commerce, in all the civilized parts of the world; and that Europe is much richer, though less wise than of old. The spirit of meekness by gradually suppressing the noble ardour of ancient heroism, and withdrawing the attention from abstract investigations, as daring and presumptuous, has given birth to innumerable discoveries in the arts, unknown to the speculative genius of antiquity. Hence the luxuries of life have received an immense improvement; and the spirit of meekness though not calculated to soar, has wandered over the surface of the earth, and diffused its *humble* blessings even to the remote regions of the poles. Penetrating and smooth, it has crept like oil, through the communities of mankind; and increased the activity, by lubricating the joints of the flexible body of Commerce. As oil too allays the fury of the sea, and calms its agitated waves; so meekness suppresses the effervescence of desire, restrains the restless spirit of enquiry, and calms the impetuosity of genius. Hence though we are no longer surprised with the daring exploits, and prodigious talents which distinguished the ancient world, yet we can boast a greater uniformity of character, a

more general equality in moderate attainments, and a more interested spirit. In consequence of this universal mediocrity, our capacity for commerce increased, and our abilities enlarged, for accumulating wealth by groveling pursuits. But the most important advantage acquired by the spirit of meekness, is that mentioned by the great apostle of the Gentiles, “of becoming all things to all men.” The benefits indeed which such a pliability of temper confers on a commercial kingdom, compose so great a part of the arcana of traffic, that revelation alone could have made mankind sensible of their importance. Meekness like Proteus assumes every possible appearance which the interest of concealment may require; and philosophy alone can trace it, through its multiform shapes, and vanquish its transforming power.

But though we excel the ancients in the virtue of meekness, and its attendant arts, we are infinitely below them in the cultivation of intellectual philosophy. By the invention of the microscope and telescope, we have indeed discovered the structure of the subtle parts of body, and beheld stars invisible to the ancient world. Hence our knowledge of particulars has received, and is continually receiving an immense increase: but we forget that particulars are infinite, and that while they produce the fleeting fabric of opinion, they are incapable of forming the steady and permanent basis of science. The doctrine of causes, was the object of ancient investigation: the enumeration of effects is the busy employment of the moderns. Experimental inquiries have enabled the philosopher of the present day to solve partial phænomena, and to deceive the importunities of doubt by the intervention of secondary causes. However, arguments derived from the modifications of matter, can only satisfy superficial enquirers; and will be indignantly rejected, by the profound and contemplative genius. So far from deriving any illumination by accumulated experiments, the professors of this philosophy confess their ignorance of principles; and neglect their investigation under the specious pretext of declining hypotheses. On the contrary the philosophers of antiquity impelled by intellectual dignity and strength, ascended to principles, as the pillars of the universe, and the sources of conviction and repose. Hence they glorified in asserting and vindicating the capacious powers of the soul: and by severe investigation, experienced the serene splendours of knowledge, and banished the anxieties of doubt. The intellectual philosophy refines the morals while it enlightens the mind, and improves the heart while it exalts the powers of imagination and thought. On the contrary the mechanical philosophy produces opposite consequences, by introducing the darkness of ignorance, and debasing the energies of the soul.

But there cannot I think be a more egregious instance of the barren state of philosophy at present, than the prevailing opinion that the most valuable knowledge is derived from common life, and the general conduct of mankind. The manners of the multitude, so far from affording any really valuable information, exhibit nothing but specimens of folly and vice, astonishingly various, and differently combined. A knowledge of this kind may indeed be necessary to the man who wishes to accumulate wealth, and acquire popular honours; but is infinitely remote from the possession of true wisdom, and the true cultivation of human understanding. The best, as well as the most exalted knowledge, is as we have already proved^[99], that which is desirable for its own sake; which confers felicity on its possessor, and gives a final respite to the arduous labour of mental investigation. The knowledge of common things, is alone the province of common, or uncultivated minds; and men of great genius in every age, have been distinguished by their happy ignorance of the trifling pursuits, and empty attainments of the vulgar. Indeed he who mixes much with the multitude, necessarily imbibes false opinions and engages in puerile occupations: the strength and activity of his mind, is continually weakened, or unworthily exerted, by a general diffusion; and he at length loses all that intellectual energy, which nature first implants, but retirement calls forth into the blossoms of elegance, and the perfection of vigor.

The late Dr. Johnson is a striking instance of the truth of these observations; and a lasting example of the wretchedness of a mind unenlightened by philosophy. His talents were indeed vast and uncommon, but degraded by false cultivation and ruined through neglect. Hence he employed himself solely on subjects of vulgar speculation and thought deeply on nothing but the vices and follies of the illiterate and the base. Like a giant in the dark, his strokes were indeed powerful, but often ineffectual; and were never directed by the hand of wisdom, or assisted by the irradiations of truth. Thus he constantly displayed strength without skill, and exertion without knowledge, abilities without genius, and grandeur without a grace. He appears to resemble indeed nothing so much as the eyeless Polypheme. Deprived of the cheering light of science and philosophy, he wandered in the caverns of sense, wretched through the want of sight, and avoided by the timid multitude who trembled at his strength. To approach him too near was generally destructive of the order of society, and often fatal to the peace of bold but ignorant individuals.

His piety too as well as his literary talents shews how little of felicity is to be expected where philosophy is wanting. For though he professed to believe in the immortality of the soul, he was a perpetual slave to the dread of death: and though he was continually exercised in the externals of religion, he could find no consolation when alone. There is nothing indeed whose certainty is so generally admitted in discourse as the soul's immortality; and yet nothing at present is so generally disbelieved. For I will not disgrace the word belief, by supposing it possible that a man can be firmly assured of this important truth, and yet continually seek for arguments in defence of its reality. This is however the case with modern believers. They profess reverence for the decisions, and faith in the doctrines of revelation; but are glad to seek for conviction in the arguments of philosophy. Faith is found sufficient to support the mind, while it reclines on the bosom of the church, or clings round the pillars of orthodox opinion. But when it is once shaken by enquiry and staggered by doubt; when it leaves the enchanted enclosure of faith, and ventures on the wide ocean of enquiry; it can alone find security in the harbour of reason, and rest in the embraces of philosophy.

Dr. Johnson is however celebrated by his female biographer as "a man good beyond the imitation of mortals." As if goodness could ever reside in a soul perpetually harassed with fears, and agitated with passion; distracted with the prospects of futurity, and afraid of retiring into itself. Is it not ridiculous to suppose that a consciousness of virtue and worth can ever be combined with misery and fear; or that the steady and serene light of truth can ever dwell enshrined in the gloom of despondence, or beam through a mind disturbed and clouded with care? "The good man says Plotinus is ever tranquil and serene, undisturbed by passion, and superior to grief:" and that religion is but of little worth, which confers on its votary nothing but the torments of anxiety from consciousness of inward folly or vice; and the dread of dissolution from the uncertainty of its result. We may rest assured that no one can be truly worthy who is wretched in himself: for to be truly good is to resemble the divinity: and to suppose that misery can be combined with such a character, is to ascribe imperfection to deity, and unhappiness to the fountain of good. For the exemplar cannot be contrary to its image, though it may be infinitely superior in excellence and dignity of nature.

And thus much for a history of the restoration of the Platonic theology by the latter Platonists. I only add, that I am in no respect a debtor to the gratitude of the public: for my writings hitherto, have neither been attentively studied, nor liberally received. Solely influenced by the love of truth, I have endeavoured to disseminate the wisdom of Greece, and to draw aside the mystic veil of recondite theology: but experience has convinced me that the period of philosophy is past; and that some fortunate revolution can alone restore its fallen honours, and establish its original sway. Should the present work survive the literary wreck, which will probably precede the revival of philosophy, I shall consider myself amply rewarded for the toil of its execution: and I am not ashamed of owning, that the pleasing hopes of such an event have inspired me with the patience and vigour requisite to so laborious an undertaking. In short whatever may be its immediate or future success, my views have been liberal in the publication, and my mental advantages considerable from the study of ancient philosophy. Amidst the various storms of a life distinguished by outrage and disease, it has been a never-failing support, and an inviolable retreat. It has smoothed the brow of care, and dispelled the gloom of despondence; sweetened the bitterness of grief, and lulled agony to rest. After reaping such valuable advantages from its acquisition, I am already rewarded, though my labours should be unnoticed by the present and future generation. The lyre of true philosophy is no less tuneful in the desert than in the city; and he who knows how to call forth its latent harmony in solitude, will not want the testimony of the multitude to convince him that its melody is extatic and divine.

E L E M E N T S

OF

T H E O L O G Y ^[100]

O N T H E O N E .

PROPOSITION I.

All multitude participates in a certain respect of *the one*.

For if it in no respect participates of *the one*, neither will the whole be one whole, nor each of the many^[102] from which the multitude is composed: but each of these will also be multitude, and this will be the case in infinitum; and each of these infinities will again be infinite multitude. For if it in no respect participates of *the one*, neither according to its whole self, nor according to each of its parts, every where, and throughout there will be infinite multitude. For each of these *manys*, which ever you assume, will either be *one* or *not one*, will either be multitude or nothing. But if each of these manys be nothing, that which is composed from them shall also be nothing. But if each be many, then each shall consist from infinite infinities. But these consequences are impossible. Since no being is composed from infinite infinities^[101]. For there is nothing greater than infinite. But that which is constituted from all is greater than each particular. Nor can any thing be composed from nothing. All multitude therefore participates in a certain respect of *the one*.

PROPOSITION II.

Every thing which participates of the one, is both *one* and *not one*.

For if it be not *the one itself* (αὐτοὺν) (since it participates of *the one*) because it is something different^[103] from *the one*, it suffers *the one*, by participation, and sustains itself to become one. If then it be nothing else besides *the one*, it is one alone, and does not participate of *the one*, but is *the one itself*. But if it be something different from *the one*, which is not *the one*, but its participant, it is both *not one* and *one*, not indeed the self-subsisting *one*, but *one being*, as participating of the *one itself*. This then is neither *one*, nor does it subsist as *the one*, but it is *one being*, at the same time participating of *the one*, and on this account, since it is not the self-subsisting one, it is both *one* and *not one*, because it is something different from *the one*. For so far as it abounds it is *not one*; but so far as it is passive from participation, it becomes *one*. Every thing therefore which participates of *one*, is both *one*, and *not one*.

PROPOSITION III.

Every thing which becomes one, becomes so through the participation of one; and is one, so far as it suffers the participation of one.

For if things which are not one, become one, it must be by a conjunction, and communication with each other: and they will sustain the presence of one, without being *one itself*. Hence they will participate one, so far as they suffer themselves to become one. For if they are already one, they will not become one: since that which is, does not become that which it already is. But if they are formed from non-one, and privation, they will in the first place possess one, from some *one*, being ingenerated in their nature.

PROPOSITION IV.

Every thing united, is different from the *one itself*.

For if it is united it will participate of one, so far as it is called united. And that which participates of one is both one, and non-one. But *one itself*, is by no means one and non-one. For if this also was both one and non-one, the one which it contains will also possess both, and this in infinitum; since there is no *one itself* in which the progression can stop; but every thing will be both one and non-one. That which is united therefore is something different from one. For if that which is united was the same with one, one would be infinite multitude; and in like manner each of the parts from which the united nature is composed.

PROPOSITION V.

All multitude is posterior to the one itself.

For if multitude is prior to the one, the one indeed will participate of multitude, but the multitude which is prior to the one, will not participate of the one: since it is multitude prior to the subsistence of the one. For it cannot participate that which is not: because that which participates of the one, is both one and non-one. But the one does not yet subsist, since multitude is the first. It is however impossible, that there should be any multitude, which in no respect participates of the one. Multitude therefore is not prior to the one. But if multitude subsists together with the one, it will be of the same order with the one: for time cannot hinder such a conjunction. Hence neither the one can be essentially many, nor multitude one, because they are at the same time contradistinguished; since neither is prior, or posterior to the other. Multitude therefore will not be essentially one, and every thing it contains will be a non-one, and this in infinitum, which is impossible. Hence it naturally participates of the one, nor can any part of it be assumed which is not one: for if any part is not one, it will be an infinite composed from infinites, as we have demonstrated. And hence it entirely participates of the one. But if the one which is one itself, in no respect participates of multitude, multitude will be perfectly posterior to the one; participating indeed of the one, but not participated by the one. But if the one should participate of multitude in such a manner as to exist as one according to subsistence, but as not one according to participation; the one itself will be multiplied, in the same manner as multitude is united by the one. Hence the one will communicate with multitude, and multitude with the one. But things which coalesce, and communicate after a manner with each other, if they are congregated by something else, that something must have a prior existence. But if they connect themselves, they are not opposed to each other: for opposites do not hasten to a mutual conjunction. But if the one, and multitude have a contrary division, and multitude, so far as multitude, is not one, and the one so far as one, is not multitude; hence the one cannot subsist in the other: for they would be at the same time both one and two. But if there be any thing prior to the one, and multitude which collects them into one, this will either be one, or non-one. And if non-one, it will either be many, or nothing. But it is not many; lest multitude should be prior to the one. Nor is it nothing: for how can that congregate which is nothing? Hence it is the one alone. For this one is not also many, lest we should advance in an infinite progression. It is therefore the one itself; and all multitude proceeds from the one.

CONCERNING UNITY.

PROPOSITION VI.

Every multitude is either composed from things united, or from unities.

For that every one of things many is not multitude alone, is evident; and it is likewise clear that each part of this multitude again, is not multitude alone. But if it be not multitude alone, it is either united, or unities. And indeed if it participates of unity it is united: but if it be composed from things primarily united, it is unities. For if there be one itself, there is also that which primarily participates the one, and is primarily united. But this is composed from unities. For if it is composed from things united, these are again united from certain others, and this will take place in infinitum. But it is requisite that a nature which is primarily united, should be composed from unities. And thus we have discovered what we proposed in the beginning.

Concerning PRODUCING CAUSES, and THINGS PRODUCED.

PROPOSITION VII.

Every thing productive of another, is more excellent than the nature of the thing produced.

For it is either more excellent, or worse, or equal. Let it be in the first place, equal.

That which is produced from this therefore, will itself also, either possess a power productive of some other, or it will be entirely barren. But if it be barren, it will on this account be worse than its producing cause: and because of its inefficacy, it will be unequal to that which is prolific, and possesses a productive power. But if it be productive of other natures, it will either produce that which is equal to itself (and this will be the case in all things, and all beings will be equal to each other, and nothing will be more excellent than another, since the productive nature always constitutes the thing produced equal to itself), or that which is unequal. But in this case, it will not be equal to its producing cause: for it is the property of equal powers to fabricate equal effects. But the productions of these are unequal to each other, since on this hypothesis the producing cause, is equal to that which is prior to itself, but that which is posterior is unequal to it. It is requisite, therefore, that the thing produced should not be equal to its producing cause.

But neither can the producing cause be over worse than the thing produced. For if the producing cause, confers essence on the thing produced, it bestows power also, according to essence. And if it is productive of all the power which that posterior to itself possesses, it can also make itself such as its production. But if it can do this, it will also make itself more powerful: for impotence cannot hinder, since a fabricative power is present, nor a defect of will: for all things, naturally desire good. Hence if it can form any thing else more perfect, it will also perfect itself, before it perfects that which is posterior to itself. The thing produced, therefore, is neither equal to, no more excellent than its producing cause: and hence the producing cause is entirely more excellent than the nature of the thing produced.

On the FIRST GOOD, which is called THE GOOD ITSELF.

PROPOSITION VIII.

The first good which is no other than good itself, precedes all the participants of good.

For if all beings desire good, it is evident that the first good is above beings. For if he is the same with any one being, either being is the same with the good, and so this particular being, will no longer be desirous of good, since it is the good itself; for that which desires any thing, is indigent of that which it desires, and by its desire is different and foreign: or being and the good are different; and the good, will indeed participate of being, and being of the good. It is, therefore, a particular good resident in some particular participant, but not good simple and universal, and which all beings desire: for this is the common object of desire to all beings. But that which is generated in another, participates alone of that, in which it is generated. The first good therefore is nothing besides good. For if you add any thing else, by addition you diminish the good itself; effecting a particular good, instead of good simple and universal. For that which is added, since it is not good itself, but something less, diminishes by its essence the good itself.

Concerning that which is SUFFICIENT TO ITSELF.

PROPOSITION IX.

Every thing sufficient to itself, either according to essence, or according to energy, is more excellent than that which is insufficient, and the cause of whose perfection depends on another cause.

For if all beings, naturally desire good, and one thing supplies itself with good, but another is indigent of something else; the former, will indeed have the cause of good present, but the latter separate and apart. By how much the nearer, therefore, that is which affords the object of desire, by so much the more excellent will it be than that which requires a separate cause, and externally receives the perfection of its being or energy. Besides, that which is sufficient, is both similar and diminished, and more similar to the good itself. It is diminished, because it participates good, and because it is not the first good. Yet it is in some respect allied to the good, because it can possess good from itself. But that which participates, and participates through another, is more distant from the first good, which is nothing else than good.

PROPOSITION X.

Every thing sufficient to itself is worse than that which is simply good.

For what is that which is sufficient, than that which from itself, and in itself, possesses good? But this is now full of good, which also it participates: but it is not the simply good. For that, as has been demonstrated, is more excellent than the participation and repletion of good. If then that which is sufficient fills itself with good, that from which it fills itself will be more excellent than that which is sufficient, and will be superior to sufficiency: for that which is simply good is not indigent of any thing. For it does not desire any other; since by desire it would be imperfect; and thus would be full of good, and not the first good.

CONCERNING CAUSE.

PROPOSITION XI.

All beings proceed from one first cause.

For either there is no cause of beings, or the causes of all terminated beings revolve in a circle, or there is an infinite ascent of causes; so that one thing is the cause of another, and the prior subsistence (προυπόστας) of essence, no where stops its progression. But if there be no cause of beings, neither will there be an order of things second and first; of the perfecting, and perfect; of the adorning, and adorned; of the generating, and generated; of the active and passive: nor will there be any science of beings. For the knowledge of causes is the employment of science: and we then assert that we know, when we know the causes of beings. But if causes revolve in a circle, the same causes will be both prior and posterior, more powerful and more debile. For every thing which produces is more excellent than the nature of the thing produced. But there is no difference, whether we conjoin the cause with the thing caused by many, or fewer mediums, and place the thing caused as subordinate. For the cause is more excellent than all the intervening natures of which it is the cause: and by how much the greater the number of mediums, by so much the more is it a cause. But if there be an addition of causes in infinitum, and one always proceeds from another; on this hypothesis likewise science cannot subsist: for there is no knowledge of infinites. But causes being unknown, neither can there be any science of things subsequent to causes. If, therefore, it is requisite that there should be a cause of beings, and causes are distinct from things caused, and there can be no infinite ascent; there will be a first cause of beings, from which as a root particulars proceed, some of which exist in propinquity, and others at a distance from his nature. For that it is necessary there should be one principle, is demonstrated; because all multitude is secondary to the one.

PROPOSITION XII.

The principle and first cause of all beings is the good.

For if all things proceed from one cause, it is requisite to call that cause, either the good, or more excellent than the good. But if it be more excellent than the good, we ask whether *any thing* emanates from this cause into beings, and into the nature of beings, or *nothing*? And indeed if nothing, it will be absurd: for we cannot, on this hypothesis, any longer preserve it in the order of a cause; since it is every where requisite that something should be present from the cause to the things caused, and especially from the first cause, from which all things depend, and through which every being exists. But if there is a participation of this first cause in beings, in the same manner as there is of the good, there will be something *more excellent* than goodness, penetrating into beings from the first cause. For since it is more excellent, and superior to the good, it cannot bestow on secondary natures any thing worse than the benefits distributed by that which is posterior to itself. But what can be more excellent than goodness itself? Since we apply the term more excellent to that which participates more of the good. If then that which is non-good, is not more excellent, it must be posterior to the good. But if, likewise, all beings desire good, how can any thing be prior to this cause? For if good also desires, how can it be good in the most eminent degree? But if it does not desire, must not all beings desire that cause of all, from which they proceed? And if it is the good itself, from which all beings depend, the good must be the principle and first cause of all.

PROPOSITION XIII.

Every Good is endued with a power of uniting its participants, and every union is good; and *the good itself*, is the same with *the one*.

For if the good itself is the preserver of all beings, and on this account is desirable by all, but the one²⁰ itself, preserves and contains the essence of each: (for all things are preserved by the one, and dispersion removes every thing from essence) hence the good causes those things to be one, to which it is present, and contains them by union. But if the one is endued with a congregating and containing power, it perfects every being by its presence: and hence it is good to all things to be united. But if union is essentially good, and good is unific (or unifying), the simply good, and the simply one is the same: at the same time uniting, and benefiting beings. Hence it is, that things which in a certain respect fall from good, are also deprived of the participation of one: and that things which are destitute of the one, because they are replete with separation, are after the same manner likewise deprived of good.

C O R O L L A R Y .

Hence both goodness is union, and union is goodness; and the good is the one, and the one is the first good.

Concerning an IMMOVEABLE, and SELF-MOTIVE PRINCIPLE OR CAUSE.

PROPOSITION XIV.

Every being is either immoveable, or moved; and if moved, it is either moved by itself, or by another.

In the first place if it is moved by itself, it is self-motive, but if by another, it is alter-motive (ἐτεροκίνητον.) Every being therefore, is either immoveable, or self-motive, or alter-motive. For it is necessary, that since there are alter-motive natures, there should be something immoveable, and between these, a self-motive nature. For if every thing alter-motive, when in motion is moved by another, motions are either performed in a circle, or in infinitum. But they can neither subsist in a circle, nor in infinitum, since all beings are terminated by a principle, and the motive nature, is more excellent than the thing moved. Something immoveable therefore will be the first mover. But if this be the case, it is necessary that there should be something self-motive. For should all things stand still, what will that be which is first moved? It cannot be the immoveable itself, for motion is not natural to this. Nor the alter-motive, for it is moved by another. It remains therefore that the self-motive, must be that which is first moved; since it is this which unites the alter-motive to the immoveable, existing as a medium, moving, and at the same time moved. For of those, the one moves alone, and the other is alone moved. Every being therefore, is either immoveable, or self-motive, or alter-motive.

C O R O L L A R Y .

From hence also it is evident, that of things which are moved, a self-motive nature is the first, but of things motive, an immoveable nature.

Concerning an INCORPOREAL ESSENCE, and its PROPERTIES.

[330]

PROPOSITION XV.

Every thing which is converted to itself, is incorporeal.

For no body is naturally adapted to be converted to itself. For if that which is converted to any thing, is conjoined to that to which it is converted, it is evident that all the parts of a body must be conjoined with all the parts of that which is converted to itself: since self-conversion then takes place, when that which is converted becomes one with that to which it is converted. But this is impossible in body, and in all partible natures. For the whole of that which is partible, is not conjoined with the whole, on account of the separation of the parts which are differently situated. No body therefore is naturally adapted to self-conversion, so that the whole may be converted to the whole. And hence whatever is self-convertive is incorporeal and impartible.

PROPOSITION XVI.

Every thing which is converted to itself, has an essence separate from all body.

For if it be inseparable from any body, it will not possess some action separable from body. For if essence is inseparable from body, it is impossible that an essential energy should be separable: since in this case energy would be more excellent than essence; because the latter would be indigent of bodies, but the former would be sufficient to itself, without requiring the assistance of body. If then any thing be inseparable according to essence,

it must be so likewise according to energy, or indeed more inseparable. But if this be the case, it is not converted to itself. For that which is converted to itself, as it is different from body, has an energy separate from body, neither subsisting through body, nor in conjunction with its nature: since action, and that to which action is directed, is not indigent of body. Hence that which is converted to itself, is entirely separable from bodies.

PROPOSITION XVII.

Every thing which moves itself primarily, is endued with a self-convertive power.

For if it moves itself its motive energy also is resident in its nature, and the thing moving is at the same time one with the thing moved. For either it moves with a part, but is moved in a part, or the contrary. But if one part is motive, and another part is moved, it will not be essentially self-motive, because it will subsist from non-self-motive natures: and it will appear indeed self-motive, but will not be so essentially. But if the whole moves, and a part is moved, or the contrary, there will be some part in each, which, according to one, will be at the same time both moving and moved^[102]: and this will be primarily self-motive. But if one and the same moves and is moved, it will possess with itself the energy of moving, because it is self-motive; but it will be converted to that in which it energises. Every thing therefore primarily self-motive is converted to itself.

PROPOSITION XVIII.

Every thing which supplies being to others, is that primarily which it bestows on the things supplied.

For if it gives being, it procures the communication from its own essence. But that which it gives is worse than its own essence: and that which it is, is more excellent and perfect. For every artificer of any thing, is more excellent than the nature of the thing fabricated: and hence that which pre-exists in the donor, is more sublime than the gift; for the one is primary, but the other secondary and subordinate. For it is necessary, either that both should be the same, and that there should be one reason of both; or that nothing should be common, or the same in both; or that this should be first, but that the second. But if there be, one and the same reason, or definition, the one will no longer be cause, and the other effect; nor this in itself, but that in the thing given; nor will this be the efficient, but that the effect. But if they have nothing the same, the remainder will not subsist, in consequence of the existence of the other, because it will communicate nothing to its being. It remains therefore, that this which bestows is first; but that which is bestowed is second; among which the being of the one is supplied from the other.

PROPOSITION XIX.

Every thing which is primarily inherent in any of the natures among beings, is present to all the beings distributed according to that nature, in one reason, and after the same manner.

For if it be not present to all after the same manner, but to these, and not to those; it is evident that it^[32] not be primarily inherent in that nature. But it will be present with some primarily, and in others which participate sometimes but not always, secondarily. For that which sometimes subsists, and sometimes not, does not exist primarily, nor essentially, but is adventitious, and extrinsically accedes to the natures in which it resides.

Concerning the GRADATION OF BEINGS.

PROPOSITION XX.

The essence of soul is superior to all bodies; and an intellectual nature is superior to all souls: and the one itself is superior to all intellectual essences.

For every body is moved by another, but is naturally incapable of moving itself. But it is moved by^[33] itself, through the participation of soul: it likewise lives through soul, and by means of its presence, is after a manner self-motive; but when soul is absent it is alter-motive; because it is essentially endued with this nature, but soul is allotted a self-motive essence. For it imparts self-mobility to whatever it supervenes. But soul is much prior to that which it essentially imparts. It is therefore above bodies, as a self-motive essence: since these become self-motive through participation. Again, soul which is moved from itself has the second order from an immoveable

nature, existing immoveable in energy; because a self-motive nature precedes all things that are moved, but an immoveable essence, all moving natures. If therefore soul which is self-motive, moves others, it is requisite that an immoveable mover, should be prior to soul. But intellect moves, existing immoveable, and always energising according to the same. For soul participates through intellect of eternal intelligence; in the same manner as body participates through soul of a self-motive nature. For if eternal intelligence, was primarily resident in soul, it would be inherent in all souls; in the same manner as a self-motive nature. And hence this is not primarily inherent in soul. It is therefore requisite, that a first-intellective nature should be prior to soul. Intellect therefore is prior to souls. But the one is prior to intellect. For intellect though immoveable, is not the one: since it understands, and energises about itself. But all beings of whatever kind, participate of the one, but all do not participate of intellect. For it is necessary that those natures should participate of knowledge, to whom a portion of intellect is present; because intellectual cognition, is the principle, and first cause of knowledge. Hence the one is superior to intellect; nor is there any thing superior to the one: for the one is the same with the good. But the good is the principle of all things, as we have demonstrated.

That INTELLECT is not the FIRST CAUSE.

PROPOSITION XXI.

Every order beginning from unity proceeds into some multitude co-ordinate to unity:
and multitude of every order is reduced to one unity.

For unity possessing the relation of a principle, generates a multitude proper to itself. Hence one series, and one universal order descends from unity into multitude. For there would neither be any order, nor series, if unity was essentially barren. But multitude is again reduced into one common cause of all co-ordinates. For that which is the same in every multitude, does not proceed from one of the things which multitude contains: since that which emanates from one out of many; is not common to all, but is alone peculiar to the property of that one. Since therefore according to every order, there is both a certain communion, coherence, and identity, on account of which *these* are said to be co-ordinate, and *those* of another order, it is evident that the sameness of every order proceeds from one principle^[103]. There is therefore in every order an unity prior to multitude, affording one reason, and series to the things ordered in itself, as well with respect to each other, as likewise to the whole. For admitting that among things contained under the same series, one thing is the cause of another, yet it is necessary that before all things, there should be a cause that the series is one, and that from it all things should be generated as co-ordinates; not that every thing may be a particular something, but may exist of this particular order.

C O R O L L A R Y .

From hence it is evident that both one and multitude, is inherent in the nature of body, and that one nature has many coherent natures, and that many natures depend on the one nature of the universe. And this property belongs to the order of souls, to begin from one first soul, and to descend into a multitude of souls, and to reduce multitude into one. And to an intellectual essence it is peculiar to possess an intellectual unity, and a multitude of intellects proceeding from one intellect, and intimately converted to its nature. And to the one prior to all things, a multitude of unities is present, and to unities themselves a return to the one. Hence after the first one, unities^[104] subsists; intellects after the first intellect; souls after the first soul; and natures after universal nature.

PROPOSITION XXII.

Every thing which subsists primarily, and according to the nature of a principle, is in
every order one; and is neither two, nor more than two, but is universally self-
begotten.

For if possible let it be two; since the same absurdity will ensue should more than two be admitted. Then if it be two, it is either that which is composed from both unities; and in this case the first will be one and not two: or it is each of the unities. But in this case, either one of each, and not both, will be the first; or both will be equally the first. But if equally, neither of them will be the first: for if the one is first, but this one is not the same with the other, what order will it possess? For that subsists primarily, which is nothing else than what it is denominated. But each of these being different, the one from the other, each at the same time is, and is not that which it is said

to be. But if these differ from each other, yet not with respect to that which is called first (for this primarily suffers identity), both will not be first; but that through the participation of which, both are said to be first.

C O R O L L A R Y .

From hence it is evident that the first being is one alone, and that there are not two, or more first beings.³⁵ And that the first intellect is one alone, but that there are not two first intellects. And that the first soul is one; and in every particular species, the same conclusion will result: as for instance the first beautiful, first equal, and similarly in all the rest. In like manner there is the same demonstration, with respect to one first form of animal and man.

Concerning the imparticipable^[105] (or that which is without participation), and that which participates.

P R O P O S I T I O N XXIII.

Every imparticipable produces from itself participants, and all participated hypostases, or subsistencies, are reduced to imparticipable essences.

For an imparticipable possessing the relation of unity, as depending on itself, and not on another,³⁶ and as separated from participants, generates things able to participate. For either it remains in itself barren, and possesses nothing honourable; or it gives something from itself. And that which receives from this imparticipable participates, and that which is given subsists in a participated manner. But every thing participating of another by which it is generated, is posterior to that nature which is similarly present to all things, and which fills all things from itself. For that which subsists in one thing is not in others: and that which is similarly present to all things, that it may illustrate all is not in one thing, but before all. For it either subsists in all, or in one of all, or before all. But that which subsists in all, because distributed into all, again requires another, which may unite its divided nature: and all things will no longer participate of the same, but this will participate one thing, and that another, if the one is distributed into many. But if it subsists in one of all, it will no longer be common to all, but to one. If therefore it is common to things capable of participating powers, and is likewise common to all, it will be prior to all things. But this is imparticipable.

P R O P O S I T I O N XXIV.

Every participant is inferior to that which it participates, and that which is participated is subordinate to an imparticipable.

For the participant being imperfect prior to participation, but becoming perfect through participation, is entirely posterior to that which it participates; so far as it is perfect through participation. For so far as it was imperfect, it is inferior to that which it participates and which is the cause of its perfection. But that which is participated because common to some one, and not to all, is on this account allotted a subsistence inferior to that which is common to all, and not to some particular one: for the latter is more allied, but the former, less, to the cause of all things. Hence an imparticipable, precedes things participated; and these last precede participants. And in short that which is imparticipable is one prior to many; that which is participated is one in many; and every participant is at the same time non-one, and one.

C O N C E R N I N G T H E P E R F E C T .

P R O P O S I T I O N XXV.

Every thing perfect proceeds to the procreation of such offspring as it is able to produce, imitating the one principle of the universe.

For as the principle of the universe, on account of his goodness is uniformly constitutive of all beings,³⁷ (for the good is the same with the one, and on this account that which is endued with the form of good, is the same with that which is uniform); so things posterior to the principle, on account of their proper perfection hasten to generate other things subordinate to their own essence: and this perfection is a certain portion of the good; and the perfect so far as perfect imitates the good. But this is constitutive of all things. Hence the perfect also is naturally productive of things within its power; and that which is more perfect, by how much the more perfect it is, by so

much the more is it the cause of more numerous productions: for that which is more perfect, participates more of the good. But this is nearer to the good, is more allied to the cause of the universe, and is the cause of more numerous productions. But the imperfect, by how much the more imperfect it is, by so much the more is it the cause of less numerous effects: for existing more remote from the producer of all things, it becomes the cause of fewer effects. For to constitute and adorn, or perfect, or contain, or vivify, or fabricate all things, and to effect each of these in many, is allied to the principle of all. But if this is accomplished in a few it becomes more foreign from the principle.

C O R O L L A R Y .

From hence it is evident that matter which is most distant from the principle of the universe, is barren, and the cause of nothing. For if it should generate any thing, it would have something posterior to itself, and it would not be the most remote. But that which it produces, would be more distant than itself, and because it produces and imitates the productive cause of all beings, it would be nearer to the principle of the universe.

CONCERNING THAT WHICH PRODUCES.

PROPOSITION XXVI.

Every cause productive of other things, abiding in itself produces natures posterior and subsequent to itself.

For if it imitates the one itself, but that immoveably generates natures posterior to itself, hence ³³⁸every productive nature will in a similar manner possess the cause of producing. But that the one itself immoveable generates, is evident from hence. For if he generates through motion, either motion will be resident or non-resident in his nature, and that which is moved will be no longer one; because it will be changed and moved from one. Hence the one will either produce in infinitum ^[106], or immoveably. And every producing nature will imitate the one productive cause of the universe. For from that which is first, that which is not first every where emanates. And hence from that which is productive of all things, that which is productive of some things will proceed. Every producing cause, therefore, produces subsequent natures, abiding in itself: and while productive natures abide in themselves undiminished, secondary natures are produced from them. For that which is in any respect diminished, cannot abide such as it is.

PROPOSITION XXVII.

Every producing nature, on account of its perfection, and abundance of power is productive of secondary natures.

For if it produces, not on account of its perfection, but through a defect of power, it cannot preserve its own proper order immovable. For that which affords being to another, through its defect and imbecillity, confers on it essence by a mutation and alteration of itself: but every producing nature abides such as it is, and while it abides, that which is posterior to itself proceeds into being. Hence existing full and perfect, it procreates secondary natures immovably, and without diminution: at the same time existing such as it is, neither changing itself into its progeny, nor diminishing its nature. For the thing produced is not a dissection of the parts of the producing cause: since this is neither proper to generated natures, nor to generating causes. Nor is it a transition; since it is not the matter of that which proceeds to generation. For the producing cause abides such as it is, and the production is different from itself. That which generates therefore abides without alteration, and without diminution, multiplying itself through its prolific power, and supplying from itself secondary subsistencies.

PROPOSITION XXVIII.

Every producing nature generates things similar to itself, prior to such as are dissimilar.

For since the producing cause is necessarily more excellent than the thing produced, these can never be mutually the same simply considered, or equal according to power. But if they are not the same and equal, but different and unequal; they are either entirely separated from each other, or they are both united and separate. But if they are entirely separate, they cannot be conciliated with each other, and the thing produced will not sympathize with its cause. Hence the one will not participate of the other, because they are entirely different. For

that which is participated, imparts a communication to its participant, with respect to that of which it participates. But it is necessary that the thing caused should participate of the cause, as that from which its essence is derived. But if that which is produced is in one respect separated, and in another united to its producing cause; if it equally suffers both, it equally participates and does not participate of its producing cause. It will therefore both possess and not possess an essence from it, after the same manner. But if it should be more separated, the thing generated will be more foreign from its generating cause than is proper, and will be to itself more inelegant than elegant, and more deprived than endowed with sympathy of nature. If then generated natures, are both allied according to essence, and in sympathy with their causes, naturally depending upon, and desiring a contact with them, pursuing good and obtaining the object of desire through cause, it is evident that things produced are more united with their producing causes than separated from them. And things which are more united are more similar than dissimilar to the natures with which they are especially united. Every productive cause therefore generates things similar prior to such as are dissimilar.

PROPOSITION XXIX.

Every progression is accomplished by a similitude of things secondary to such as are first.

For if a producing cause generates similars prior to dissimilars, similitude generates things produced from their producing causes. For similars become similar through similitude, and not through dissimilitude. If, therefore, progression in its diminution preserves the identity of the thing generated to its generator, and exhibits that which is posterior to itself secondarily, such as itself is primarily, it will possess its essence through similitude.

PROPOSITION XXX.

Every thing immediately produced from another, both abides in its producing cause, and proceeds from it.

For if every progression is effected while things first abide, and is perfected through similitude, things³⁴⁰ similar subsisting prior to the dissimilar, the thing produced will abide in a certain respect in its producing cause: since that which has entirely proceeded from its cause, possesses nothing the same with that which abides, but is perfectly separated. But if it possesses any thing in common, and united with its abiding cause, it will also abide in its cause, in the same manner as that abides in itself. But if it abides only without proceeding, it differs nothing from its cause, nor will it while that abides, be effected something different: for if it is something else, it will be distinct and separate. But if separate, and its cause abides, it proceeds from its cause that it may be separated from its abiding generator. So far, therefore, as the thing produced possesses any thing the same with its producing cause, it abides in this cause: but so far as it possesses something different, it proceeds from its cause. But on account of its similitude, it is at the same time in a certain respect both the same with, and different from its producing cause. It abides therefore, and at the same time proceeds with its cause, and neither is separate from the other.

PROPOSITION XXXI.

Every thing proceeding from another essentially, is converted to that from which it proceeds.

For if it should indeed proceed, without being converted to the cause of its progression, it will not desire this cause: since every desiderative nature (τὸ ὀρεγόμενον) is converted to the object of its desire. But every being desires good; and its acquisition takes place through a cause proximate to particulars. Hence particulars desire their cause. For well-being is distributed to any particular being, through the same cause as being itself. But desire is primarily directed to that cause from which well-being proceeds: and conversion is directed to that, to which desire primarily tends.

PROPOSITION XXXII.

Every conversion is effected by a similitude of the converted natures to the object of their conversion.

For every thing converting itself to another, hastens to conjoin itself with the object of its conversion, and desires its communion and conjunction. But similitude collects all things, in the same manner as dissimilitude

separates and divides. Conversion therefore is a certain communion and contact. But every communion and every contract is caused by similitude. And hence every conversion is effected by similitude.

PROPOSITION XXXIII.

[107] Every thing proceeding from, and returning to another, has a circular energy.

For if it is converted to that from which it proceeds, it conjoins the beginning with the end. And therefore alone and a continued motion: this commencing from, and that proceeding to the abiding nature. Hence all things proceed circularly, from causes to causes. But these circles of regression are greater and less: since some conversions proceed to things proximately placed above them, but others to things more superior, and so on to the principle of all. For all things proceed from this, and to this finally return.

PROPOSITION XXXIV.

Every thing which is naturally converted, converts itself to that from which it derives the progression of its peculiar subsistence.

For if it is naturally converted, it possesses an essential desire towards that to which it is converted, and it directs all its being towards that to which it makes an essential conversion. And it is also essentially similar to the object of its conversion; and on this account, is in sympathy with it according to nature, because allied to it by essence. But if this be the case; either the essence of both is the same; or the one proceeds from the other; or both derive their similitude from some other one. But if the essence of both is the same, how can the one be naturally converted to the other? And if both proceed from one; both will be naturally converted to this one. It remains therefore that the one must derive its essence from the other. But if this be the case, progression also must originate from that to which there is a conversion according to nature.

C O R O L L A R Y .

From hence it is evident that intellect is the object of desire to all things: that all things proceed from intellect; and that the whole world, though eternal, derives its essence from intellect. For it is not because eternal excluded from proceeding from intellect. Nor is it because established in perpetual order, excluded from conversion. But it both perpetually advances and is eternal according to essence; and it is perpetually converted and indissoluble according to its invariable order.

PROPOSITION XXXV.

Every thing caused (or produced by a cause), both abides in its cause, proceeds from, and is converted to it.

For if it alone abides, it will differ in nothing from its cause; from which it will be indistinct. For progression subsists together with distinction. But if it alone proceeds, it will be unconjoined with its cause, and in no respect communicate with it according to a sympathetic affection. But if it is alone converted, how can that which does not derive its essence from its cause, be naturally converted to that which is foreign from its nature? But if it abides and proceeds, and is not converted, how can the natural desire of every thing to well-being, and to good, and an excitation to its generator, arise? But if it proceeds indeed, and is converted, but does not abide, how, since it is distant from its cause, can it hasten to be conjoined with it? Since it would be unconjoined prior to its departure. For if it was conjoined, according to this conjunction it would entirely abide. But if it should abide and be converted, but should not proceed, how, since not separated, can it be converted? For every thing returning to its cause, in the act of returning is assimilated to that from which it is essentially divided. But it is necessary that the thing caused, should either alone abide, or be alone converted, or alone proceed; or that the extremes should be conjoined with each other; or that the medium between these, should be united with each extreme; or that all these should take place together. It remains, therefore, that every thing caused must abide in, proceed from, and be converted to its cause.

PROPOSITION XXXVI.

Of all things which are multiplied in progression, such as are first are more perfect than such as are second, and such as are second than those of a posterior order, and so on in continual succession.

For if progressions distinguish things produced from their causes, and are the subordinations of secondary natures to such as are first; first progressions will be more conjoined with their causes, from which they are produced, and of which they may be considered as the blossoms. But secondary progressions are more remote from their causes; and this will be the case in a continual succession. But things nearer, and more allied to their causes, are more perfect: for causes are more perfect than things caused. But such as are more remote are more imperfect, because on this account dissimilar to their causes.

PROPOSITION XXXVII.

Of all things which subsist according to conversion, the first are more imperfect than the second; and the second than those in succession. But the last are the most perfect.

For if conversions are produced in a circle, and conversion tends to that from which the progression began; but progression is from the most perfect, conversion also will tend to the most perfect. And if that to which progression tends is the last, the first conversion will originate from this. But progression to the last is the most imperfect; and conversion commences from the most imperfect. In things, therefore, subsisting according to conversion, the first are the most imperfect, but the last the most perfect.

PROPOSITION XXXVIII.

Every thing proceeding from a certain number of causes, is converted to them by the same number as it proceeds from them; and every conversion subsists through the same cause as progression.

For since both are produced through similitude, that which immediately proceeds from any cause, is immediately converted to it: for the similitude was immediate. But that which requires a medium in its progression, requires also a medium in its conversion. For it is requisite that both progression and conversion should subsist about the same. It will, therefore, be first converted to the medium, and afterwards to that which is more excellent than the medium. Hence well-being is distributed to every thing through the same number of causes as being; and the contrary of this is likewise true.

PROPOSITION XXXIX.

Every being is either alone essentially converted, or vitally, or according to a gnostic mode (γνώσις).

For it either possesses being alone from its cause, or life together with being, or it receives from thence a gnostic power. So far, therefore, as it is *being* alone, it makes an essential conversion. But so far as it likewise *lives*, a vital conversion. And so far as it *knows*, a gnostic conversion. For according to its progression, such is its conversion; and the measures of its conversion are defined by the measures of its progression. Hence some are endued with desire according to being alone; this desire being adapted to the participation of causes. But others according to life; and this vital desire is a motion to more excellent natures. And others according to cognition, which desire is a perception of the goodness of causes.

PROPOSITION XL.

Self-subsistent natures antecede all things proceeding from another cause.

For if every thing sufficient, is more excellent, either according to essence, or according to energy, ¹³⁴¹that which depends on another cause, but that which produces itself, because productive of its own being, is sufficient to itself. But that which is alone produced from another is not sufficient. Likewise since that which is sufficient is more allied to good; and things more allied and similar to causes, subsist from cause prior to dissimilars: hence things self-productive, and self-subsistent, are more ancient, than such as proceed to being from another alone.

For either nothing is self-subsistent; or the good itself is such; or things which are the first subsistents from the good. But if nothing is self-subsistent, there will not be a true sufficiency in any thing. For this cannot reside in the good, since that is more excellent than sufficiency, subsisting as the one; and being *the good*, but not possessing good. But neither on this hypothesis can sufficiency reside in natures posterior to the good. For all things will be indigent of that which is prior to their nature. But if the good is self-subsistent, because it produces itself, it will not be one. For that which proceeds from one, is not one: and it will proceed from itself, if it subsists by itself. And hence the one itself will be at the same time one, and not one. It is necessary, therefore, that a self-subsistent nature should be posterior to the first: and it is evident, that it must likewise be prior to things alone proceeding from another cause. For it is more principal than these, and, as we have demonstrated, is more allied to the good.

PROPOSITION XLI.

Every thing residing in another is alone produced by another. But every thing residing in itself, is self-subsistent.

For that which abides in another, and is indigent of a subject, can never be generative of itself. For that which naturally generates itself, does not require a foreign seat, since it is contained by itself, and is preserved in itself separate from a superior. But that which is capable of abiding, and of being established in itself, is productive of itself: since it proceeds into itself, and contains its own nature; and abides in itself; as a thing caused in its cause. For it does not abide as in place, nor as in a subject. For place is different from that which subsists in place, and that which resides in a subject is different from its subject. But this is the same with itself. For it is self-subsistent, and abides in itself, as the thing caused in its cause.

PROPOSITION XLII.

Every thing self-subsistent is converted to itself.

For if it proceeds from itself, it will also return to itself. For that which is the source of progression to particulars, is likewise the end of a conversion co-ordinate to the progression. For if it should alone proceed from itself, but should not be converted by a progression into itself, it will never desire its own proper good, and that which it is able to afford itself. But every cause is capable of conferring on its progeny, together with the essence it affords, well-being, which is conjoined to the essence which it distributes. Hence it can confer this on itself. And, therefore, this is the proper good of a self-subsistent nature. But this, according to the hypothesis, will not be desired by that which is converted to itself: and because it does not desire this, neither will it pursue it; and in consequence of not pursuing, it will be imperfect and insufficient. But if sufficiency and perfection belong to any thing, they must be proper to a self-subsistent nature. And hence it will pursue and desire its own proper good, and will be converted to itself.

PROPOSITION XLIII.

Every thing converted to itself, is self-subsistent.

For if it be naturally converted to itself, and is perfect in its self-conversion, it will also possess essence from itself. For that to which conversion according to nature tends; from this also the essential progression of every thing proceeds. If, therefore, it affords to itself *well-being*, it will also indeed afford to itself *being*; and it will be the lord of its own subsistence. Hence that which is capable of being converted to itself, is self-subsistent.

PROPOSITION XLIV.

Every thing converted to itself according to energy, is also converted to itself according to essence.

For if it can be converted to itself according to energy, but is not converted essentially, it will be more excellent according to energy than according to essence; since the former is convertible, and not the latter. For that which depends on itself, is more excellent than that which depends on another. And that which preserves itself, is more perfect than that which is only preserved by another. If, therefore, it is converted to itself, according to that energy which proceeds from essence, it will likewise be allotted a convertive essence; so that it will not only energize to itself, but will likewise depend on itself, and will be contained and perfected by itself.

PROPOSITION XLV.

Every thing self-subsistent is without generation.

For if it be generated, because generated it will be essentially imperfect, and indigent of that perfection which proceeds from another. But because it produces itself, it is perfect and sufficient. For every thing generated is perfected by another, which brings it into existence from a non-existent state. Since generation is the passage from that which is imperfect to its contrary, the perfect. But if any thing produces itself, it is always perfect; because it always coheres to its own cause, or rather inheres in that which is perfective of its essence.

PROPOSITION XLVI.

Every thing self-subsistent, is incorruptible.

For if it may be corrupted, it may desert itself, and exist separate from itself. But this is impossible. For, on account of the unity of its nature, it is at the same time both a cause and the thing caused. But every thing which is corrupted, is corrupted by a departure from its cause. For so far as any thing depends on that which contains and preserves it, so far it is contained and preserved. But that which is self-subsistent will never desert its cause, because it will not desert itself. For it is its own cause. And hence every thing self-subsistent is incorruptible.

PROPOSITION XLVII.

Every thing self-subsistent is impartible (i.e. without parts) and simple.

For if that which is self-subsistent is partible, it will constitute itself partible, and the whole will be converted to itself^[108], and all will be in all itself. But this is impossible. Hence that which is self-subsistent is impartible. But it is likewise simple. For if composite, it will contain both that which is excellent, and that which is base^[109]; and the more excellent will proceed from the more base, and the more base from the more excellent, since, according to hypothesis, the whole proceeds from the whole itself. Besides it will not be sufficient to itself, because it requires the elements of itself, from which it is composed. Every thing therefore which is self-subsistent is simple.

Concerning that which is Eternal, for the Purpose of demonstrating the Eternity of the World.

PROPOSITION XLVIII.

Every thing not eternal, is either a composite, or subsists in another.

For it is either capable of being dissolved into its component parts, and is entirely composed from the parts into which it is dissolved; or being indigent of a subject, and deserting its subject, it passes into non-entity. But if it is simple, and abides in itself, it will be indissoluble and incapable of dissipation.

PROPOSITION XLIX.

Every thing self-subsistent is eternal.

For there are two modes according to which it is necessary, that any thing should be non-eternal. The one flows from composition, and the other from residing in a subject. But that which is self-subsistent, is neither compound, but simple; nor does it abide in another, but in itself. And hence it is eternal.

PROPOSITION L.

Every thing which is measured by time is *generation*, either according to essence, or according to energy, so far as it is measured according to time.

For if it is measured by time, essence or energy, according to time, is proper to its nature. And it is likewise proper that the terms, *it was*, and *it will be*, should be different from each other. For if *it was*, and *it will be*, were numerically the same, that which is measured by time would suffer nothing from time proceeding, and always having something prior and posterior. If then *it was*, and *it will be* differ from each other, that which is measured

by time is *in generation* (γινόμενόν ἐξί) and never truly is^[110], but proceeds together with time, by which it is measured, existing in a state of tendency to being, or in becoming to be (ἐν τῷ γίνεσθαι ὄν). Nor does it stop in the same state of being, but always receives different being; so that its *now* becomes perpetually different on account of the progression of time, and prevents its existing totally at the same period of time. For it subsists in a dispersion of temporal extension, and is co-extended with time. But this is no other than to possess being, in non-being. For that which is in generation, or a state of becoming to be, is not generated. *Generation*, therefore, is that which subsists in a flowing existence.

PROPOSITION LI.

Every thing self-subsistent is exempt from things measured by time according to essence.

For if that which is self-subsistent is without generation, it cannot be measured by time according to essence. For generation subsists about a nature measured by time. Hence nothing self-subsistent subsists in time.

PROPOSITION LII.

Every thing eternal, is at once total.

For whether it only possesses an external essence, it will possess the whole at once present. Nor will it have one of its parts already constituted, but another which remains to be constituted, because not yet in existence; but as much as is possible it possesses the whole without diminution, and without extension. Or whether it possesses an energy with respect to essence, and this collected into one, and abiding in the same measure of perfection, and established as it were according to one and the same bound, immovably, and without progression, it will still possess the whole at once present. For if it be eternal, as its name denotes, it is a perpetual being (τὸ ἀεὶ ὄν). But *to be* sometimes, and to have an existence in *becoming to be*, is different from that which always is. And hence it is requisite that an eternal nature should not possess any thing prior and posterior: for in this case it would become generation and non-entity. But where neither prior, nor posterior, not *it was*, and *it will be* have any subsistence, but being alone, there an essence at once total abides^[111]. And every thing energizes according to its essence.

C O R O L L A R Y .

From hence it is evident that eternity is the cause of total existence. Since every thing eternal, has either its essence or its energy totally present with itself, either according to essence, or according to energy.

PROPOSITION LIII.

Eternity *itself* has an existence prior to all eternal natures, and *time itself* exists before all temporal natures.

For if every where participated natures are prior to their participants, and imparticipables before such^[112] are participated, it is evident that an eternal being is different from the eternity in an eternal nature, and different from eternity itself. For the first of these subsists as a *participant*, the second as a thing *participated*, and the third as an In like manner with respect to time, one thing exists as a *participant*, and the time which it contains, as a thing *participated*, and the time prior to this, as an *imparticipable*^[112]. And each of these consists from imparticipables every where, and in all things the same. But participated time alone exists in the natures by which it is participated. For there are many eternal and temporal natures, in all of which eternity abides according to participation. And since such a time is indivisible, there is also a divided time. And there is one time prior to these. And *eternity itself* is an *eternity of eternities*, but *time itself*, is a *time of times*; and they are sustainers of participated natures.

PROPOSITION LIV.

Every *eternity* is the measure of eternal natures, and every *time* of temporal natures; and these are the only two measures of life and motion in beings.

For every thing which measures, either measures according to a part, or measures the whole, when it is accommodated to that which is measured. But that which measures according to the whole is eternity; and that which measures according to parts is time. There are therefore only two measures, this of eternal, and that of temporal natures.

PROPOSITION LV.

Every thing subsisting according to time, either subsists in an eternal time, or has its subsistence in some part of time.

For if all progressions subsist by similitude, and prior to things perfectly dissimilar, things similar are more proximate to first natures than such as are dissimilar: and if it is impossible to conjoin with eternals, things formed in a part of time: (for as things generated differ from such as are self-subsistent, and things which have a partial, from such as have a perpetual existence, but the middle of these and those, are partly similar to them, and partly dissimilar.) Hence between things which are sometimes generated, and such as are eternal, the medium must either be that which is always in generation, or that which is sometimes, or that which is not truly, or does not possess true being. But it is impossible that the medium should be that which sometimes truly is. And that which is not true being, is the same with that which is sometimes in generation. Hence the medium cannot be that which *sometimes is*. It remains therefore that that which is always in generation, or in becoming to be, must be the middle of both: for on account of its passing, or flowing existence, it is conjoined with the worse nature, but on account of its perpetuity it imitates an eternal nature.

C O R O L L A R Y .

From hence it is evident that eternity is two-fold: for some things are of themselves eternal, but others according to time. And the former of these, is an *abiding* eternity; but the other a *flowing* eternity, or such as exists in becoming to be. And the former of these has its being united, and at once total; but the latter diffused, and unfolded according to temporal extension. And the former of these is essentially total; but the latter is composed from parts, each of which are separated according to prior, and posterior.

PROPOSITION LVI.

Every thing which is produced from secondary causes, is also produced from those prior and more principal causes, from which secondary causes are produced.

For if that which is secondary possesses its whole essence from that which is prior to itself, its power of producing emanates also from thence. For productive powers reside in producing causes according to essence, and replenish the essence of these. But if they are allotted a productive power from a superior cause, they possess from this the cause of being, measured from thence according to an hypostatic, or fabricative power. But if this be the case, the productions of this secondary cause, are caused on account of that which is prior to its nature: for that which perfects a cause perfects also the thing caused. And that the thing caused is more perfected from thence, is manifest. For if that which is first gives to the second, the cause of producing, it will primarily possess this cause; and on this account that which is second generates, receiving from thence a secondary generative power. But if the one becomes productive through participation, but the other by communication; on this hypothesis likewise that will be the primary and more principal cause, which bestows a power of generating on another proximate to its nature.

PROPOSITION LVII.

Every cause both energizes prior to the thing caused, and is productive of more effects posterior to the things caused.

For so far as it is cause, it is more perfect and powerful than that which is posterior to its nature. And if this be the case it is the cause of more effects. For it is the property of a greater power to produce more, of that which is equal, equal, and of that which is less, less effects. And that which in things similar can effect greater things, can also accomplish such as are less. But that which can accomplish less effects, cannot necessarily effect greater. If, therefore, the cause is more powerful than the thing caused, it is also productive of more effects. But whatever the thing caused can accomplish, the cause is much more capable of effecting. For every thing which is produced

from secondary causes, is much more produced from prior and more principal causes. Whatever, therefore, the thing caused is naturally adapted to produce, co-exists with the cause. But if the cause produces prior to the thing caused, it is evident that it energizes before the thing caused, according to its productive energy. Every cause therefore energizes prior to the thing caused; and in conjunction with, and posterior to its nature constitutes other effects.

C O R O L L A R Y .

From hence it is manifest, that whatever is caused by soul, is also caused by intellect, but whatever is caused by intellect, is not also caused by soul. For intellect energizes prior to soul; and whatever the soul confers on secondary natures, intellect also confers in a more ample manner. And when soul no longer energizes, intellect illuminates with its gifts, natures to which soul does not communicate its essence. For that which is inanimate, so far as it participates of form, participates of intellect, and the formation of intellect. Besides, this likewise follows that whatever is caused by intellect is also caused by the good, but not the contrary. For the privations of forms emanate from the good: since all things flow from this. But intellect since it is form, is not the fabricator of privation.

P R O P O S I T I O N L V I I I .

Every thing produced from many causes, is more compounded, than that which is produced from a few.

For if every cause confers something on that which proceeds from it, many causes will confer many gifts, but fewer causes will bestow fewer gifts. Of participants, therefore, some will consist from many, but others from a few of the things which each participates; the former indeed on account of their progression from many causes, but the latter on account of their progression from a few. But the former proceeding from many causes are more composite: and things proceeding from a few, are more simple than those which proceed from many causes. Hence every thing produced from many causes, is more compounded; but that which proceeds from a few is more simple. For the more compounded participates of that which the more simple participates, but the contrary to this, is not true.

P R O P O S I T I O N L I X .

Every thing essentially simple, is either more excellent, or worse than composite natures.

For if the *highest* of beings are produced from things fewer and more simple, but such as are in the *middle*, from a many, these will be composite. And with respect to the *extremes*, some are more simple, according to that which is more excellent, but others according to that which is worse. But that the highest beings are produced from fewer causes, is evident from their being superior, and originating prior to inferiors, and extending themselves over beings, beyond the progressions of subordinate natures, on account of their diminution of power. For on this account the last of beings ^[113], is most simple, as well as the first, because it proceeds from the first alone. But one kind of simplicity subsists according to a nature more excellent, but another kind, according to that which is more base than every compound; and there will be the same proportion in all things.

P R O P O S I T I O N L X .

Every thing which is the cause of a multitude of effects, is more excellent than that which is allotted a power productive of a few; when the few are parts of the many.

For if this is the cause of a few, but that of a many, and the few are parts of the many, that which forms the one, will also form the rest, if fabricative of a many. It is, therefore, more powerful and comprehensive of a greater multitude. For as production is to production, so is one producing cause to another, according to a mutual relation. But that which is capable of accomplishing more, possesses a greater, and more universal power. And this is nearer to the cause of all: but that which is nearer is a greater good: since the cause of all, is the good itself. Hence that which is the cause of many effects, is essentially more excellent than that which produces but a few.

P R O P O S I T I O N L X I .

Every power when impartible is greater, but when divided becomes less.

For if it is divided, it passes into multitude. And if this is the case, it becomes more distant from unity; and on this account is diminished in power; since it departs from unity by which it is contained, and acquires imperfection. Since the good of every thing subsists through the benefit of union.

PROPOSITION LXII.

Every multitude which is near to unity, is less in quantity than things farther distant, but is greater in power.

For that which is near is more similar to unity. But unity is constitutive of all things without multiplication. The cause, therefore, of many effects, is more similar to unity. Since the cause of all is the most uniform and impartible of all things; if the cause of all is *one*. As, therefore, that which is less multiplied is more allied to the one; so that which is productive of a multitude of effects, is more allied to the cause of all. But a nature of this kind is more powerful.

COROLLARY.

From hence it is evident, that there are more corporeal natures than souls; more souls than intellects; and more intellects than divine unities. And in all other natures there is the same proportion.

PROPOSITION LXIII.

Every imparticable produces two-fold orders of things participated; one in things which sometimes participate; but the other in such as participate always, and in a connate manner.

For that which is always participated, is more similar to an imparticable than that which is sometimes participated. Hence that which is always participable, will subsist prior to that which is sometimes participated. Because it is participated indeed, differing from that which is posterior to itself, but because it is always more allied, it is also more similar to an imparticable. Nor are there alone things, which are sometimes participated: for prior to these are the natures which are always participated; through which they are conjoined with impartibles according to a certain well ordered progression. Nor are there alone natures which are always participated: for these, since they possess an unextinguishable power (on account of their perpetuity), bear other natures in their essence, viz. the natures which are sometimes participated. And as far as to these diminution and subjection extends.

COROLLARY.

From hence it is manifest, that of the unions, which illustrate beings from the one, some are always participated, but others sometimes: and that intellectual participations are in the same manner two-fold; and likewise the animations of souls, and of other forms. For beauty, and similitude, and station, and identity, are imparticable, but they are participated, through things which always participate, and by things which sometimes participate in a secondary manner, according to the same order.

PROPOSITION LXIV.

Every principal unity produces a two-fold number; one indeed of self-perfect substances; but the other of illuminations, possessing their subsistence in others.

For if its progression takes place by subjection, and through things proper to fabricative causes; and if perfect natures proceed orderly from the perfect, and things imperfect through these as mediums: hence some will be self-perfect substances, but others imperfect; and these last will become the forms of participants. For since they are imperfect, they will be indigent of subjects to their existence. But the perfect natures will make themselves their own participants: for since they are perfect, they will replenish and establish themselves. But they will require nothing of inferior natures, to their proper subsistence. Self-perfect substances, therefore, on account of their distinction into multitude, are diminished from their principal unity; but on account of their self-perfect essence, they are in a certain respect assimilated to it. But imperfect substances because they reside in others, are remote from that which is self-subsistent; and because they are imperfect, they are distant from that which

perfects all things. But progressions are accomplished by things similar, as far as to things perfectly dissimilar. Hence every principal unity produces a two-fold number.

C O R O L L A R Y .

From hence it is evident, that with respect to unities, some are self-perfect proceeding from *the one*: but that others are illuminations of unities and intellect. And again, that some of these are self-perfect essences, but others nothing more than resemblances of souls which are animated. And hence, neither is every unity a god, but this is peculiar to a self-perfect unity alone. Nor is every intellectual property an intellect, but that which is essential alone. Nor is every illustration of soul, a soul: but there are likewise images of souls.

PROPOSITION LXV.

Every thing which subsists in any manner whatever, either subsists according to cause, in a primary manner (or possessing the form of a principle ἀρχοειδῶς) or according to hyparxis^[114], or according to participation, after the manner of an image (εἰκονικῶς.)

For either the thing produced is beheld in its producing cause, as in a pre-existent cause: (because every cause previously assumes in itself, the thing caused, being that primarily, which its effect is secondarily) or the producer, is beheld in the thing produced. For since that which is produced participates of its producing cause, it exhibits in itself, in a secondary manner, that which its producer is primarily. Or every thing is to be considered in its own order, and is not to be contemplated either in its cause, or in the thing caused. For the one so far as it exists, subsists after a more excellent mode; but the other, so far as it is, in a subordinate manner. But it is requisite that this last also, should be such as it is. And every thing subsists in its own order, according to hyparxis.

PROPOSITION LXVI.

All beings are to one another either wholes, or parts, and are either *the same*, or *different*.

For either one comprehends, but the remainder are comprehended; or they neither comprehend, nor are comprehended. And they either suffer that which is the same, as participating one; or they are distinguished from one another. But if they comprehend they are wholes: and if they are comprehended they are parts. But if many things participate one, they are the same according to one. But if they are many only; so far as many they are different from each other.

PROPOSITION LXVII.

Every totality is either prior to parts, or composed from parts, or contained in a part.

For we either contemplate the form of every thing in its cause, and affirm that the whole which subsists in its cause is prior to parts; or we contemplate the form of a thing in the parts which participate of that form. And this in a two-fold respect. For the form is either collectively in all the parts; and this is a whole composed from parts, any one of which when absent diminishes the whole itself. Or it is in each of the parts; so that every part according to participation becomes a whole, i.e. a partial whole. But the whole composed from parts subsists on account of essence. But that which is prior to parts according to cause: and that which subsists in a part, according to participation. For this is a whole according to ultimate subjection, so far as it imitates the whole consisting from parts; since it is not any part indifferently, but that which is capable of being assimilated to a whole, whose parts also are wholes.

PROPOSITION LXVIII.

Every whole contained in a part, is a part of that whole which is composed from parts.

For if it is a part, it is a part of some whole: and is either a part of that whole which abides in itself, according to which it is called a whole in a part. But on this hypothesis the whole would be a part of itself, and a part would be equal to the whole, and each would be the same. Or it will be a part of some other whole; and if of some other,

it is either only a part of that other, in such a manner as again to differ in no respect from the whole. Or it will be a part together with some other whole. For the parts of every whole, are more than one; and this will be a whole composed from many parts. And thus the whole contained in a part, is a part of that whole which is composed from parts.

PROPOSITION LXIX.

Every whole composed from parts, participates of that totality which is prior to parts.

For if it is composed from parts, it becomes passive to a whole. For the parts since, they are made one, suffer a whole, on account of their union: and this is a whole subsisting in parts which are not wholes. But that which is imparticipable has an existence prior to every thing participated. An imparticipable totality, therefore, exists prior to a participated totality. And hence there is a certain species of totality, prior to that whole, which is composed from parts. And this is not a passive whole, but is an essential totality; from which the totality resulting from parts proceeds. Since likewise that whole which is composed from parts, subsists in many places, and in various ways, in many other things composed from parts. But it is requisite that there should be an essential monad or unity of all totalities. For each of these wholes is not sincere, because indigent of the parts from which it is composed, and which are themselves different from wholes. Nor if this whole was generated in any thing particular, could it be the cause that all others are wholes. The cause, therefore, by which all things are wholes, is prior to parts. For if this also was composed from parts, it would be a *certain whole*, and not that which is *simply whole*. And this again would subsist from another whole: and this must either be the case in infinitum, or there must be a first whole; a whole not composed from parts, but that which is a perfect totality.

PROPOSITION LXX.

That which is more universal subsists in principal causes, and prior to particulars illuminates participants; and leaves that which participates as second in order from principal causes.

For a more universal cause begins its energy in secondary natures, prior to that which is posterior to a more universal cause: and it is present not only when that which is posterior is present, but even when the energy of that which is posterior is no more; and it energises in a more causal manner, and this not only in different subjects, but also in each of the things which sometimes participate. Thus for example it is requisite, that being should be first, afterwards animal, and then man. And the species man no longer exists, when deserted by the rational power: but animal, breathing and sentient, will still subsist. And again, though life is taken away, being remains: for when man ceases to live, being is present. And the same reasoning may be adopted in all things. But a cause which is more efficacious, and which is on this account more causal, energises first in a more causal nature: for it suffers the same from a cause more powerful, and prior to itself; and it co-energises with that which is secondary when in energy. For every thing which generates that which is secondary, congenerates also that which is more causal: and when that which is secondary deserts the more causal nature, that which generates the secondary nature is present. For the communication of a more powerful cause, when it is more efficacious, leaves that which participates it, the last of all. For through the communication of that which is second, it strengthens its own illumination.

PROPOSITION LXXI.

[364]

All things which abide in principal causes, and which possess a more universal and superior order in effects, according to the illuminations proceeding from them, become in a certain respect the subjects of the communications of particulars. And the illustrations emanating from superior, receive the progressions of secondary natures. And thus some participations antecede others, and representations, or resemblances (ἐμφάσεις) supernally coalesce one after another in the same subject: things more universal energizing first, but particulars posterior to the energy of universals, bestowing their communications on their participants.

For if things which partake more of cause, energize prior to things secondary, on account of their exuberance of power, and are present with, and illuminate things endued with a more imperfect aptitude: but if things more subordinate, and which are second in order, receive their communications from these; it is evident that the illustrations of superior natures pre-occupy that which participates of both, and establish the communications of subordinate natures. But these illustrations use the resemblances emanating from superior natures, as supports and foundations, and operate in a participant prepared by superior natures.

PROPOSITION LXXII.

All things which in their participants have the relation of a subject, proceed from more perfect and universal causes.

For the causes of many effects are more powerful and universal, and nearer to *the one*, than the causes of a few effects. But things producing the subjects of others, are the causes of many effects, because they likewise produce aptitudes, before forms are present. And hence these are more universal and perfect in the order of causes.

COROLLARY.

From hence it is evident why matter which derives its subsistence from *the one*, is essentially destitute of form. And why body is effectually destitute of soul, although it participates of being. For matter which is the subject of all things proceeds from the cause of all^[115]: but body which is the subject of animation, subsists from that which is more universal than soul, because it participates of being in a certain respect.

PROPOSITION LXXIII.

Every whole is at the same time a certain being, and participates of being: but every being is not a whole.

For either *being* and *whole* is the same; or the one is prior, and the other posterior. But a part also, so far as a part, is a certain being (for that which is a whole consists from partial beings), yet is not an essential whole. And hence *being* and *whole* is not the same: for on this hypothesis, a part would be a *non-entity*. But if the part is a *non-entity*, neither can the whole be *being*. For every whole is a whole of parts, either considered as existing prior to the parts^[116], or as inherent in the parts. But the part being a non-entity, it is impossible that the whole can exist. But if the whole is prior to being, every being will be immediately a whole; and so a part, will not be a part, which is impossible. For if a whole is a whole, and is a whole of parts; the part also existing as a part, will be a part of the whole. It remains therefore that every *whole* is *being*, but not every *being* a *whole*.

COROLLARY.

For hence it is evident that the first being is above totality, since being is present to a multitude of things: for it affords essence to parts, so far as parts. But totality is present to fewer natures. For the cause of a multitude of effects is more excellent: but that of a few is subordinate, as is demonstrated.

PROPOSITION LXXIV.

Every form is a certain whole.

For it is composed from a multitude of things, each of which replenishes the form. But every whole is not a form. For that which is particular, and an indivisible, is indeed a whole, so far as it is an indivisible; but is not a form: For every whole consists from parts. But form or species, is that which may be divided into many particular forms. Hence *whole*, and *form* differ from each other: and the former is inherent in more natures than the latter. That which is *whole*, therefore, is above the *forms* of beings.

COROLLARY.

From hence it is evident that *whole* possesses a middle order between being, and forms: and hence it follows^[161] that being is prior to forms, and that forms, are beings; and yet every being is not a form. From hence likewise in

effects, privations are after a certain manner beings, yet they are not forms^[117]. But on account of the unifying power of being, they likewise receive a certain debile representation of being.

PROPOSITION LXXV.

Every thing which is called a cause properly, is exempt from its effect.

For if it subsisted in its effect, it must either replenish its effect, or be indigent of it in a certain respect, in order to its being; and thus be more imperfect than its effect. But that which abides in its effect, is more an assistant-cause than cause itself: because it is either a part of that which is made, or an instrument of that which makes. For the part subsisting in that which is made, is more imperfect than the whole: and the instrument which supplies the measures of fabrication to the efficient, is not able to separate itself. Every thing, therefore, which is properly a cause, if it is more perfect, than that which proceeds from it, affords likewise a measure to generation, and is exempt from instruments, and elements, and from every thing which is simply called an assisting cause.

PROPOSITION LXXVI.

Every thing which is produced from an immoveable cause, possesses an immutable hyparxis: but every thing which emanates from a moveable cause, possesses a mutable hyparxis.

For if every thing which fabricates is entirely immovable, it produces that which is second from itself, ³⁶⁷not by motion, but by being. But if this be the case, that which emanates from it, concurs with its essence. And if this be the case, as long as it exists, it will produce. But it always is, and therefore will always produce that which is posterior to itself. Hence too that which is posterior always emanates from thence, and always is; conjoining its own *progressive ever*, with the *ever* according to energy, of an immoveable cause. If, therefore, the cause is moved, that also which is produced by it will be essentially mutable. For that which derives its essence through motion, when the motion is changed, changes its being. For if that which is produced from motion abides immutable, it will be more excellent than its producing cause: but this is impossible. It will not, therefore, be immutable; and will consequently be changed, and moved according to essence, imitating the motion by which it is produced.

PROPOSITION LXXVII.

Every being in *capacity*, emanates from that which is *energy*; and that which is in capacity proceeds into energy. But that which is in a certain respect in capacity, so far as it is in capacity, emanates from that which is in a certain respect in energy. And that which is all things in capacity proceeds from that which is all things in energy.

For that which is in capacity is not adapted to produce itself into energy because it is imperfect. For if that which is imperfect should become the cause of perfection to itself, and this in energy, the cause will be more imperfect than that which it produces. Hence that which is in capacity, so far as in capacity, will not be to itself, the cause of subsisting in energy. For on this hypothesis, so far as it is imperfect, it will be to itself the cause of the perfect: since every thing in capacity, so far as in capacity, is imperfect. But every thing which is in energy, so far as in energy, is perfect. If then that which is in capacity becomes in energy, it will inherit perfection from something else. And this again, will be either in capacity; (but then again, the imperfect will be generative of the perfect), or it will be in energy. And either something else, or this which is in capacity, will rise into energy. But if something else in energy operates, acting according to its propriety, it will produce into energy, that which is in capacity, in another. Nor will this again be in energy; unless it rises into energy from capacity. It remains, therefore, that from that which is in energy, that which is in capacity must be changed into energy.

PROPOSITION LXXVIII.

All power or capacity is either perfect, or imperfect.

For that which produces energy is a perfect power: for it makes other things perfect through its energies. But that which is perfective of others, is greater, because it is more self-perfect. But the power which is indigent of something pre-existing in energy, according to which it is something in capacity, is imperfect: for it is indigent of the perfect abiding in another, that it may become perfect through its participation. And hence such a power is essentially imperfect. Hence too the power which subsists in energy is perfect, because prolific of energy. But the power which subsists in capacity is imperfect; deriving its perfection from power in energy.

PROPOSITION LXXIX.

Every thing which is generated, is generated from a two-fold power.

For it is requisite that it should be adapted to generation, and that it should possess an imperfect power. It is likewise requisite that the agent being in energy, such as that which is generated is in capacity, should previously assume a perfect power. For every energy proceeds from inherent power. For if the agent does not possess power, how can it energise, and operate in another? But if that which is generated, does not possess power according to aptitude, how can it be fabricated? For that which produces, produces every thing, in that which possesses a passive power; but not in every thing; nor in that which is not naturally passive to the energies of the producing cause.

PROPOSITION LXXX.

Every body is naturally adapted to passivity: but every thing incorporeal is naturally adapted to fabricate. And the former is essentially inefficacious, but the latter is impassive. Yet that which is incorporeal becomes passive through its communion with body: just in the same manner, as bodies are enabled to fabricate, through the participation of incorporeals.

For body, as body is divisible alone, and through this becomes passive; being every way partible, and this every way in infinitum. But that which is incorporeal, because it is simple, is impassive. For neither can that which is impartible be divided, nor can that which is incomposite be altered. Hence nothing will be fabricative, or this must be affirmed of an incorporeal: since body so far as body, does not operate, because it is alone exposed to division and passivity; while on the contrary every agent possesses an active power. Hence it will not fabricate so far as body, but according to a power of operating, which it contains. But body is essentially inefficacious, and impotent: and hence when it fabricates, it fabricates by a participation of power. But incorporeals likewise participate of passions, when they abide in bodies; because in this case they are divided in conjunction with bodies, and enjoy their partible nature, though at the same time, they are impartible according to their proper essence.

PROPOSITION LXXXI.

Every thing which is participated in a separable manner, is present by a certain inseparable power, which it inserts in its participant.

For if it is separable from its participant, and does not abide in it, as that which possesses a subsistence in itself, a certain medium is requisite, which may connect the one with the other, and which is more similar to that which is participated, and to that which participates. For if this medium is separable, how can it be participated by the participant? Since neither the participant contains the medium nor any thing proceeding from this medium. A power, therefore, and illumination emanating from this medium, into its participant, conjoins both. One thing, therefore, is that through which participation subsists, but the second is that which is participated, and the third is the participant.

LXXXII.

Every thing incorporeal, because converted to itself, when it is participated by others, is participated in a separable manner.

For if in an inseparable manner, its energy will not be separable from its participant, any more than its essence. But if this be the case, it will not be converted to itself. For if it is converted it will be separable and

different from its participant. If, therefore, it is capable of being converted to itself, it is participated in a separable manner, when it is participated by others.

PROPOSITION LXXXIII.

Every thing endued with a self-gnostic power, is entirely converted to itself.

For that which is converted to itself, in energy, manifestly knows itself: for that which knows is one^[370] the same with that which is known; and the knowledge of itself reverts to itself, as that which is known. And as this knowledge belongs to that which knows, it is a certain energy: but it is an energy of itself to itself, because it possesses a power of knowing itself. But that this also subsists in essence if in energy has been demonstrated. For every thing which is converted to itself in energy, contains also an essence verging to itself.

PROPOSITION LXXXIV.

Every thing which always *is*, possesses an infinite power.

For if its essence is never-failing, its power also, according to which it is what it is, and is able to be, must be infinite. For if the power according to which it subsists was finite, it would some time or other fail. But if it should fail, the being also of that which possesses this power must fail; nor would it be any longer an eternal being. It is requisite, therefore, that the power belonging to, and containing that which always is, should be essentially infinite.

PROPOSITION LXXXV.

Every thing which is always *in generation* (ἀεὶ γινόμενον), possesses an infinite power of *being generated* (τοῦ γίνεσθαι).

For if it is always in the act of becoming to be, it contains a never-failing power of generation. For^[371] if its power was finite, it would cease in a infinite time. But if its power of being generated ceases, that also which is in generation will cease; viz. that which is in generation according to this power will cease; nor will it any longer be always in generation^[118]. But it is always in generation, according to the hypothesis: and consequently it possesses an infinite power of being generated.

PROPOSITION LXXXVI.

Every *true being* is infinite, not according to multitude, nor according to magnitude, but according to power alone.

For every infinite is either in multitude, or in magnitude, or in power. But true being is indeed infinite, as possessing an inextinguishable life, a never-failing essence, and an undiminished energy. Nor is it infinite on account of magnitude; since that which is true being, is without magnitude, and self-subsistent. For every self-subsistent being, is impartible, and simple. Nor is it infinite, on account of multitude, for it is most uniform, on account of its vicinity, and alliance to the one. But the one is infinite according to power: and hence through this, true being will be impartible and infinite. And indeed by how much the more it is one and impartible, by so much the more will it be infinite. For power when distributed into parts, becomes debile, and finite. And powers entirely partible, are entirely finite. Far such as are last, and most distant from the one, are after a certain manner finite: but first powers are on account of their impartibility infinite. For partition dissipates and dissolves the power of every thing. But impartibility, from its binding and collective nature, contains in itself, that which is never-failing and undiminished. But infinite according to magnitude and multitude, is entirely a privation, and desertion of impartibility. For that which is finite proximately approaches to that which is impartible; while that which is infinite is most distant from an impartible nature, because it on all sides departs from the one. Hence that which is infinite according to power, does not belong to the infinite, either of multitude, or magnitude: since infinite power is co-existent with impartibility. But infinite, either in multitude or magnitude is most distant from an impartible nature. If, therefore, true being was infinite, either in magnitude, or multitude, it would not be endued with infinite power. But it is endued with infinite power, and is, therefore, not infinite, either according to magnitude, or multitude.

PROPOSITION LXXXVII.

Every thing eternal, is *being*; but not every being is *eternal*.

For in generated natures the participation of being, is after a certain manner inherent, so far as they ³⁷² are not that, which is in no respect being. But if that which is in generation, is not that which is *in no respect being* (οὐδαμῶς ὄν,) it is being, *in a certain respect* (ἔστι πῶς ὄν.) But that which is eternal, or eternity itself, is *in no respect* inherent in generated natures; and is particularly separated from things which do not participate of eternity, according to the whole of time. But every thing eternal always is; for it participates of eternity itself, which confers perpetual being, on the natures by which it is participated. *Being*, therefore, is participated by more natures than *eternity*: and hence *being* is above *eternity*. For things which participate eternity, participate also of being; but not all that participate of being, participate also of eternity.

PROPOSITION LXXXVIII.

Every true being, is either prior to eternity, or abides in eternity, or participates of eternity.

But that it is above, or prior to eternity, is demonstrated in the preceding proposition. And it likewise abides in eternity: for eternity possesses perpetuity together with being. And this is also true of that which participates of eternity: for every thing eternal, is called eternal from its participation of perpetuity and being. For this according to participation possesses both perpetuity and being. But *eternity* possesses perpetuity the first of all; but *being*, through participation ^[119]. And being itself, is the first being.

PROPOSITION LXXXIX.

Every *primary being* (πρώτως ὄν) consists of *bound*, and *infinite*.

For if it is endued with infinite power, it is evident that it is infinite, and through this subsists from ³⁷³ *infinite*. But if it is impartible and uniform, through this it participates of *bound*. For that which participates of the one, is bounded. But that which is impartible, is at the same time endued with infinite power. Every true being, therefore, consists from *bound* and *infinite*.

PROPOSITION XC.

First bound, and *first infinity*, have a self-subsistence prior to all things which consist from *bound* and *infinite*.

For if beings which subsist from themselves, subsist prior to certain beings, because common to all, and primary causes, and this not to some in particular, but simply to all; it is requisite that there should be a first bound, and a first infinity prior to that which consists from both. For in that which is mixed, *bound* participates of *infinity*, and *infinity* of *bound*. But the first of each, is no other than that which it is. It is requisite, therefore, that that which is *primarily infinite* should not possess the form of *bound*, and that that which is *primarily bound*, should not possess the form of *infinite*. And hence these subsist primarily prior to that which is mixed.

PROPOSITION XCI.

Every power is either bounded, or infinite. But every terminated power subsists from infinite power: and infinite power from *first infinity*.

For powers which have a partial existence; or subsist sometimes, are bounded; because they have fallen from the infinity of perpetual being. But the powers of eternal beings, are infinite, because they never desert their own hyparxis.

PROPOSITION XCII.

Every multitude of infinite powers, depends on one first infinity, which is not as a participated power, and which does not subsist in things endued with power, but is

essential; not existing as the power of any participant, but as the cause of all beings.

For though the first being possesses power, yet it is not power itself: for it likewise possesses bound.^[341] But the first power is infinity: for infinite powers, are infinite, through the participation of infinity. Infinity itself, therefore, will be before all powers; through which being also possesses infinite power, and all things participate of infinity. For *that which is first*, is not infinity: for that is the measure of all things, because it is the good; and the one. Nor is *being* infinity: for this is infinite, but not infinity (or infinite itself.) Hence between that which is first, and being itself, infinity subsists, as the cause of all things endued with infinite power, and of all the infinity in beings.

PROPOSITION XCIII.

Every infinite subsisting in beings, is neither infinite with respect to superior natures; nor is it infinite to itself.

For that by which every thing is infinite, by this also it is without circumscription. But every thing in natures superior to beings, is bounded in itself, and in all things prior to itself. It remains, therefore, that the infinite belonging to inferior natures, belongs to them alone, above which it is expanded in such a manner, that it is incomprehensible by them all. For however they may extend themselves towards this infinite, yet it possesses something entirely exempt from their nature. And though all things enter into this infinite, yet it possesses something occult, and incomprehensible by secondary natures. And again, though it expands its powers, yet it contains something on account of its union, invincible, convolved, and surpassing their involutions. Likewise containing and bounding itself, it will not be infinite to itself; and much less will it be infinite with respect to superior natures, because it possesses a portion of the infinity which they contain. For the powers of more universal natures are more infinite^[120], because they are more universal, and are placed nearer to the first infinity.

PROPOSITION XCLV.

Every eternity is a certain infinity, but every infinity is not an eternity.

For many infinities possess *the infinite*, not on account of their perpetuity; as is evident in the^[375] infinity according to multitude, and according to magnitude, and in the infinity of matter; and whatever else may be infinite, either because it cannot be passed over, or on account of the indefinite nature of its essence. But that eternity is an infinity, is evident: for that which never fails is infinite: and this because it possesses a never-failing subsistence. Infinity, therefore, is prior to eternity. For that which constitutes a greater multitude, and is more universal, is a more causal nature. *First infinity*, therefore, is above eternity, and *infinity itself*, is prior to eternity.

PROPOSITION XCV.

Every power which possesses more of the nature of the one than of multitude, is more infinite than the power which is multiplied.

For if the first infinity is the nearest to the one, hence of the powers which are more allied to the one, that which is less distant from the one, is more infinite. For when multiplied it loses its uniform nature; in which when it abides, it will possess a superiority among other natures, being contained by impartibility. For among partible natures, collected powers are multiplied, but such as are divided, are debilitated.

PROPOSITION XCVI.

The power of every finite body, when infinite is incorporeal.

For if this power is corporeal, since in this case it would be an infinite body, infinite would reside in that which is finite. But if this power is a finite body, it will not be an infinite power, on account of body, but on account of something else. For if through body it is finite, but through power infinite; it will not be power, on account of body. Hence the infinite power, which resides in a finite body is incorporeal.

PROPOSITION XCVII.

Every principal cause in every series, communicates its property to all that series: and the series is that by remission, or subjection, which this cause is after a primary manner.

For if it is the leader of the whole series, and all kindred natures, are co-ordinated to this cause, it is manifest, that it confers on all one idea, through which they are allotted an order under the same series. For either all things participate of similitude with this principal cause, without a cause; or the sameness which is in all, proceeds from this principal cause. But that this should be the case, without a cause, is impossible: for that which is without a cause is fortuitous. But among things in which there is order, and a connection with each other, and which always abide in the same state, chance can never take place. From this principal cause, therefore, the whole series receives the property of its subsistence. And if from this cause, it is evident that it receives this property with remission, and a descent accommodated to secondary natures. For either this property subsists in a similar manner, in that which is principal, and in things secondary, and the former presides, but the latter are allotted a subsistence posterior to the principal; or it subsists in a dissimilar manner. And if this is the case, it is manifest that identity in multitude proceeds from one, and not the contrary; and that the property which primarily pre-exists in one, is secondarily in multitude, and is exempt from the series.

PROPOSITION XCVIII.

Every separate cause, is at once, every where, and no where.

For by the communication of its power, it is every where. For this is a cause, which replenishes things naturally adapted to participate of its nature, and is the leader of all secondary natures, and is present to all the prolific progressions of illustrations. But, on account of an essence unmingled with things in place, and through its excellent purity, it is no where. For if it is separate from effects, it is placed above all things. In like manner it resides in none of the natures subordinate to itself. For if it was alone every where, it would not indeed be hindered from being a cause, and it would be in all participants: but it would not be in a separate manner prior to all. But if it was no where, without being every where, it would not indeed be restrained from being prior to all things, and it would not be any one of subordinate natures, but it would not be in all things; as causes are naturally in things caused, through their abundant communications. On account of its being a cause, therefore, it is in all things which are able to participate its nature: and from its being separate in itself, it is prior to all the natures which it replenishes; and is at once every where and no where. And indeed it is not according to a part every where, and according to a part no where: for thus it would suffer a divulsion and separation from itself: since one part of itself would be every where in all things, and another part would be no where, and prior to all things. But it is total, every where and no where, after the same manner. For things which are able to participate of this cause, abide in the *whole*, and find the *whole* present with their nature; while this whole is exempt from its participants. For its participant does not establish this whole in itself, but participates of it as much as it is able to receive. Nor in communicating does it contract itself, through the participations of a multitude of things: for it is separate. Nor do the participants participate in a defective manner: for that which communicates is every where.

PROPOSITION XCIX.

Every imparticipable, so far as it is an imparticipable, does not subsist from another cause, but is the principle and cause of all participated natures. And in consequence of this every principle in every series is without generation.

For if it is imparticipable, it is allotted a principality in its own proper series, and does not proceed from others: for it would not be the first, if it received that property on account of which it is imparticipable, from any other. But if it is worse than others and proceeds from them, it is not allotted a progression, so far as it is imparticipable, but so far as it is a participant. For in this case it participates of the natures from which it proceeds, and the things which it participates, have not a primary subsistence. But that which is imparticipable has a primary subsistence. And hence so far as it is imparticipable it does not flow from a cause^[121]. For if it proceeded from a cause, it would be a participant, and not an imparticipable. But so far as it is an imparticipable, it is the cause of participants, and not that which participates of others.

PROPOSITION C.

Every series of wholes is extended to an imparticipable cause and principle. But all imparticipables depend on one first principle of all.

For if every series suffers a certain sameness, there is something in every ruling nature, which is the cause of identity. For as all beings proceed from *the one*, so likewise every series emanates from *one*. But all imparticipable unities are reduced to the one itself because all of them are analogous to the one. So far, therefore, as they suffer a certain sameness, through their analogy to the one, so far they are reduced to the one itself. And so far as they all proceed from *the one*, none of them is a principle, but they flow from *the one*, as from a principle. But so far as each of them is imparticipable, so far each of them is a principle. Because, therefore, they are principles of certain things, they depend on the principle of all: for that is the principle of all, of which all participate. But the first is alone participated by all things; while some things only participate of the rest. And hence *the one* is that which is simply the first. But others are firsts with relation to a certain order, but are not simply firsts.

PROPOSITION CI.

The leader of all things participating of intellect, is an imparticipable intellect: of all things participating life, an imparticipable life: and of all things participating being, an imparticipable being. But of these, being is prior to life, and life is prior to intellect.

For since in every order of beings, imparticipables are prior to participants, it is requisite that there should be an intellect prior to intellectuals, life prior to things vital, and being prior to beings. But since that which is the cause of more effects precedes that which is the cause of a fewer; hence among these being will be the first: for it is present to all things, to which life and intellect is present. For every thing vital and intellectual participates of being; but the contrary is not a necessary consequence: since all beings are not endued with life, and intellect. But the second in order is life: for all things to which intellect is present, participate also of life; but the contrary is not true. For many things are endued with life, but are destitute of cognition. But the third is intellect. For every thing which is endued in any respect with cognition, both lives, and possesses being. If, therefore, being is the cause of more effects; but life of fewer; and intellect of still fewer: hence *being* is the first in order, *life* the second, and *intellect* the third.

PROPOSITION CII.

All beings, in whatever manner they may possess being, consist from *bound* and *infinite*, through the first being. But all vital natures, are self-motive, through the first life. And all gnostic natures, participate of cognition, through the first intellect.

For if that which is in every series imparticipable, communicates its peculiar property, to all things under the same series; it is evident that being first communicates to all things *bound* together with *infinite*; since it is primarily mixed from these. And life imparts the motion resident in its nature: for life is the first progression and motion, from the stable subsistence of being. And lastly, intellect imparts cognition: for the summit of all cognition, is in intellect; and intellect is the first gnostic nature.

PROPOSITION CIII.

All things are in all, but subsist peculiarly in each.

For in being there is both life and intellect; and in life, being and intellection; and in intellect, being and life. But in intellect all things subsist intellectually, in life vitally, and in being, according to true beings. For since every thing subsists, either, according to cause, or according to essence, or according to participation: and since in the first the rest subsist according to cause; and in the second, the first subsists through participation, and the third through cause: and in the third, natures prior to its own, subsist through participation; hence in *being*, *life* and *intellect* preside. But since every thing receives its characteristic, according to hyparxis, and not according to cause (for cause pertains to other things, or to effects); nor yet according to participation (for it receives

externally that which it participates): hence in being there is *true*^[122] life and *true* intelligence, essential life and essential intellect. And in life, there is being according to participation; but intelligence according to cause. But both of these, are vitally inherent in life: for its hyparxis is according to life. And in intellect there is both life and essence, through participation: but both these subsist intellectually. For the being and life of intellect is knowledge.

PROPOSITION CIV.

Every thing which is primarily eternal, has both its essence, and energy eternal.

For if it is primarily allotted the perpetuity of eternity, it does not partly participate of this, and partly^[120] but it entirely participates of perpetuity. For either participating according to energy, it does not participate according to essence: but this is impossible, since in this case, energy would be more excellent than essence. Or participating according to essence, it does not participate according to energy. And thus that which is primarily eternal, will be the same with that, which primarily participates of time^[123]. And time will primarily measure the essence of some things, but eternity which is more excellent than all time, will be the measure of nothing; since that which is primarily eternal, will not be contained by eternity according to energy^[124]. Every thing, therefore, primarily eternal, has both its essence and energy eternal.

PROPOSITION CV.

Every thing immortal is eternal, but every thing eternal is not immortal.

For if that is immortal which always participates of life, and that which always participates of life, participates also of being, and that which is always vital is perpetual; hence every thing immortal is eternal. For that is *immortal* which does not receive death, and perpetually *lives*: but that is *eternal* which cannot receive non-being, and which always *is*. But if there are many beings more excellent and worse than life, but which are not susceptible of immortality, though they are perpetual beings; it follows that every thing eternal is not immortal. But that many perpetual beings; are not immortal is evident. For there are certain beings, destitute indeed of life; yet perpetual and incorruptible^[125]: since as being is to life, so is that which is eternal to that which is immortal. For that life which cannot be taken away, is *immortality itself*. And being which cannot be destroyed, is *eternity itself*. But being is more comprehensive than life: and hence that which is eternal is more comprehensive than that which is immortal^[126].

PROPOSITION CVI.

The medium of every thing entirely eternal, both according to essence, and according to energy, and of that which has its essence in time; is that which is in a certain respect eternal, but which in a certain respect is measured by time.

For that which has its essence comprehended by time is entirely temporal: for this in a most primary manner, is allotted a temporal energy. But that which according to all things is temporal, is perfectly dissimilar to that which is according to all things eternal. But all progressions subsist through similars. There is, therefore, some medium between these. Hence either that is the medium which is eternal in essence, but temporal in energy; or the contrary. But this is impossible: for energy on this latter hypothesis would be more excellent than essence. It remains, therefore, that the former hypothesis must be the medium.

PROPOSITION CVII.

Every thing which is in a certain respect eternal, but in a certain respect temporal, is at the same time *being* and *generation*.

For every thing eternal, is being, and that which is measured by time is generation. And hence, if the same thing participates both of time and eternity, yet not according to the same, or after the same manner; this same thing will be, both being and generation, yet will not be both according to one of these alone^[127].

C O R O L L A R Y .

From hence it is evident that generation, since it has a temporal energy, depends on that which partly participates of being, and partly of generation; and which at once participates of eternity and time. But this is related to that which is eternal according to all things. But that which is eternal according to all things is related to eternity itself: and eternity itself is related to *being*, which is prior to eternity.

PROPOSITION CVIII.

Every thing which is particular in each order, is capable of participating in a two-fold manner, that unity which is placed in a proximate superior disposition; either by its own proper totality, or through that which it contains, of a partial nature, and which is allied to something particular, according to an analogy to the whole series.

For if all things are converted through similitude, the particular nature which subsists in an inferior order, is dissimilar to that which in a superior order is monadic, and total; and is as that which is particular to that which is universal, and as different orders are to each other. But this particular nature is similar to a *whole* of the same series, on account of a communion of peculiarity; and to that superior proximately co-ordinated property, through an analogous subsistence. It is, therefore, evident that through these mediums a conversion from one to the other is effected, as through similars, to that which is similar^[128]. For *particular*, is similar to that which is *particular*, but that which is of the same series is peculiar. And the universal, or whole which is placed above the series, is dissimilar according to each of these.

PROPOSITION CIX.

Every particular intellect participates of that unity, which is above intellect, and is the first; both through that which is *universal*, and through a *particular* unity, co-ordinated to its nature. And every particular soul, participates of universal intellect; both through *universal* soul, and a *particular* intellect. And every particular nature of body, participates of universal soul, both through *universal* nature, and a *particular* soul.

For every thing particular (by the preceding proposition) participates of that unity placed in an order above it; either by its own proper universality; or by that particular nature which it contains, and which is co-ordinated to something particular.

PROPOSITION CX.

Of all things placed in order, according to every series, such as are first, and are conjoined with their unity, may participate of those which are proximately established in a superior order, through analogy. But such as are more imperfect, and remote from their proper principle, are not naturally adapted to participate of their superiors.

For because some things are allied to their superiors being allotted in their proper order, a more excellent and divine nature; but others are more distant, because they are allotted a secondary and ministerial, but not a primary and principal progression in every series: hence some things are naturally conjoined with those of a superior order; but others are not conjoined with a superior order. For all things are not of equal dignity, though they belong to the same distribution. For there is not one reason, of all things: but all things proceed from one, and return to one, from their own proper unity. And hence they are not allotted the same power. But some are able to receive continually the participations of their superiors. But others of a dissimilar nature, are deprived of such a power, through their far distant progressions from their principles.

PROPOSITION CXI.

Of every intellectual series, some are divine intellects receiving the participations of the gods; but others are intellects alone. And of every animastic series (i.e. a series composed of souls) some are intellectual souls, depending on their proper intellects; but others are souls alone. And of every corporeal nature, some have supernally-presiding souls, but others are natures alone, destitute of the presence of souls.

For in every series the whole genus is not naturally adapted to depend on that which is prior to itself; ¹⁸⁴but this belongs to that more perfect nature which the genus contains, and which is sufficient to coalesce with superior natures. Hence every intellect is not connected with a god; but this belongs to the highest and most uniform intellects, (i.e. to intellects which participate most of unity). For these are allied to the divine unities. Nor do all souls participate of participable intellect; but this belongs to such as are most intellectual. Nor do all corporeal natures enjoy the presence of soul, and of that soul which is participated; but such only as are more perfect and rational. And the mode of demonstration is the same in all.

PROPOSITION CXII.

First natures in every order, possess the form of things prior to themselves.

For the highest genera in every order, are conjoined with their superiors by similitude, and through a continuation of the progression of universals. Hence such as superior natures are primarily, such is the form which things first in every order are allotted, and which is allied to the nature of superiors. And through the peculiarity of their subsistence, they appear such as natures prior to themselves.

PROPOSITION CXIII.

Every divine number is *unical* (ἐνιαῖος), i.e. possesses the form of unity.

For if a divine number has the one itself, as its preceding cause, in the same manner as an intellectual ¹⁸⁵number has intellect, and an animastic number (ψυχικὸς, or number possessing the form of soul) soul as its preceding cause, and if multitude is every where analogous to its cause; it is evident that a divine number also, is uniform. Since *the one itself*, is *the deity*: and this because *the good itself* is the same with *the one*. For the *good itself*, and *the deity* are the same: since that above which there is nothing, and which all things desire, is *the deity*. And that from which all things proceed, and to which all things tend, is *the good*. If, therefore, there is a multitude of gods, it is an uniform multitude. But that there is a multitude is evident: since every principal cause is the leader of a proper multitude ^[129]; and to this multitude it is similar and allied.

PROPOSITION CXIV.

Every god is a self-perfect unity: and every self-perfect unity is a god.

For if there is a two-fold number of unities, as we have previously demonstrated, and some of these are self-perfect, but others illuminations emanating from the self-perfect unities; and if a divine number is allied, and of a similar nature to the good; hence the gods are self-perfect unities. And on the contrary every self-perfect unity is a god. For as unity is most excellently allied to the one itself, and that which is self-perfect to the good, and through both the one, and the good, participates of a divine property; so likewise that which is self-perfect is a god.

C O R O L L A R Y .

But if a god was a unity, yet not a self-perfect unity; or a self-perfect hypostasis, yet not a unity, he would be placed in another order, on account of a mutation of his property.

PROPOSITION CXV.

Every god is super-essential, supervital, and super-intellectual.

For if every god is a self-perfect unity, but each of these (viz. essence, life, and intellect) is not a unity ¹⁸⁶; but united; it is evident that every god, is above essence, life, and intellect. For if these differ from each other, but all

are in all, every one of these being *all things* will not be *one* alone. Besides, if the first deity is super-essential, but every god, so far as a god is of the first series^[130]; hence every god will be super-essential. But that the first deity is super-essential is evident: for *essence* is not the same with *unity*; nor is *to be*, and *to be united* one and the same. But if *essence*, is not the same with *unity*, that which is first will either be both of these, and so will not be *one* alone, but something besides *one*, and will participate of *the one*, without being *the one itself*; or it will be either of these. But if indeed it is essence, it will be indigent of the one. But it is impossible that *the good*, and *the first* should be indigent. It will, therefore, be *the one* alone; and will consequently be super-essential. But if every thing subsisting in a primary manner, confers the property of its primary subsistence on the whole series; hence every divine number is super-essential. For every principal cause produces similars prior to dissimilars. If, therefore, the first god is super-essential, all the gods, will be super-essential: for by this means they will be perfectly similar. But if they were essences they would be produced from the first essence, as the unities of essences.

PROPOSITION CXVI.

Every deity, except *the one*, is participable.

For that *the one* is imparticipable, is evident; since if he participated any thing, and thus became dependent on some other nature, he would no longer be the cause of all things; both of such as are prior to beings, and of beings themselves. But that other unities are participants we shall now demonstrate as follows: For if there is another imparticipable unity after the first, in what does it differ from *the one*? For it either subsists in the same manner as that: but how in this case, is the one second, and the other first? Or it does not subsist in the same manner. And so *that* will be the one itself, but *this* will be both *one*, and *non-one*. But this non-one, if it is no hypostasis (or subsistence) will be one alone. But if it is some other hypostasis besides the one, *the one* will be participated by non-one: and that will be a self-perfect one, by which it is conjoined. Hence this again, will be the deity. But that which subsists as non-one, will subsist in the participation of the one. Every unity, therefore, which subsists after *the one* is participable, and every god is participable.

PROPOSITION CXVII.

Every god is the measure of beings.

For if every god possesses the form of one (ένιαόος), he defines and measures all the multitudes of beings. For since all multitudes, are naturally indefinite, they are bounded by *the one*. But that which is one, measuring and bounding whatever it supervenes, is willing to lead into bound, by its terminating power, whatever is indefinite. For that which is one becomes uniform through participation: but that which is indefinite recedes from the one, through its interminable and infinite nature. And by how much the less it is uniform, by so much the more is it indefinite and immense. And hence every multitude of beings, is measured by the divine unities.

PROPOSITION CXVIII.

Every thing which is in the gods, according to their idioms (or properties), pre-exists in their natures. And the property of the gods, is uniform and super-essential. And hence all things are contained in the gods, uniformly, and super-essentially.

For if every thing subsists in a three-fold manner, either through cause, or through hyparxis, or ^[131]through participation, but the first of all numbers is the divine number; hence nothing will be inherent in the gods according to participation. But all things will reside in them, either through hyparxis, or through cause. But likewise, whatever the gods, as the authors of all things previously receive, they previously receive in a manner convenient, and apposite to their union. For every thing which presides over secondary natures according to cause, naturally contains the cause of inferiors. All things, therefore, are in the gods uniformly, and super-essentially.

PROPOSITION CXIX.

Every god subsists according to a super-essential goodness, and is good neither through habit, nor through essence.

For both habit and essence are allotted an order secondary and remote from the gods: but these have a super-essential subsistence. For if the first one, is also the good, and so far as the one is the good itself, and so far as the good the one itself; hence every series of gods, is both uniform, and beneficent (ἀγαθοειδής), on account of one property alone, and not through more than one. But every god, so far as a unity is also a goodness, and so far as a goodness is also a unity; and on account of progression from the first is also beneficent and uniform. For the first cause of all, is both the one itself, and the good; and consequently all the gods are unities and goodnesses. As therefore *the one* of the gods is super-essential, so likewise *the good*, which they contain, is super-essential, and is nothing else than *one*. For every god is not first of all something different from good, and afterwards *good*; but is good alone. Nor is first of all something besides *one*, and afterwards *one*; but is *one* alone.

C O N C E R N I N G P R O V I D E N C E .

P R O P O S I T I O N C X X .

Every god contains in his hyparxis a providence of the universe; and primary providence resides in the gods.

For all things posterior to the gods, provide through the communion of the gods: but providence is connate with the gods. For if to communicate good to things provided for, is the peculiar employment of a providential property, but all the gods are goodnesses; hence they will either communicate themselves to nothing, and so there will be no good in secondary natures (for whence can that which subsists by participation emanate, but from natures which are primarily endued with properties); or if they communicate, they will communicate good, and through this communication provide for the universe. Providence, therefore, primarily subsists in the gods. For where can an energy prior to intellect abide, but in super-essential natures? And hence providence as the name indicates, is an energy prior to intellect. The gods, therefore, on account of their being, and because they are goodnesses, provide for all things^[131]; and fill all things with that goodness which is prior to intellect.

P R O P O S I T I O N C X X I .

Every thing divine has for its hyparxis goodness itself, and possesses an uniform power, and a knowledge occult, and incomprehensible by all secondary natures.

For if it provides for the universe, it contains a power comprehensive of the things for which it provides^[130], and by this invincible and indescrivable power, it fills all things with itself; and subjects every thing to its own nature. For every principal and ruling cause, rules through its abundance of power, and contains according to nature. There is, therefore, a primary power in the gods, which does not govern some things, and not others, but it equally assumes in itself in a primary manner, the powers of all beings: and this is neither an essential power, nor much more unessential, but it is connate to the hyparxis of the gods, and is super-essential. But likewise the boundaries of all cognitions, pre-exist uniformly in the gods. For all other cognitions subsist, on account of divine cognition, which is abstracted from the universality of things. And this cognition is neither intellectual, nor much less does it belong to the cognitions posterior to intellect; but according to its divine property, it is constituted above intellect. If then this knowledge is divine, it is an occult and uniform cognition. But if the power is divine, it is uncircumscribed by all things, and, in a similar manner, comprehensive of all things. But if the goodness is divine, it gives bound to the hyparxis of the gods. For if all things are contained in the gods, and among these knowledge, power, and goodness; and if their hyparxis is characterized with that which is best, the subsistence also of the gods will take place according to the best: and this is no other than goodness.

P R O P O S I T I O N C X X I I .

Every thing divine both provides for secondary natures, and is separated from the things for which it provides; providence neither remitting its unmixed and uniform excellence, nor a separate union obscuring providence.

For the gods abiding in their uniform nature, and hyparxis, fill all things with their power. And every thing which is able to participate, enjoys the goods, which it is capable of receiving according to the measure of its proper subsistence; the gods in the mean time, through being itself, or rather through a nature prior to being,

pouring their illuminations on every thing which exists. For since they are no other than goodnesses, they abundantly confer good upon all things, through being itself; not making a distribution according to a reasoning energy, but because these receive according to their dignity, and those confer according to their hyparxis. Hence, in their providential operations, they receive no impediment from the natures for which they provide: for they benefit all things through their very being itself. But every thing which operates essentially, operates without habitude or respect: for respect, is an addition to being itself: and is on this account contrary to nature. Nor again, because they are separate, do they take away their providential care; for thus they would remove (which it is unlawful to say) their peculiar hyparxis whose property is goodness. For the communication of good extends to every thing capable of its participation: and that which is greatest, is not that which is endued with a form of good, but that which is beneficent. This beneficent nature, therefore, either no being will possess, or the gods will possess it prior to beings. For to goods subsisting through communication, it is impossible that a greater good should be present, but a less good only, to such goods as are first.

PROPOSITION CXXIII.

Every thing divine, on account of its super-essential union, is ineffable and unknown to all secondary natures; but it is comprehensible, and knowable by its participants. And hence *that which is first*, is alone entirely unknown, because it is imparticipable.

For all knowledge subsisting through reason, belongs to *beings*, and in *beings* possesses the apprehension of its truth: for it is conversant with *conceptions*, and subsists in intellections. But the gods are above all beings. Hence that which is divine, is neither to be apprehended by opinion, nor by a rational energy, nor by intellection. For every being, is either sensible, and on that account the object of opinion, or true being, and on that account intelligible. Or it subsists between these, and is at the same time *being*, and *generable*, and is on this account the subject of a rational energy. If, therefore, the gods are super-essential^[132], and prior to beings, there can neither be any opinion of their natures, nor science, nor cogitation, nor intellection. But they are known by dependant natures in a manner correspondent to their properties: and this by a necessary consequence. For the diversities of participants are divided together with the properties of the things participated. Nor does every thing participate every thing: for neither is there a composition of things perfectly dissimilar, nor does any thing participate fortuitously of another: but a kindred nature is conjoined with every thing kindred, and derives its progression from that to which it is allied.

PROPOSITION CXXIV.

Every god knows partible natures, in an impartible manner, things subsisting in time without time, things not necessary, necessarily, things mutable, immutably; and universally, all things, in a manner more excellent than the order of the things known.

For if every thing which is present with the gods is present, according to their *characteristic*; it is evident^[133] that the knowledge of the gods will not subsist according to the nature of things inferior, but according to the singular excellence which the gods contain. Their knowledge, therefore, of multiplied, and passive natures, will be uniform and without passion. Likewise if that which is the object of cognition, is partible, divine knowledge will be impartible. If the subjects are mutable, the gnostic energy of the gods will be immutable: if contingent, divine knowledge will be necessary; and if indefinite, definite. For that which is divine does not receive knowledge into itself from subordinate natures, that so cognition may correspond to the object of knowledge; but inferiors receive their indefinite subsistence, about the terminated nature of the gods, are changed about their immutability, receive with passivity that which is impassive, and temporally, that which subsists without time. For it is possible that subordinate, may be surpassed by more excellent natures: but it is not lawful for the gods to receive in themselves any thing from natures inferior to their own.

PROPOSITION CXXV.

Every god proceeds through all secondary natures, in the order from which he begins to indicate himself. Always indeed multiplying and dividing his communications, yet preserving the characteristic of his own proper subsistence.

For since progressions are produced through remission, things first, every where multiply into the decrements of secondary natures. But proceeding according to a similitude to their producing causes, they receive the same ordination; so that the whole, is in a certain respect the same and different, and that which proceeds, with that which abides. For on account of its remission, it appears different, but on account of its coherence with the whole, it does not depart from identity. But such as *that* is among first natures, such is the subsistence of *this* among things secondary, and such is its preservation of the indissoluble communion of the series. Every god, therefore, appears in a manner adapted to the orders in which he exhibits his presence: but he proceeds from thence even to the last of things, through the generative power of primary natures. But he always multiplies the progression from one into multitude: but preserves identity, in the progression, on account of the similitude of the progressions to the governing and first operative cause of every series.

PROPOSITION CXXVI.

Of every deity, he is the more universal, who is nearer to the one: but he is more particular, who is more distant.

For he who is the cause of more effects, is nearer to the cause of all, but he who produces fewer effects, is more distant. And he who is the author of many is more universal, but he who produces fewer effects, is more particular. And each of these is a unity. But the one is greater, and the other less according to power. And the more particular goods are generated from such as are more universal, without the latter receiving any division, (for they are unities) of alteration (for they are immoveable) or being multiplied according to habitude (for they are unmixed). But they generate from themselves through an abundance of power, secondary progressions, diminished from such as are first.

PROPOSITION CXXVII.

Every thing divine is primarily, and especially simple, and through this is most sufficient.

For that it is simple, is evident from its unity: for the whole is eminently uniform. But a nature of this kind, is most eminently simple. But that it is likewise most sufficient, may be learnt by any one who considers, that a composite, is indigent; though not of things external to its nature, yet of those from which it is composed. But that which is most simple, and uniform, and one, is the same with *the good*, in which good establishing itself, it becomes most sufficient. But every thing divine, is of this kind. And hence it is neither indigent of externals, because it is goodness itself, nor of things requisite to composition, because it is uniform.

PROPOSITION CXXVIII.

Every god, who is participated by natures nearer to his own, is *immediately* participated: but when he is participated by far distant natures, this is effected through *mediums* more or less numerous.

For the former, since they are by their alliance uniform, are on this account enabled to participate the divine unities. But such as through their diminution, and extension into multitude become far distant, are indigent of other things more united, that they may participate such as are no longer united, but are essential unities. For multitude united, subsists between essential unity, and divided multitude. And thus united multitude is able to coalesce with unity, through *union*; but is at the same time allied to divided multitude, through *the manifest appearance of multitude*.

PROPOSITION CXXIX.

Every divine body is divine, through a divine soul. But every soul, is divine, through a divine intellect. And every intellect is divine through the participation of a divine

unity. And *unity* indeed, is a god from itself (αὐτόθεν θεός) but intellect, is most divine: and soul is divine, but body deiform, or endued with a divine form.

For if every number of gods is above intellect, but participations are effected through kindred and similar¹³⁵ natures, an impartible essence will first of all participate the super-essential unities. But in the second place things conjoined with generation. And in the third place, generation itself. And each particular will participate through its proximate superior; the peculiarity of the gods proceeding even to the extremities of things in participants, through mediums allied to their natures. For unity confers on the first intellect, its own illustrious power among divine concerns, and causes this intellect to be like itself, according to an uniform multitude. But through intellect it is present also to soul, adapting and inflaming its conjunction with intellect, when this intellect is participable. And by the resounding echo^[133] as it were of soul, it imparts its idiom or peculiarity to body, if it is a body participating in any respect of soul. And thus body becomes not only animated, and intellectual, but also divine. For it receives life and motion from soul, but indissoluble permanency from intellect, and divine union from participated unity. For each of these communicates its subsistence to subsequent natures.

PROPOSITION CXXX.

In every divine order, things first, are more exempt from the natures proximately placed under them, than these last are from thing subsequent: and secondary natures are more dependant on their proximate superiors, than following natures are dependant on these.

For by how much the more uniform, and universal any thing is, by so much the more it is allotted an excellence greater than subsequent natures: and by how much the more it is diminished according to power, by so much the more is its alliance encreased with things posterior to its nature. And sublimer natures are indeed more united with their more principal causes: but inferiors are less united. For it is the property of a greater power to be more exempt from its inferiors, and to be more united with more excellent natures. And on the contrary to recede more, and to be passive together with these, implies a diminution of power. And this indeed happens to secondary natures in every order, but not to such as are first.

PROPOSITION CXXXI.

Every god begins his own proper energy from himself.

For he first exhibits in himself the peculiarity of his presence in secondary natures, because he likewise communicates himself to others, according to his own exuberant plenitude. For neither is defect, nor plenitude alone, peculiar to the gods: since every thing deficient is imperfect; and it is impossible that the imperfect, can cause any thing to be perfect. But that which is full, is alone sufficient, and is not yet prepared for communication. It is requisite, therefore, that the nature which fills, and extends its beneficence to others, should be above measure full. Hence, if that which is divine, fills all things through itself, with the goods which it contains in itself, every thing divine is beyond measure full. And if this be the case, it will primarily possess in itself, the property which it confers on others. For thus it will extend to others the communications of overflowing goodness.

PROPOSITION CXXXII.

All the orders of the gods, are bound in union, by a medium.

For all the progressions of beings, are effected through similars; and much more is it necessary that the orders of the gods, should possess an indissoluble continuity, because they subsist uniformly, and are terminated according to one principal cause of their subsistence. Their remissions, therefore, take place in an united manner, and through that similitude alone which is found among beings, of things secondary to such as are first: and this because the subsistence of the gods, much more consists in union, than the subsistence of beings. All the divine genera, therefore, are bound together by proper mediums; so that first natures do not immediately proceed into progressions entirely different, but through genera common to each, and of which they are the immediate causes. For these genera combine the extremes into one union, being subjected to some, through an alliance of nature, but proximately separated from others: and they preserve the well-ordered progeny of divine causes.

PROPOSITION CXXXIII.

Every god is a beneficent unity, or a goodness unific (ἐνοποιός); and each possesses this hyparxis, so far as a god. But the first god is simply *good*, and simply *one*. And every god posterior to the first, is a certain goodness, and a certain unity.

For a divine property or idiom distinguishes the unities and goodnesses of the gods: so that every god confers goodness on all things, according to a certain characteristic of goodness; such as that of *perfecting*, or *containing*, or *defending*. For each of these is a certain good, but not every good. But that which is first primarily establishes a uniform cause. And this is no other than the good, constitutive as it were of all goodness. For all the hyparxes of the gods are not together equal to *the one*; so great is the super-eminence of *the first*, with respect to the multitude of the gods.

PROPOSITION CXXXIV.

Every divine intellect, *understands* as *intellect*, but *provides* as a *god*.

For to possess a knowledge of beings, and a perfection in intellectual conceptions, is the property of ~~intellect~~^[307]. But it is the province of a god to exercise a providential care, and to fill all things with good. But this communication, and replenishing, subsists through a union of the things replenished, with natures prior to their own. And intellect imitating this, becomes in its intellections the same with intelligibles. So far, therefore, as a divine intellect provides, it is a god; because providence is an energy prior to intellect. Hence, as a god, it communicates itself to all things; but as intellect, it is not present to all things. For a divine unity, extends beyond the progressions of an intellectual property. And this will be evident by considering, that natures void of intelligence, desire to provide, and to participate something of good: and this because all things do not desire intellect, even among such as are capable of its participation; but all things desire good, and hasten to acquire its possession.

PROPOSITION CXXXV.

Every divine unity is immediately participated by some being; and every thing which is deified, is extended to one divine unity; and the number of the participating genera of beings is the same as that of the participated unities.

For neither two, or more unities, are participated by one being^[134]. For how is it possible, that when the properties, which the unities contain, are changed, that which is connate to each, can remain without alteration; since conjunction subsists through similitude? Nor is one unity participated in a divisible manner by many beings: for many beings are unconjoined with unity; both considered as beings, with respect to that which is prior to beings, and as multitude to unity. But it is requisite that the participant, should be partly similar to that which it participates, and partly different and dissimilar. Since, therefore, that which participates, is something belonging to beings, but unity is super-essential, and the two are on this account dissimilar; it is requisite that *that* which participates should be one, that by this means it may become similar to the participated one; though the latter is one, because it is a unity, but the former is one, because it is passive to the communications of one, and is united through its participation.

PROPOSITION CXXXVI.

Every god having a more universal subsistence, and being placed nearer to *the first*, is participated by a more universal genus of beings. But every god who is more particular and remote, is participated by a more particular genus of beings. And as being is to being, so is unity to divine unity.

For if the number of unities, is the same with that of beings, and on the other hand one unity is participated by one being; it is evident that the order of beings proceeds according to the order of unities, assimilated to an order prior to that of beings. And more universal beings coalesce with more universal unities, but more particular beings, with more particular unities. For if this be not the case, dissimilars will again be joined with dissimilars, and distribution will not subsist according to dignity of nature: but both these cases are impossible; since the one

itself, and a proper measure, through the divine unities illuminates and supervenes all other natures. Much more, therefore, will there be an order of participation in the divine unities; similars depending on similars according to the power which they contain.

PROPOSITION CXXXVII.

Every unity, together with *the one* constitutes being participating of its nature.

For *the one*, as it is hypostatic, (ὕποστατικός) or constitutive of all things, so likewise it is the cause of participated unities, and of beings depending on unities. But the unity belonging to every being, produces the property, which shines forth to view in that particular being. And *the one*, indeed, is the cause of *simple being*; but *unity* is the cause of alliance, because it is connate to *the one*. Hence unity, is that which of itself defines the being, which is its participant, and *essentially* exhibits in it a *super-essential* characteristic. For universally, from that which is primary, that which is secondary obtains its subsistence. If, therefore, there is any super-essential property of deity, it must belong to being, which participates it essentially.

PROPOSITION CXXXVIII.

Of all things which participate of a divine property, and which are deified, the first and highest is *being itself*.

For if being is above intellect and life, as we have demonstrated, and is the most abundant cause after the one; hence *being* will be the highest after *the one*. For it is more uniform than intellect and life, and is on this account more venerable. But there is no other prior to this, except *the one*: for what besides *the one*, can be prior to uniform multitude? But *being itself* is uniform multitude; because it consists from *bound* and *infinite*. And universally, super-essential being^[135] is prior to essence. For in the illuminations which are imparted to secondary natures, *the one* alone extends beyond *being*. But *being* subsists immediately after *the one*. For that which is *being* in capacity, and is not as yet *being* in energy, is nevertheless according to its nature one. And the being, which subsists after *the one*, is being in energy. Among the principles of being, therefore, *non-being*^[136] subsists immediately above being, as something more excellent, and no other than the *one itself*.

PROPOSITION CXXXIX.

All the participants of the divine unities originate from *being*, and end in a *corporeal nature*.

For being is the first of participants, but body the last: for we say that there are divine bodies. For the highest of all genera are attributed to the gods, whether they are bodies, souls, or intellects; as in every order, things analogous to the gods, contain and preserve secondary natures, and every number is a whole, containing all things in itself according to that whole which is contained in a part, and possessing before all things a divine characteristic. The divine genus, therefore, subsists both *corporeally*, and *animastically* (or according to the nature of soul ψυχικῶς) and *intellectually*: and it is evident that all these are divine through participation. For that which is primarily divine subsists in the unities. The participants, therefore, of the divine unities, originate from being, but end in a corporeal nature.

PROPOSITION CXL.

All the powers of divine natures, having a supernal origin, and proceeding through proper mediums, extend to the extremity of things, and to places situated about the earth.

For nothing intercepts these powers, and restrains their universal presence; because they are in no respect indigent of places and intervals, on account of their invincible excellence in all things, and a presence every where pure and unmixed. Nor is that which is adapted to the participation of these powers, prohibited from participation: but as soon as any thing is prepared for their communications, they are immediately present, neither then approaching, nor being prior to this absent, but always possessing themselves in the same uniform manner. If, therefore, any terrene nature is adapted to the participation of these divine powers, they are present to this; and fill all things with themselves. And indeed they are more present to superior natures, but they are present to such

as are middle according to the order of these, and to last natures, in an ultimate respect. They supernally, therefore, extend themselves to the extremities of things: and on this account last natures contain the images of such as are first; and *all things sympathise with all*^[137]. For secondary pre-exist in first natures; and first natures manifestly appear in such as are second. For every thing subsists in a three-fold manner; either through *cause*, or through *hyparxis*, or through *participation*.

PROPOSITION CXLI.

Every providence of the gods, is partly exempt from the natures for which it provides, and is partly co-ordinated with them.

For one kind of providence is entirely extended above the things which are illuminated, according to its subsistence, and the characteristic of its order. But another kind provides for subjects of the same co-ordination with itself; these subjects themselves imitating the providential energy of the gods, who are separated from the concerns for which they provide; and desiring to fill secondary natures with the goods, they are capable of receiving.

PROPOSITION CXLII.

The gods are present to all things after the same manner, but all things are not after the same manner present to the gods. For every thing participates of their presence according to its order and capacity. And this is accomplished by some things uniformly, and by others variously; by some things eternally, and by others according to time; and by some things incorporeally, and by others in a corporeal manner.

For it is necessary that the different participation of these, should either proceed from the participant,^[402] from the thing participated. But every thing divine always possesses the same order: and with respect to all things, is without restraint, and without mixture. It remains, therefore, that mutation must subsist through the participant; and that in these that which is not perpetually the same must abide; and that these are differently present to the gods. Hence the gods are present to all things, in the same uniform manner, though all things are not equally present to them, But particulars are present according to their ability; and they enjoy the divinities, agreeable to the manner in which they are present to their illuminations. For the participation of these is according to the measure of their presence.

PROPOSITION CXLIII.

All inferior natures fail before the presence of the gods, though a participant among these may be adapted to participation. Indeed every thing foreign departs from divine light, but all things are at once illuminated by the gods.

For divine natures always possess a more comprehensive capacity, and are more powerful than their progressions. But the inaptitude of the participants, is the cause of the privation of divine light: for it obscures divine light by its debility^[138]. But this obscured light, appears to receive another domination, not according to its own power, but according to the impotency of the participant, which seems to fail and die away, before the illumination of a divine form.

PROPOSITION CXLIV.

All beings, and all the distributions of beings, extend as far in their progressions as the orders of the gods.

For the gods produce beings together with themselves, nor is any thing able to subsist, and to^[403] receive measure, and order beyond the influence of the gods. For all things are perfected, disposed, and measured through the power of the gods. Hence the gods have a subsistence prior to the last genera of beings; who also dispose these, and impart to them life, formation, and perfection; who convert them to *the good*, and who are in like

manner prior to middle, and primary natures. And all things are bound, and stably rooted in the gods, and through this derive the continuance, and preservation of their being. But when any thing apostatizes, or recedes from the gods, and becomes on this account solitary and destitute, it entirely departs into non-entity, and perishes: because perfectly deprived of those natures, by which it was contained.

PROPOSITION CXLV.

The characteristic of every divine order, pervades through all secondary natures, and imparts itself to all the subordinate genera of beings.

For if the distributions of beings, extend as far as the orders of the gods, there must be in every genus of beings, a supernally-illuminated property of the divine powers. For every thing receives from its proximate cause, that characteristic, or property, by which it is allotted its peculiar subsistence. I say, for example, if any deity possesses a cathartic, or purgative power, there will also be a purgation in souls and in animals, in plants and in stones. And in the same manner with respect to a defensive, converting, perfective, and vivifying power. And a stone indeed participates of a purgative virtue, but in a corporeal manner only. But a plant participates it more clearly according to life. An animal possesses this form, according to the motion of appetite: but a rational soul, in a rational manner; and intellect, intellectually. But the gods possess this super-essentially, and uniformly. And the whole series is endued with this power, from one divine cause: and there is the same mode of reasoning in the rest. For all things depend on the gods. And different natures are illuminated by different gods; the divine series, descending even to the extremity of things. And some things are connected with the gods immediately, but others through more or fewer mediums; *while all things in the mean time are full of gods*. And whatever any being naturally possesses it possesses from the gods.

PROPOSITION CXLVI.

The extremities of all the divine progressions, are assimilated to their principles; preserving a circle without beginning and end, through a conversion to their principles.

For if every progression returns to the principle from which it proceeds, much more must total orders, proceeding from their summit, be converted to it again. But the conversion of the extreme to its principle, forms one whole, finite, and verging to itself; and exhibiting through its inclination uniformity in multitude.

PROPOSITION CXLVII.

The summits of all the divine orders, are assimilated to the extremes of their superiors.

For if it is requisite that there should be a coherence, and continuity in a divine progression, and that every order should be connected by proper mediums; it is necessary that the summits of secondary orders, should be conjoined with the extremes of such as are first. But conjunction subsists through similitude: and hence there will be a similitude of the principles of an inferior order, to the extremes of one superior.

PROPOSITION CXLVIII.

Every divine order is united to itself in a triple respect; by the summit which it contains; and by its middle, and end.

For its *summit* possessing a most united power, transmits this power into a total union, and unites every thing supernally flowing into itself. But its *middle* extending to each extreme, connects every thing about itself: transfusing the gifts of primary natures, but extending the powers of such as are last; and inserting in all things a communion and connection with each other. For by this means one co-ordination is produced from replenishing and replenished natures, mutually verging to the middle, as to a certain centre. But the *end* returning again to the beginning, and reducing to this the progressive powers, affords similitude and a mutual inclination to the whole order. And thus the whole order is one, through the unifying power of its primary parts; through the coherence subsisting in its middle; and through the conversion of the extreme, to the principle of the progressions.

PROPOSITION CXLIX.

Every multitude of divine unities, is bounded according to number.

For if it is proximate to the one, it is not infinite; since that which is infinite is not connate to the ^{6405]}one, but foreign from its nature. For if multitude essentially recedes from *the one*, it is evident that infinite multitude is perfectly destitute of the one: and hence it is likewise impotent and inefficacious. The multitude of the gods, therefore, is not infinite: and consequently, it is uniform and bounded, and more bounded than any other multitude, because it is more allied to the one. If, therefore, multitude was the principle of things, it would be requisite that every thing nearer to, should be a greater multitude than that which is more distant from the principle: for that which is nearer is more similar. But since that which is first is *the one itself*, the multitude conjoined with it must be less multitude than that which is more remote from *the one*. But infinite is not a less multitude, but multitude in the, most eminent degree.

PROPOSITION CL.

Every thing progressive in the divine orders, is not naturally adapted to receive all the powers of its producing cause. Nor do secondary natures entirely receive all the powers of natures prior to themselves: but these possess some powers abstracted from inferiors; and incomprehensible by things posterior to themselves.

For if there is a difference in the characteristics of the gods, those of the inferior must pre-exist in the ^{6406]}superior gods: but the characteristics of the superior, as being more universal, do not reside in the inferior divinities. But the more excellent characteristics impart some powers to their productions, but eminently pre-occupy others in themselves. For it has been demonstrated that those are more universal, which are nearer to the one, but more particular, which are more distant. But if the more universal possess powers comprehensive of the more particular characteristics; hence those which possess a secondary, and more particular order, will not contain the power of such as are more universal. Hence in the superior, there is something incomprehensible, and uncircumscribed by the inferior properties. For every thing divine is truly infinite; nor does it exhibit itself to itself; nor to things of a much prior superiority to itself: but to all such as are posterior to its nature. But infinity resides in these last, according to capacity. And infinite is incomprehensible by those to whom it is infinite. Hence inferiors do not participate of all the powers, which more excellent natures pre-occupy in themselves. For the latter are incomprehensible by the former. Hence things secondary, from their more particular subsistence, will neither possess the whole of superior natures, nor will they contain the properties which they possess, in the same manner, as their superiors; on account of that infinity through which superior excel subordinate natures.

PROPOSITION CLI.

Every thing paternal^[139] in the gods has a primary subsistence, and pre-exists in the order of *the good*, according to all the divine distributions.

For that which is paternal, produces the hyarxes of secondary natures, and universal powers, and essences, according to one ineffable excellence. And on this account it is denominated *paternal*, indicating the uniform and beneficent power of *the one*, and the hypostatical, or procreative cause of secondary natures. And in every order of the gods, that which is paternal, obtains the principality, producing and adorning all things from itself; because it is established analogous to *the good*. And with respect to these divine fathers, some are more universal, but others more particular; just as the orders of the gods differ in the proportion of cause, according to more universal, and more particular. As many therefore as are the universal progressions of the gods, so many also, are the differences of fathers. For if in every order there is something analogous to the good, it is requisite that *the paternal* should reside in all, and that each order should proceed from a paternal union.

PROPOSITION CLII.

Every thing generative in the gods proceeds according to the infinity of a divine power, multiplying itself, penetrating through all things; and eminently demonstrating a never failing energy, in the progressions of secondary natures.

For what else but the infinite power of the gods, through which all divine natures are filled with prolific good, can multiply progressions, and produce them into offspring from their occult comprehension in causes? For that which is universally full, produces other things from itself, through its overflowing power. Hence a dominion of power, is the characteristic of generative deity: and this absolute dominion multiplies the powers of generated natures, causes them to be prolific, and excites them to the generation and production of others. For if every thing imparts its primary characteristic to others, every thing prolific must insert in natures posterior to itself, a prolific progression, and form a figurative representation of that infinity, which is the first progeny of the universe; from which every generative power proceeds, and which eminently scatters as from a fountain, the perennial progressions of divine natures.

PROPOSITION CLIII.

Every thing perfect in the gods, is the cause of divine perfection.

For as with respect to hypostases, or subsistences, some belong to beings, and others are super-essential; so likewise of perfections, some belong to the gods themselves according to hyparxis, but others to secondary beings posterior to the gods. And the former indeed are self-perfect, and first-artificers, because in these good is contained in a primary manner: but the latter possess perfection through participation. On this account, therefore, the perfection of the gods is different from the perfection of things deified. But that which is primarily perfect in the gods, is not only the cause of perfection to things deified, but to the gods themselves. For if every thing perfect is converted to its domestic principle, the cause of every divine conversion, is the perfective genus of the gods.

PROPOSITION CLIV.

Every thing in the gods endued with a protecting power, preserves every thing in its proper order; uniformly separating secondary natures, and establishing them in such as are first.

For if the preservation of every order, preserves measure in an immutable manner, and contains⁴⁰⁰ the protected natures, in their proper perfection, divine protection will insert in all things an eminence above their inferiors, and will permanently establish in itself every thing, without mixture. It will likewise be the cause of immaculate purity, to protected natures, and will establish them in their superiors. For every thing adhering to primary natures is perfect; but at the same time it abides in itself, and is extended above inferior natures.

PROPOSITION CLV.

Every thing vivific in the divine genera, is a generative cause; but every prolific order is not also vivific.

For a generative power is more universal than that which is vivific, and is nearer to the principle of all. For generation manifests a cause producing beings into multitude: but vivifying (ζωογονία) represents deity the supplier of universal life. If, therefore, the former multiplies the hypostases of beings, but the latter the progressions of life; it will be as *being* is to *life*, so is the *generative* order to the *vivific* series. And hence the generative order will be more universal, and the cause of more effects, and on this account nearer to the principle of all.

PROPOSITION CLVI.

Every cause of purity, is contained in the protecting order. But the protecting is not the same with the purifying genus.

For purity inserts an unmixed nature in every thing inferior to the gods, and an unpolluted power⁴⁰¹ the providence of secondary natures. But protection likewise produces this, comprehending all things in itself, and firmly establishing them in their superiors. Hence the protecting is more universal than the purgative genus. For it is simply the property of protection, to preserve the order of every thing, both with respect to itself, and to things prior and posterior to its nature. But it is the property of purity to separate things more excellent from such as are

more base^[140]; and the former of these are primarily contained in the gods. For it is requisite that there should be one antecedent cause of that which is contained in all things. And universally the uniform measures of every thing good, are first received from the gods; and there is no good in secondary natures, which does not pre-exist in the gods according to cause. For what other origin, or cause, can this possess? In the gods, therefore, purity is likewise a primary good, together with protection, and every thing of this kind.

PROPOSITION CLVII.

Every paternal cause supplies every thing with being, and constitutes the *hyparxes* of beings. But every demiurgic, or fabricative cause of forms, precedes composite natures, together with their order, and division according to number: and is of the same order with a paternal cause, in the more particular genera of things.

For each of these belongs to the order of bound, because both *hyparxis*, and *number*, and *form*, are all of them endued with the form of bound: and hence through this they are co-ordinate to one another. But that which is a *demiurgic cause*, deduces fabrication into multitude. And that which is *uniform*, supplies the progressions of beings. And the former indeed is the artificer of forms, but the latter produces essence. In whatever respect, therefore, *form* and *being*^[141] differ from each other, in the same respect that which is *demiurgic* differs from that which is *paternal*^[142]. But form itself, is a certain one. A paternal cause, therefore, is both more universal and causal, and is superior to the demiurgic genus; in the same manner as being itself is more universal than form.

PROPOSITION CLVIII.

Every reductorial cause (τὸ ἀναγωγὸν) in the gods differs both from a cathartic or purifying cause, and from convertive genera.

For that a reductorial cause, ought to be primarily resident in the gods, is evident; as in these all the ~~causes~~ ^{effects} of universal good pre-exist. But it subsists prior to a cathartic cause; because that liberates from baser, but a reductorial cause connects with more excellent natures. It has, however, an order more particular than the convertive genus; because every thing convertive, is either converted to itself, or to a more excellent nature. But the operation of that which is reductorial, is characterized according to a conversion to that which is more excellent; because it leads that which is converted to something superior, and more divine.

PROPOSITION CLIX.

Every order of the gods consists from the first principles, *bound* and *infinity*. But one order consists more from the cause of *bound*, and another from that of *infinity*.

For every order indeed proceeds from both, because the communications of primacy, penetrate through all secondary causes. But in some orders *bound* predominates in the mixture, and in others *infinity*. And hence that in which *bound* prevails, becomes a genus possessing the form of bound; but that in which *infinity* has the dominion, becomes a genus endued with the form of infinity.

C O N C E R N I N G I N T E L L E C T .

PROPOSITION CLX.

Every divine intellect is uniform, and perfect; and is a primary intellect subsisting from itself, and producing other intellects.

For if it be a god, it is full of divine unities, and is uniform. But if this be the case it is also perfect, being full of divine goodness. And again, if this be the case, it is a primary intellect, as being united to the gods: for deified intellect is more excellent than every intellect. But since it is a primary intellect, it also confers subsistence on other intellects: for from first entities, all secondary beings obtain their *hyparxis*.

PROPOSITION CLXI.

Every true being depending on the gods, is a divine intelligible, and is imparticipable.

For since true being as we have demonstrated is that which first participates a divine unity, it also fills intellect, from itself. For intellect is being, as that which is replenished with being: and consequently true being is a divine intelligible. It is divine indeed, as that which is deified; but as that which is filled with intellect, which it also participates, it is intelligible. And intellect indeed is being, through the first being. But the first being is separated from intellect, because intellect is posterior to being. And imparticipables are prior to things participated. Hence being united with intellect, pre-exists by itself, and is imparticipable. For it is intelligible, not as co-ordinated with intellect, but as eminently perfecting intellect; because it communicates being to intellect, and fills it with essence substantial and real.

PROPOSITION CLXII.

Every multitude of unities illustrating true being, is occult and intelligible. *Occult* indeed, as conjoined with the one; but *intelligible*, as participated by being.

For all the gods are denominated from their dependants, because the different hypostases of the gods may be known from these. For every thing divine is of itself ineffable and unknown, because connate to *the ineffable one*. But by the permutation of participants, it happens that the properties of the gods become known to subordinate natures. Indeed the unities which illustrate true being are intelligible; because true being is a divine intelligible, and is likewise imparticipable, as subsisting prior to intellect. For this would not depend on the first gods, unless they possessed a primary hypostasis, and a power perfective of other gods: since as participants are to each other, so likewise are the hyparxes of participated natures.

PROPOSITION CLXIII.

Every multitude of unities participated by imparticipable intellect, is intellectual.

For as intellect is to true being, so are these unities, to intelligible unities. So far, therefore, as they illustrate divine and imparticipable intellect, they are intellectual: but they are not so intellectual, as subsisting in intellect, but as subsisting through cause prior to, and generating intellect.

PROPOSITION CLXIV.

Every multitude of unities participated by imparticipable soul, is super-mundane.

For since imparticipable soul, is primarily super-mundane, the gods also participated by this soul, are super-mundane; possessing the same proportion to the intellectual, and intelligible gods, which soul has to intellect, and intellect to true being. As therefore every soul is extended to intellect, and intellect is converted to that which is intelligible; so likewise the super-mundane depend on the intellectual gods, in the same manner as these last, on such as are intelligible.

PROPOSITION CLXV.

Every multitude of unities participated by any sensible body, is mundane.

For it supernally illuminates the parts of the world, through the mediums of intellect and soul. For neither is intellect present without soul to any mundane body; nor are deity, and soul immediately conjoined: for participations subsist through similars. And intellect according to the intelligible which it contains, and the summit of its nature, participates of unity. Unities, therefore, are mundane, so far as they fill the whole world, and deify apparent bodies. For each of these is divine, not through soul; (for soul is not the first god) nor through intellect; (for this is not the same with *the one*), but is animated and self-motive, through soul. But it always contains itself in the same manner, and is carried in the best order through intellect; being at the same time divine through a divine unity. And if it possesses a providential power, it is such through unity as the cause.

PROPOSITION CLXVI.

Every intellect is either imparticipable, or participable. And if participable, it is either participated by super-mundane, or mundane souls.

For an imparticipable intellect possessing a primary hyparxis presides over every multitude of intellects. But of participated intellects, some are super-mundane, and illustrate imparticipable soul; but others are mundane. For multitude emanating from an imparticipable, is not immediately mundane; since progressions subsist through similars. But that which is separated from the world, is more similar to an imparticipable, than that which is divided about it. And there is not only a super-mundane, but likewise a mundane multitude. Since there is likewise a mundane multitude of gods, and the world is at the same time animated and endowed with intellect. And the participation of super-mundane gods by mundane souls, subsists through mundane intellects as the connecting mediums.

PROPOSITION CLXVII.

Every intellect understands itself. But the first intellect understands itself alone^[143].

And in this, intellect and that which is intelligible is one in number. But all succeeding intellects, understand both themselves and prior intellects.^[144] And the intelligible to this first intellect, is partly that which it is itself, and partly that from which it proceeds.

For every intellect either understands itself, or that which is above, or that which is posterior to itself. But if it understands that which is posterior to itself; since it is intellect it will be converted to a worse nature, and will not even know that to which it is converted, because the object of its intellection will not reside in its nature, but will be external. And thus it will only possess in itself a type, or figure, of this external object. For it knows that which it possesses, and that to which it is passive, but not that which it does not possess, and by which it is not affected. But if it understands that which is above itself, since this is accomplished by the knowledge of itself, it will both understand itself, and the nature superior to its own. But if it knows that alone, it will at the same time that it is intellect, be ignorant of itself. But by knowing that which is superior to itself, it knows also that it is a cause, and of what it is the cause: for if it is ignorant of these, it will also be ignorant of that superior nature. And hence by knowing that which is prior to itself, it will also know itself. If therefore any intellect is intelligible, this by knowing itself will understand an intelligible, and will be itself its own intelligible. But each of the intellects posterior to this, will at the same time understand that which is intelligible in itself, and that which is prior to itself. There is, therefore, in intellect, that which is intelligible, and in that which is intelligible intellect. But the one, is the same with that which is intelligible^[145]; and the other is the same with the intelligible in itself, but is not the same with the intelligible prior to itself. For the one is simply intelligible, and the other is an intelligible in an intelligent nature.

PROPOSITION CLXVIII.

Every intellect knows in energy that which it understands. And it is not the property of one part of its nature to know, and of another to understand that which it knows.

For if it is intellect in energy, and knows itself as not different from the object of its intellection^[146]; it will both know and perceive itself. But beholding that which is intelligent, and knowing that which beholds, it will know that it is intellect in energy. And knowing this, it will know that it understands, and will not alone know the object of its intellection. It will, therefore, at the same time both know that which is intelligible, and that it understands this: and by intellection it will be understood by itself.

PROPOSITION CLXIX.

Every intellect possesses in eternity, its essence, power, and energy.

For if it understands itself, and intellect is the same with that which is intelligible; intellection also is the same with intellect, and intelligible. For since intelligence is a medium between that which knows, and that which is known; and since these two are the same, intelligence also will be one and the same with each of these. But since the essence of intellect is eternal (for the whole subsists at once) intelligence also will be eternal: for it is the same with the essence of intellect. But if intellect is eternal, it will by no means be measured by time, neither according

to essence, nor according to energy. And since these subsist in the same manner, the power also of intellect is eternal.

PROPOSITION CLXX.

Every intellect, at once understands all things. But an imparticipable intellect understands all things simply. And each of the intellects posterior to this understands all things according to one.

For if every intellect establishes its essence in eternity, and together with its essence, its energy^[416] will understand all things at once. And all things indeed exist according to parts, and a successive energy, which do not subsist in eternity. For every thing successive subsists in time; since it possesses prior and posterior, which are successive, and do not subsist all at once. If, therefore, all intellects understand similarly, they will not differ from each other: for if they understand all things similarly, they are all things similarly; since they are no other than the things which they understand. But if they are all things similarly, one intellect will not be imparticipable, and another not: for their essences are the same with the objects of their intellections; since the intellection of each is the same with its essence, and every intellect is both intelligence and essence. It remains, therefore, either that every intellect does not equally know all things but one or more, and not all things together; or that it knows all things according to one^[147]. But to assert that intellect does not understand all things, is to make it ignorant of some particular being. For if it is affected with transition, and does not understand *at once*, but according to prior and posterior, at the same time possessing an immoveable nature, it will be inferior to soul, understanding all things according to motion, or a mutable energy; because intellect on this hypothesis, will only understand one thing by its permanent energy. It will, therefore, understand all things according to one. For it either understands all things; or one thing; or all things according to one. And the intelligence indeed of all things perpetually subsists in all intellects: but they terminate all things, according to one intelligence of all. Hence there is something pre-dominant in intellection, and the objects of intelligence; since all things are at once understood as one, through the dominion of one, which characterizes all things with itself.

PROPOSITION CLXXI.

Every intellect is an impartible, or indivisible essence.

For if it is without magnitude, incorporeal, and immoveable, it is impartible. For every thing in any respect partible, is either partible on account of magnitude, or multitude, or on account of energies subsisting in time. But intellect is eternal according to all things, and is beyond a corporeal nature; and the multitude which it contains is united. It is, therefore, impartible. But that intellect is incorporeal, is manifest, from its conversion to itself: for no body possesses a self-convertive power. But that intellect is also eternal, the identity of its energy with its essence evinces: for this we have already demonstrated. And that its multitude is united, is evident from the coherence of intellectual multitude, with the divine unities: for these are the first multitude, and after these intellects subsist. Hence though every intellect is a multitude, yet it is an united multitude. For prior to that which is divided, that which is collected, and is nearer to the one, subsists.

PROPOSITION CLXXII.

Every intellect is the proximate sustaining cause of natures eternal, and immutable according to essence.

For every thing produced from an immoveable cause, is immutable according to essence. But immoveable intellect being all things eternally, and abiding in eternity, essentially produces that which it produces. But if it is perpetual, and subsists after the same manner, it will always produce, and according to one uniform energy. Hence it is not the cause of things which are sometimes beings, and sometimes not, but it is the cause of eternal beings.

PROPOSITION CLXXIII.

Every intellect is intellectually both the things which are prior and posterior to itself.

For it is the same with things posterior to itself according to cause, and with things prior to itself by participation; but still it is intellect, and is allotted an intellectual essence. Hence it defines all things according to its essence; both such as subsist according to cause in another, and such as subsist according to participation. For every thing according to its natural constitution, participates of more excellent natures; but not according to the subsistence of its superiors. For these indeed are participated by all things, though in a different respect, according to the various natures of the participants. And hence participations subsist according to the characteristic and power of the participants: and consequently in intellect things prior to its nature, subsist in an intellectual manner. But intellect is likewise intellectually things posterior to itself: for it does not consist from its effects, nor does it contain these, but the causes of these in itself. But intellect is the cause of all things by its essence, and its essence is intellectual; and consequently it contains the causes of all things intellectually. Hence every intellect possesses all things intellectually: both such as are prior and such as are posterior to itself. As, therefore, every intellect contains intelligibles intellectually, so likewise it contains sensibles according to an intellectual subsistence.

PROPOSITION CLXXIV.

Every intellect constitutes through intelligence natures posterior to itself: and its fabrication is contained in intellection, and its intelligence in fabrication.

For if intelligible and intellect is the same; hence the being of every intellect consists in self-intellection. But it fabricates that which it fabricates by its essence, and produces that which is, according to being; and consequently its productions arise from intelligence. For in intellect being and intelligence are one; because intellect is the same with every being which it contains. If, therefore, intellect fabricates by its essence, and its essence is intellection, it will operate through intelligence, and intelligence will subsist in energy in intellection. But this is the same with its essence: and its essence consists in operating. For that which operates immoveably, always possesses its essence in operating: and consequently intellection consists in fabrication.

PROPOSITION CLXXV.

Every intellect is primarily participated by those natures, which are intellectual, both according to essence, and according to energy.

For it is necessary that it should either be participated by these, or by other natures, which possess indeed an intellectual essence, but are not always intelligent. But it is impossible that it should be participated by these latter. For the energy of intellect is immoveable. And hence the natures by which intellect is participated, always participate of an intellectual energy, which always causes the participants to be intellectual. For that which possesses its energy in any part of time, cannot be conjoined with an eternity of energy. But as in essences themselves, so also in the variations of energies, between every eternal energy, and that energy which receives its perfection in some period of time, that energy intervenes which possesses its perfection through the whole of time. For progressions subsist nowhere immediately, but are produced through kindred and similar natures, both according to hypostases, and the perfections of energies. Every intellect, therefore, is primarily participated by those natures which are able to understand through the whole of time, and which possess a perpetual intelligence; though their intellection may subsist according to time, and not according to the stability of eternity.

C O R O L L A R Y .

From hence it is evident that the soul which sometimes understands, and at other times is void of intellection, cannot proximately participate of intellect.

PROPOSITION CLXXVI.

All intellectual forms subsist in one another, and each is at the same time separate and distinct from the rest.

For if every intellect is impartible, and the multitude which it contains is united through an intellectual impartibility: hence all that intellect contains will entirely subsist in one, and impartibles will be united to each other, and all intellectual forms will penetrate through all. But if all intellectual forms subsist immaterially, and incorporeally, they are without confusion with respect to each other, and each separately preserves its own purity,

and abides that which it is. But the characteristic participation of each distinct participant, declares the unconfused subsistence of intellectual forms. For if participated natures were not distinguished, and separate from each other, neither would their participants participate them distinctly, but there would be a much greater indistinct confusion in subordinate natures, from their subsisting in a more degraded order. For from whence could distinction arise, if the natures which constitute and perfect these, should be indistinct and confused? Again, the hypostasis of that which contains impartibly, and an uniform essence, attest the union of forms. For things possessing their hyparxis, in that which is impartible and uniform, subsist impartibly in the same. For how can that be divided, which is impartible and one? For natures of this kind subsist together, and penetrate totally through each other, without distance: since that which contains, is not distant; and one thing is not in this place, and another in that, as in things separated by interval from each other. But every thing at once subsists in that which is impartible and one: and consequently they all subsist in each other. All intellectual forms, therefore, subsist unitedly in each other, and each is at the same time distinctly separate from the rest.

C O R O L L A R Y .

But if any one besides the above demonstrations requires examples, let him contemplate the theorems resident in one particular soul; all which subsist truly in the same soul, in an essence destitute of magnitude, and are united to each other. For the soul does not contain the things resident in its nature, according to magnitude, and locally, but impartibly, and without distance, unitedly, and distinct. For the soul produces all things distinctly, and each at the same time separate and apart, without attracting any thing to itself from the rest, which unless they were always distinguished according to habit, would not be distinguished by the energy of the soul.

P R O P O S I T I O N C L X X V I I .

Every intellect since it is a plenitude of forms, comprehends either more universal or more particular forms. And superior intellects contain in a more universal manner, whatever posterior intellects contain in a more particular manner. But inferior intellects, contain according to a more partial mode, whatever prior intellects contain more universally.

For superior intellects employ greater powers, because they are more uniform than secondary intellects. But inferior intellects, from their being more multiplied, diminish the powers which they possess. For such as are more allied to the one, being contracted in quantity, are superior in power to such as are posterior; while such as are more distant from the one possess a contrary property. Superior intellects, therefore, establishing a greater power, but a less multitude, produce more effects through forms, less according to quantity. [But ^[148] intellects posterior to these, produce fewer effects, through a greater multitude of forms, on account of their deficiency in power. If, therefore, superior intellects produce more effects, through a less number of forms, the forms which they contain are more universal.] And if inferior intellects, produce fewer effects through a greater multitude of forms, the forms which they contain, are more particular.

C O R O L L A R Y .

From hence it happens that the natures which are generated from the superior orders according to one form, are produced in a divided manner from secondary orders, according to a greater multitude of ideas. And on the contrary those natures which are produced from things subordinate, through many, and distinct forms, are produced by superior natures, through fewer, and more universal forms. And that which is universal and common, supernally accedes to all participants. But that which is divided, and peculiar proceeds from secondary natures. Hence secondary intellects by the more particular separation of characteristics, articulately distinguish, and attenuate the formations of primary intellects.

P R O P O S I T I O N C L X X V I I I .

Every intellectual form, is the framer of eternal natures.

For if every intellectual form is eternal, and immoveable, it is essentially the cause of immutable and eternal hypostases; but not of such as subsist in generation, and are corruptible. And hence every thing fabricated according to an intellectual form, is an *intellectual eternal*. For if it produces all forms posterior to such as are

intellectual, through *being*; and if the being of intellectual forms is eternally the same, their productions also will subsist after the same manner, and will be eternal. Hence neither the genera which according to some particular time, are fabricated by a formal cause, nor things corruptible, so far as corruptible, possess a pre-existent intellectual form. For they would be void of corruption and generation, if they possessed their hypostasis, according to a pre-existent intellectual form.

PROPOSITION CLXXIX.

Every intellectual number is bounded.

For if there is another multitude posterior to this diminished according to essence, and so more remote from the one, while intellectual number is nearer to the one: and if that which is nearer to the one, is less according to quantity, and that which is far distant is more according to quantity; intellectual number also will be less than every multitude posterior to its nature. It is not, therefore, infinite: and so the multitude of intellects is bounded. For that which is less than another, is not infinite: because infinite, so far as infinite, is not less than any thing.

PROPOSITION CLXXX.

Every intellect is a whole, as composed from parts, and is united with others, and at the same time distinguished from them. But imparticipable intellect is simply universal; and contains in itself, as it were all parts universally. But each particular intellect possesses the whole as in a part; and thus contains all things particularly.

For if it is all things according to one thing; and if that which is all things according to one, is something particular alone: hence, the whole subsists in each of these particularly, on account of something particular, determinately predominating in them all.

PROPOSITION CLXXXI.

Every intellect which is participated is either divine, as depending on the gods, or is intellectual only.

For if there is a divine and imparticipable intellect, that which is primarily allied to this, does not differ from it in both these respects; that it is not divine, and that it is not imparticipable. For things dissimilar in both these respects, cannot be conjoined with each other. It is evident, therefore, that the medium between these, is partly similar to the first intellect, and partly dissimilar. Either, therefore, it is imparticipable, and not divine; or it is participated, and divine. But every thing imparticipable is divine, as being allotted an order in multitude, analogous to the one. And hence there will be some one intellect, divine, and at the same time participated. But it is requisite that there should be an intellect, not participating the divine unities, but intelligent only. For in every series, first natures, and which are conjoined with their unity, are able to participate their proximate superiors. But such as are far distant from their primary unity, cannot depend on the natures placed in an order proximately superior to their own. There is, therefore, both a divine intellect, and an intellect intellectual alone. And the latter subsists according to an intellectual characteristic which it possesses from its own unity, and from imparticipable intellect: but the former according to a union, which it receives from participated unity.

PROPOSITION CLXXXII.

Every divine intellect, which is participated, is participated by divine souls.

For if participation renders the participant similar, and causes it to be allied to that which is participated; it is evident that that which participates a divine intellect, must be a divine soul^[149]. It is likewise evident that it must depend on a divine intellect, and that it must participate the deity which it contains, through intellect as a medium. For^[150] intellect connects with deity (θεότης) its participant soul, and conjoins one divine nature with another.

PROPOSITION CLXXXIII.

Every intellect, which is participated indeed, but is intellectual alone, is participated by souls neither divine, nor subsisting in a mutation from intellect, into a privation of

intellect.

For neither are divine souls of this kind; nor such as participate of intellect. For souls participate of the gods through a divine intellect, as we have already demonstrated. Nor are such as participate of an intellectual intellect susceptible of mutation. For every intellect is participated by natures, which are always intellectual, both according to essence, and according to energy; as is evident from the preceding propositions.

C O N C E R N I N G S O U L .

PROPOSITION CLXXXIV.

Every soul is either divine, or capable of being changed from intellect into a privation of intellect; or it always remains as a medium between these, and is at the same time inferior to divine souls.

For if a divine intellect is participated by divine souls, but an intellectual intellect, by those souls alone, which are neither divine, nor susceptible of a mutation from intellect into a privation of intellect (for there are souls of this kind, which sometimes understand, and are sometimes destitute of intelligence), it is evident that there are three genera of souls. And the first indeed are divine. But the second are not divine, yet they always participate of intellect. And the third are those, which are sometimes changed into an intellectual condition, and sometimes into a privation of intellect.

PROPOSITION CLXXXV.

All divine souls are gods animastically, (ψυχικῶς, or according to the nature of soul). But all souls participating an intellectual intellect, are the perpetual attendants of the gods. And all souls susceptible of mutation, are some time or other attendants of the gods.

For if some souls possess a divine light, supernally illustrating their nature, but others are endued with perpetual intelligence, and others again, are sometimes only allotted this perfection: hence the first of these will among the multitude of souls, be analogous to the gods; but the second, will perpetually attend the gods, on account of their perpetually energizing intellect, and will depend on divine souls, to which they will have the same proportion, as that which is intellectual to that which is divine. And those which are sometimes endued with intelligence, will also sometimes attend the gods; but they will neither always participate intellect after the same manner, nor will they always be conversant with divine souls. For that which is only sometimes allotted intellect, cannot by any means always attend the gods.

PROPOSITION CLXXXVI.

Every soul is both an incorporeal essence, and separable from body.

For if it knows itself, and if every thing self-gnostic, is converted to itself, and every thing converted to itself is not a body (for every body is incapable of self-conversion), nor inseparable from body; for every thing inseparable from body, is not naturally adapted to be converted to itself, since through this, it would be separated from body; hence every soul, is neither a corporeal essence, nor inseparable from body. But that the soul, knows itself, is manifest. For if it knows things superior to itself, and is naturally adapted to know itself, it will much more know itself, through causes prior to its own nature^[151].

PROPOSITION CLXXXVII.

Every soul is immortal and incorruptible.

For every thing which is capable in any respect of dissolution and dispersion, is either corporeal and a composite, or is allotted an hypostasis in a subject. And that indeed which is dissolved, is corrupted, as subsisting from many things. But that which is naturally adapted to subsist in another, when separated from its subject, vanishes into non-entity. But soul is both incorporeal, and external to every subject, residing in itself, and being converted to itself. It is, therefore, immortal, and incorruptible.

PROPOSITION CLXXXVIII.

Every soul, is both life, and vital.

For that to which soul accedes necessarily lives; and that which is deprived of soul, is immediately left destitute of life. For it either lives through soul, or through something else, and not through soul. But it is impossible, that it should live through something else alone. For every thing which is participated, either communicates itself, or something of itself to its participant. But if it should do neither of these, neither will it be participated. But soul is participated by that to which it is present: and that is called animated, which participates of soul. If, therefore, that which is participated confers life on animated natures, it is either life, or vital alone, or at the same time both life and vital. But if it is vital alone, and not also life, it will be composed from life, and non-life^[152]; and thus it will neither know, nor be converted to itself. For *life is knowledge*^[153]; and that which is gnostic, or endued with knowledge, so far as it is gnostic, lives. If, therefore, there is any thing in soul destitute of life, this something will not essentially possess a self-gnostic power. But if soul, is life alone, it will no longer participate an intellectual life. For that which participates of life, is vital, and not life alone; since that which is life alone, is first and impartible life. But life posterior to this, is vital, and at the same time life. And soul is not impartible life. It is, therefore, both life, and vital.

PROPOSITION CLXXXIX.

Every soul is self-vital.

For if it is converted to itself, and every thing self-convertive, is self-subsistent, soul also is self-subsistent, and sustains itself. But it is also both life, and vital, and its hyparxis is according to vitality. For to whatever natures it is present, it communicates life, through its essence. And if the participant is adapted to participation, it immediately becomes animated and vital; soul neither reasoning nor choosing, nor vivifying by reasoning and judgement, but by its essence alone communicating life to the participant. Hence the being of soul, is the same with its life. If, therefore, it possesses being from itself, and this is the same with its life, it will essentially possess life, and will afford life to itself, and will possess life from itself. But if this be the case, soul will be self-vital.

PROPOSITION CX.

Every soul is a medium between natures impartible, and such as are divisible about bodies.

For if it is self-vital, and self-subsistent, and has an hyparxis separate from bodies, it is separated from, and is more excellent than all partible natures, subsisting about bodies. For these are entirely inseparable from their subjects; because they are divided together with divisible weights, depart from themselves, and their own impartibility, and are co-extended with bodies. And though they subsist in vital natures, yet these are not the lives of partible essences, but of their participants: and though they abide in essence and forms, yet these are not their own forms, for they are forms of formed natures. Soul, therefore, is a self-subsisting, and self-vital essence; it is likewise a knowledge, gnostic of itself, and according to all these separable from bodies. But it likewise participates of life: and if this be admitted, it likewise participates of essence. But it participates also of knowledge from other causes. And hence it is evident, that it is worse than impartibles, because it is filled with life externally: and if with life, it is evident that it is also externally replenished with essence. For prior to every particular life, impartible life, and impartible essence subsists. But it is likewise manifest that soul, is not the first gnostic nature. For every soul so far as soul, possesses life indeed, but not knowledge also from its existing as soul. For certain souls, while they remain as souls, are at the same time ignorant of beings. Soul, therefore, is not the first gnostic nature, nor does it possess knowledge on account of its essence. And hence it possesses an essence the second from those, which are primarily, and essentially gnostic. Since, therefore, the essence of soul is divided from its knowledge, it does not rank among natures purely impartible. But it has been demonstrated, that neither does it subsist in the order of things divisible about bodies. It is, therefore, situated between both.

PROPOSITION CXI.

Every participable soul possesses an eternal essence, but its energy subsists in time.

For either it possesses both eternally, or both temporally; or one eternally, but the other temporally. But it cannot possess both eternally: for on this hypothesis, it would be an impartible essence; and the nature of soul would differ nothing from an intellectual hypostasis; viz. a self-motive from an immoveable nature. Nor can it possess both its energy and essence in time: for thus it would be generated alone; and would neither be self-vital, nor self-subsistent. For nothing measured by time is essentially self-subsistent. But soul is self-subsistent. For that which is converted to itself according to energy, is also essentially converted to itself, and proceeds from itself. It remains, therefore, that every soul is partly eternal, and partly a participant of time. It is either, therefore, eternal according to essence, but participating of time, according to energy; or the contrary. But this latter hypothesis is impossible. Every participable soul, therefore, is allotted an eternal essence, but possesses an energy according to time.

PROPOSITION CXCI.

Every participable soul, ranks in the number of eternal beings, and among the first of generated natures.

For if it is eternal according to essence, it is true being according to its hyparxis, and is a perpetual being. For that which participates of eternity, participates likewise of perpetual being. But if it subsists in time according to energy, it is generated. For every thing participating of time, is always in generation (or in becoming to be) according to the prior and posterior of time, and is not at once, that which it is, but the whole of it is generated. But if every soul, is in a certain respect generated according to energy, it will be the first of generated natures. For that which is entirely generated, is more remote from eternal natures.

PROPOSITION CXCI.

Every soul subsists proximate to intellect.

For if it possesses an eternal, and immutable essence, it proceeds from an immoveable essence: since that which proceeds from a moveable essence, is entirely changed according to essence. The cause, therefore, of every soul is immoveable. But if it is proximately perfected by intellect, it is also converted to intellect, and participates the knowledge, which intellect confers on the natures able to participate cognition. For all knowledge proceeding from intellect, is inherent in all the natures, in which intellect resides. But that to which all things are naturally converted, is the source of their progression according to essence. Every soul, therefore, proceeds from intellect.

PROPOSITION CXCI.

Every soul possesses in a secondary manner, all the forms, which intellect primarily contains.

For if it proceeds from intellect, and intellect is the fabricator of soul; and if intellect subsisting immoveably produces all things; it will also impart to soul, which it constitutes, the essential reasons of all things which it contains. For every thing which operates through essence, imparts secondarily to its production, that which it is itself primarily. Soul, therefore, contains in a secondary manner the representations of intellectual forms.

PROPOSITION CXCI.

Every soul is all things, containing sensible natures, after the manner of an exemplar; (παράδειγματικῶς) but intelligibles after the manner of images (εἰκονικῶς).

For subsisting as a medium between natures impartible, and such as are divided about bodies; it produces and constitutes the latter of these; but establishes in itself the prior causes from which it proceeds. Hence it previously receives after the manner of an exemplar the natures to which it is prior as their cause: but it possesses through participation, and as the blossoms of first natures, the causes of its subsistence. It previously receives in its essence, therefore, through cause all sensible natures, and contains immaterial reasons of things material, incorporeal of such as are corporeal, and indistant of such as are distinguished by interval. But it contains intelligibles after the manner of an image, and receives partly, their impartible forms, such as are uniform variously, and such as are immoveable according to a self motive condition. Soul, therefore, is all beings; containing such as are first through participation, but such as are posterior to its nature, after the manner of an exemplar.

PROPOSITION CXCVI.

Every participable soul primarily uses an eternal body, which possesses an unbegotten and incorruptible hypostasis.

For if every soul is eternal according to essence, and through its essence first animates some particular body, it will always animate this body: for the essence of every soul is immutable. But if this be admitted, that which is animated must be always animated, and must always participate life. But that which always lives, is perpetual by far the first of all things^[154]. And that which is perpetual is eternal. Hence that body which is first animated, and which first depends on soul, is eternal. But every participable soul is primarily participated by some particular body; since it is not impartible, and essentially animates its participant. Every participable, or participated soul, therefore, uses a body primarily eternal, without generation, and incorruptible according to essence.

PROPOSITION CXCVII.

Every soul is *an essence* vital and gnostic, and a *life* essential, and gnostic, and is both knowledge, essence, and life. It likewise contains all things together, the essential, the vital, and the gnostic; and all in all, and each separate and apart from the rest.

For if it has a middle subsistence between forms impartible, and such as are divisible about bodies, it is neither so impartible as all intellectual natures, nor so partible, as corporeal forms. Since, therefore, both essences, lives, and cognitions, are distributed in corporeal natures; all these subsist impartibly in souls, unitedly, and incorporeally, and are at the same time all things, on account of their immateriality, and impartibility. And since all things subsist in intellects according to union, they are distinguished and divided in souls. All things, therefore, subsist together, and apart in soul. But if all impartibles subsist together and in one, they mutually penetrate through each other: and if separate they are again divided without confusion; so that each subsists by itself, and all in all. For in essence there is both life and knowledge: since if essence was essentially deprived of life and knowledge, every soul would not know itself. And in life there is both essence and knowledge. For life without essence and knowledge belongs to material lives, which are neither able to know themselves, nor are sincere and pure essences. And knowledge which is both destitute of essence and life, is incapable of self subsistence. For all cognition belongs to that which is vital, and which is allotted essence essentially.

PROPOSITION CXCVIII.

Every thing which participates of time, and is always moved, is measured by periods.

For since it is measured by time it both participates a measure and bound of motion, and proceeds according to number. But because it is always moved, and this *always*, is not eternal^[155], but temporal, it is necessary that it should use periods. For motion is a certain mutation from some things into others. But beings are terminated by multitudes and magnitudes. And these being terminated, there can neither be an infinite mutation, according to a right line, nor can that which is always moved proceed according to a finished progression. Hence that which is always moved will proceed from the same to the same; and will thus form a period in its progression.

PROPOSITION CXCIX.

Every mundane soul uses periods and restitutions of its proper life.

For if it is measured by time it operates transitively, and possesses a proper motion. But every thing which is moved and participates of time, when it is eternal, uses periods, revolves periodically, and proceeds from the same to the same. And hence every mundane soul, possessing motion and energizing according to time, will both possess periods of motion, and restitutions into its pristine state. For every period of eternal natures, returns to its pristine state.

PROPOSITION CC.

Every period of soul is measured by time. But the period of particular souls, is measured by some particular time: and the period of the first soul, since it is

measured by time, is measured by universal time.

For if all motions contain prior and posterior, they participate of a period, and on this account of time. And that which measures all the periods of souls is time. But if the periods of all souls were the same, and about the same; the time of all would be the same. But if the restitutions of different souls are different, the periodic time of their restitutions also, is different. That the soul, therefore, which is first measured by time, is measured by universal time, is evident. For if time is the measure of every motion; the first motion, will entirely participate of time, and will be measured by the whole of time. For if universal time, did not measure its first participant, neither would it measure any thing else, according to the whole of itself. But that all other souls are measured by the more particular measures of universal time, is evident from what we have now demonstrated. For if they are more particular than the soul which first participates of time, they cannot accommodate their periods to universal time. But the multitude of their restitutions, will be parts of that one period and restitution, by which the first participant of time, returns to its pristine state. For the participation of a lesser power is more particular, but of a greater, more universal. Other souls, therefore, are not naturally adapted to receive a universal temporal measure, through one life; since they are allotted an order more remiss than that which is first measured by time, because they are allotted an inferior order.

PROPOSITION CCI.

All divine souls possess triple energies; one kind as souls; another as receiving a divine intellect; and a third kind, as depending on the gods. And they provide indeed for the universe, as gods; but they know all things through an intellectual life; and move bodies through a self-motive essence.

For since they naturally participate super-mundane natures, and are not simply souls, but divine souls, bearing before themselves an order analogous to the gods, in an animastic latitude; they will energize not only animastically, but also divinely; because they are allotted a deified summit in their essence, and possess an intellectual hypostasis, through which they are spread under intellectual essences. They energize, therefore, not only divinely, but also intellectually; possessing one energy according to *the one*, which they contain in the recesses of their natures, but another according to an intellectual operation. There is likewise present to these divine souls, an energy according to their proper hyparxis; which is motive of natures moved by others, but vivific of such as possess an adventitious life. For this is the proper employment of every soul; but such energies as intelligence and providence, they receive through participation.

PROPOSITION CCII.

All souls attending upon, and always following the gods, are inferior to divine, but more eminent than particular souls.

For divine souls participate of intellect and deity. They are, therefore, at the same time intellectual and divine, and preside over other souls, in the same manner as the gods preside over the universality of things. But particular souls are deprived of a suspension from intellect, because they are not able to participate proximately of a divine essence. For they would not fall from an intellectual energy, if they essentially participated of intellect, as we have previously demonstrated^[156]. Hence the souls, which always follow the gods, are of a middle condition; participating indeed a perfect intellect, and through this surpassing particular souls, yet not depending on the divine unities. For the intellect which they participate is not a divine intellect.

PROPOSITION CCIII.

Of every animastic multitude, (i.e. a multitude belonging to souls) divine souls since they are greater than others in power, are contracted according to number. But such as always follow the gods, retain a middle order among all souls, both in power, and quantity. And particular souls, are inferior to others in power, but proceed according to a greater number.

For divine souls are more allied to the one, on account of a divine essence; but those of a middle order, through the participation of intellect. And those of the last order, are essentially dissimilar to those of the middle and first kind. But among eternal natures such as are nearer to the one, are more united in number, and are more contracted in multitude, than such as are more distant. But such as are more remote, are more multiplied. Hence the powers of superior souls, are greater, and have the same proportion to secondary souls, as that which is divine to that which is intellectual, and as the intellectual to the animastic nature. And the quantities of inferior souls, are more in number. For that which is more distant from the one, is a greater, and that which is nearer a less multitude.

PROPOSITION CCIV.

Every divine soul presides over many souls, the perpetual attendants on the gods; and over a still greater number of such as sometimes receive this order.

For if it is divine, it is requisite that it should be allotted an order, generative of all things, and first-operative among souls. For that which is divine, throughout all beings, presides over the universality of things. And it is requisite that it should neither alone preside over such souls, as perpetually follow the gods; nor alone over such as are sometimes their attendants. For if any divine soul alone presides over such souls as sometimes attend the gods, how can these be united with a divine soul; since they are entirely different from this, and neither proximately participate intellect, nor (by a much stronger reason,) the gods? But if it alone presides over such as perpetually follow the gods, how can the series proceed to souls, the partial attendants on the gods? For thus intellectual natures will be the last, and will be unable through their barrenness, both to perfect other natures, and reduce them to their original. It is necessary, therefore, that such souls as follow the gods, and energize through intellect, and are reduced to intellects more partial than divine intellects, should first depend from every divine soul. But the second to these are partial or particular souls, which are able through the former, as mediums, to participate intellect, and a divine life. For through those which always participate, those which sometimes participate a more excellent condition, are perfected. And again, it is necessary, that about every divine soul, there should be a greater number of souls which sometimes follow, than of those which always attend on the gods. For the power of unity, always proceeds into multitude, according to remission, and subjection; failing indeed in power, but excelling in number. Since in a similar manner every soul perpetually following the gods, presides over a greater multitude of particular souls, imitating a divine soul; and elevates many souls to the first-operative unity of the whole series. Every divine soul, therefore, presides over a multitude of souls, the perpetual attendants on the gods: but presides over a still greater multitude of such as are sometimes allotted this order.

PROPOSITION CCV.

Every particular soul has the same proportion to the soul to which it is subjected according to essence, as the vehicle of the one to the vehicle of the other.

For if there is a natural distribution of vehicles in all souls, it is necessary that the vehicle of every particular soul, should have the same proportion to the vehicle of a universal soul, as the essence of the one, to the essence of the other. But the distribution of vehicles is according to nature: for first participants are naturally conjoined with the things participated^[157]. If, therefore, as a divine soul is to a divine body, so is a particular soul to a particular body, each being participated essentially; hence that is true, which was asserted in the beginning, that vehicles also have the same proportion, as their correspondent souls.

PROPOSITION CCVI.

Every particular soul, possesses a power of descending infinitely into generation, and of ascending from generation to *being*.

For if it sometimes follows the gods, but sometimes falls from its pursuit of a divine nature, and alternately participates of intellect, and a privation of intellect; it is evident that it is conversant by parts in generation, and with the gods. But since it does not reside with the gods, through an infinite time, neither will it be conversant with bodies, through the whole succeeding time. For that which has no temporal beginning, cannot have any end: and that which has no end, is necessarily without a beginning^[158]. It remains, therefore, that every soul must

perform periods, both of ascensions from generation, and of descensions into generation; and that this will never fail, through an infinite time. Every particular soul, therefore, is capable of descending and ascending in infinitum: and this passion never ceases to take place about every particular soul.

PROPOSITION CCVII.

The vehicle of every particular soul is fabricated by an immoveable cause.

For if it eternally depends on the soul, by which it is used, and is by a natural sympathy immutable according to essence, it is allotted a subsistence from an immoveable cause. For that which is produced from moveable causes, is wholly changed according to essence. But every soul possesses an eternal body, which is the first participant of its nature. Hence the cause of every particular soul^[159], and consequently of its vehicle, is immoveable, and on this account super-mundane.

PROPOSITION CCVIII.

The vehicle of every particular soul, is immaterial, indivisible according to essence, and impassive.

For if it proceeds from an immoveable fabrication, and is eternal, it possesses an immaterial and impassive hypostasis. For such things as are naturally passive according to essence, are all of them changed, and material: and from their subsisting differently at different times depend on mutable causes. And on this account they receive an all various mutation, because they are moved with their primary causes. But that this vehicle is indivisible is manifest. For every thing which is divided, is corrupted so far as it is divided, because it relinquishes the whole, and departs from continuity and conjunction. If, therefore, the vehicle is essentially immutable, it will also be impassive, and indivisible.

PROPOSITION CCIX.

The vehicle of every particular soul descends indeed with the addition of material^[436] vestments^[160]; but is conciliated with the soul, by an ablation of every thing material, and by returning to a form proper to its nature, and analogous to the soul by which it is employed.

For the vehicle indeed descends, assuming irrational lives, but in its ascent, casts aside all the powers of generation, with which it was invested in its descent, and becoming *[pure returns to its proper form, and the pristine condition of its nature. It likewise^[161]* imitates the lives of the souls which employ it as an instrument, and is every where moved in conformity, with their motions. And by its circulations, it represents the intellections of some souls, but the falling of others, through their inclination to the realms of generation; and the purgations of others through the revolutions which lead to an immaterial nature. But because it is essentially vivified by, and is connate with souls, it is all-variously changed along with their mutations; follows them every where; becomes passive, when they are exposed to passivity; returns with them when they are purified; and is elevated when they are elevated, and pursues its proper perfection. For every thing is perfected, when it pursues the perfection of its nature.

PROPOSITION CCX.

Every connate vehicle of the soul, possesses both a form and magnitude perpetually the same. But it appears to be both greater and less, and endued with a dissimilar figure, through the additions and ablations of other bodies.

For if it derives its essence from an immoveable cause, it is evident that both its figure and magnitude is derived from this cause: and each is immutable and invariable. But it appears differently at different times, as likewise greater, and less. Hence through the intervention of other bodies added from the material elements and again taken away, it exhibits a different appearance both in quantity and form.

PROPOSITION CCXI.

Every particular soul, descending into generation descends totally. Nor does any part of it remain on high, and another part descend.

For if any thing belonging the soul remains in the intelligible world, it either perpetually understands^[437] without transition, or transitively. But if without transition, it will be intellect, and not a part of the soul; and this particular soul will be that which proximately participates of intellect. But this is impossible. And if transitively, that which is perpetually, and that which is only sometimes intelligent, will form one essence. But this likewise is impossible: for all these differ, as we have previously shewn. Add too, the absurdity which results from supposing that the summit of the soul is perpetually perfect, and yet does not rule over the other powers, and give them perfection. Every particular soul, therefore, totally descends.

T H E E N D .

[438]

A P P E N D I X .

[439]

When I first determined to give my labours to the public, in hopes of contributing to the restoration of the Platonic philosophy, I embraced the resolution of Dr. Johnson and Goldsmith, *to set the Reviewers at defiance*. For I was fully convinced that neither able criticism, nor candid attention could be expected, where composition is dictated by the spirit of malevolence, and influenced by the views of pecuniary reward. However, though contempt is the most philosophical mode of revenge, yet as a certain author well observes severe retaliation is sometimes requisite, in order to convince the subjects of our revenge, that we do not stoop to the meanness of abject submission. This mode of retaliation the defamation of the *Monthly Reviewers* in their *bundle of criticism* for August last obliges me to adopt: and they have afforded me in this review the most favourable opportunity I could desire, of exposing their *malevolence*, *ignorance*, and *pride*. I shall begin, therefore, with instancing their *malevolence*, as it is the first in our list of their bad qualities, and is the general characteristic of these assuming critics. In my preface to the translation of Orpheus, after representing the difficulty of well translating the compound epithets of the Greek, into English, and the necessity of possessing the philosophic genius for this purpose. I add: “*If some sparks of this celestial fire, shall appear to have animated the bosom of the translator, he will consider himself as well rewarded for his laborious undertaking.*” Upon which these *candid* reviewers observe, (p. 138.) “Mr. Taylor was aware of this difficulty, though he seems to claim the merit of subduing it.” In the second place they assert, (p. 138.) that after lamenting, that the Commentary of Proclus on Plato’s *Cratylus* is not likely to be published, “I comfort myself with the hope that my own labours will in some measure supply its place, by opening the pure sources of genuine wisdom. And that to this end I promise copious and truly philosophic notes.” Now the passage which furnished this malevolent assertion is the following: “What farther light we have been able to throw on these mysterious remains of antiquity, will appear in our following notes. If the valuable Commentary of Proclus on the *Cratylus* of Plato, was once published, I am persuaded we should find them full of the most recondite theology: but as this is not to be expected in the present age, the lovers of wisdom will I doubt not gratefully accept the preceding and subsequent elucidations. *For on a subject so full of obscurity as the present, a glimmering light, is as conspicuous, and as agreeable to the eye of the mind, as a small spark in profound darkness, is to the corporeal sight.*” Dissertation, p. 106. The infamy of such misrepresentation is too glaring to require any illustration, too shameful to admit of any excuse, and in any other cause than that of verbal criticism, too contemptible either to rouse resentment, or deserve the most trifling attention. Let us now examine the specimens of *ignorance* which these Reviewers afford in great abundance; and which as I presume will appear much to the credit of my translation. In the first place I am charged with “universally translating the epithets *φιλένθεος*, *φίλοιτρος*, and *μανικός*, by the word *fanatic*, which I have employed in the sense of the Latin word, from which it is derived.” To which I reply, that the former part of this charge is false. For in the hymn to Minerva *φίλοιτρος* is translated *rage*; in the hymn to Diana, *fierce*; and in the hymn to Dionysius Bassareus, *μανικός* is translated *furious*. [The latter part of this assertion is true. For as the word *fanatic* is immediately derived from the Latin word *fanaticus*, which according to their own confession means *numine afflatus*, or *one inspired by a divine power*; and as the great Scaliger, whose authority is always decisive, constantly translates *φιλένθεος*, *fanaticus*, I made no scruple of adopting it in my translation. That, *fanatic* is never used in a good sense by any author of repute may perhaps be true: but I see no reason why it should not be employed according to the meaning of its original, especially as there is no other word in our language so expressive of the words to which it corresponds in the Greek. The example of Aristotle, and the greatest men of antiquity sufficiently justifies both the invention of new terms when the poverty of a language requires a supply, and the adoption of old ones in a different sense, when the difficulty of the subject demands verbal innovation. After this I am accused of *totally mistaking* the meaning of various passages, the greater part of which I shall expose to the view of the reader with a literal translation, and comment; that the ignorance of the *Reviewers* may appear without that veil which at present screens it from the eyes of the unlearned in Greek. In the hymn to Pluto then, I have translated the following line:

Μοῦνος ἔφους ἀφανῶν ἔργων φανερῶν τε βραβευτής·
Of unapparent works thou art alone
The dispensator visible and known.

That is, literally, “Thou art alone the dispensator of apparent and unapparent works.” Now there is nothing in my version can be objected to, but the omission of the word *apparent*, which the measure of the verse obliged me to neglect; and which the addition of *visible* and *known* in the second line renders superfluous, as the following observations will evince. According to the Orphic theology, *Pluto* belongs to the same order as the *sun*, and from his subsisting in occult union with this deity, he is celebrated as one and the same: a custom frequent with the Orphic theologists, as is well known to those who are skilled in their writings. Hence considered as the *sun*, he is the dispensator of *apparent*, and as *Pluto*, of *unapparent works*: and thus I presume, I have not *totally mistaken* the meaning of this line, in celebrating *Pluto* as a deity *visible and known*. But that the reader may be fully convinced of the truth of this assertion, concerning the occult union between *Pluto* and the *sun*, let him attend to the following Orphic verse, preserved by Justin Martyr, (in Cohortat. ad Gentes).

Εἷς ζεὺς, εἷς ἄδης, εἷς ἥλιος, εἷς διόνυσος·——
i.e. “Jupiter, Pluto, the Sun, and Bacchus are one.”

Again, in the epithet ἀγλαότιμς, it seems I have *totally mistaken* the meaning of my author, by translating it *honor'd light*. This word means literally *exceedingly honoured*: and the preceding exposition sufficiently proves the propriety of calling *Pluto*, *lucid*. Every reader knows the necessity there is in poetical translations of adding something to the original: and this is always allowed, when the addition is not contrary to the sense of the text, but either expands it, if condensed, or enlightens it, if obscure. I am likewise charged with mistaking the meaning of λόγου θνητοῖσι, προφῆτα, or, *prophet of discourse to mortals*, which I have rendered,

——‘Prophet of discourse.’

Now as this is literal, the mistake must consist in not substituting another word for *prophet*, which might express what the author meant; the *Reviewers* never dreaming that this word, when properly understood, is perfectly sufficient for the purpose. As they appear, therefore, to be *totally* ignorant of the original signification of a prophet, I shall subjoin its definition from Festus. “*Prophetas* dicebant veteres antistites fanorum, oraculorumque interpretes:” i.e. “the ancients called prophets the priests of fanes, and the *interpreters* of oracles.” *Prophet of discourse*, therefore, means *interpreter of discourse*: and as this epithet is applied to Mercury, it is doubtless highly proper; if we consider that he first reduced the infinity of voice into bound, by dividing letters into species; and thus truly became the interpreter of speech to mankind. In the hymn to Venus, I have translated,

Εἴτ' ἐν Κύπρῳ, ἄνασσα τροφῶ σέο——
“Or if in Cyprus with thy mother fair.”

And it is literally “*Or if in Cyprus O queen, with thy nurse*”. Fortunately for me, the metaphrase of Scaliger agrees with my version, “*Sive in Cupro, matre tua*”. Perhaps the *Reviewers* forgot, or perhaps they are ignorant, that a mother and a nurse are frequently synonymous terms! I shall not trouble the reader with any more instances of my *mistakes*, as I can faithfully assure him, that the remaining passages adduced by the *Reviewers*, betray *if possible*, more malevolence and ignorance than the present. I shall, therefore, proceed to a defence of some epithets, and expressions which I have employed; and in which these *exquisite critics*, can neither discover *beauty*, nor even *propriety*.

In the first place then, they confess that they have too little *taste*, or too little *knowledge* to discover either *beauty*, or *propriety*, in my translation of the following line:

Νύμφαι θυγατέρες μεγάλῆτορς Ὠκεανοῖο——
‘Nymphs, who from *ocean's stream* derive your birth.’

i.e. literally, ‘Nymphs, daughters of the mighty ocean.’ Now as the exceptionable part of this line, is *ocean's stream*, as appears by its being printed in *italics*; I can only assure the reader that I can plead no less authority than that of both *Homer*, *Hesiod*, *Plato*, and *Milton* for its *propriety* and *beauty*. Thus *Homer*, (*Iliad* xviii. l. 606.) speaking of the fabrication of Achilles' shield by Vulcan, says:

Ἐν δ' ἐτίθει ποταμοῖο μέγα σθένος ὠκεανοῖο.
i.e. 'But he placed in it the mighty strength of the *ocean's stream*.'

So likewise: (Iliad xx. l. 7.)

Οὔτε τις οὖν ποταμῶν ἀπέην νόσφ' ὠκεανοῖο.
i.e. 'No *stream* was absent, except the *stream of the ocean*.'

Thus again, in the Odyssey: (lib. xi. l. 637.)

Τὴν δὲ κατ' Ὠκεανὸν ποταμὸν φέρε κῦμα ῥόοιο.

i.e. 'But the waves of the current bore it (the vessel) through the *ocean stream*.' And Milton had doubtless an eye to this last passage, when, speaking of the Leviathan, (Paradise Lost, book I.) he says:

——or that sea beast
Leviathan, whom god of all his works
Created hugest, that swim th' *ocean stream*.

For here, as the reader must observe, he uses the very same expression with Homer. But Milton was not only a great poet, but a man of great learning; and was doubtless much better acquainted with Homer than the Reviewers.

Thus too Hesiod: (in Theog. l. 241. &c.):

——καὶ Δωρίδος ἠὺκόμοιο,
Κούρης Ὠκεανοῖο τελήεντος ποταμοῖο.

i.e. 'and from the fair haired Doris, the daughter of the perfect *stream* of the *ocean*.' And the same epithet is used in l. 959. of the same work. And lastly, Plato in the Phædo, thus speaks of the *ocean*, as one of the four great rivers, of which Tartarus is the source: τὰ μὲν οὖν δὴ ἄλλα πολλά τε καὶ μεγάλα καὶ παντοδαπὰ ρεύματά ἐσι. τυγχάνει δ' ἄρα ὄντα ἐν τούτοις τοῖς πολλοῖς τέτταρ' ἄττα ρεύματα, ὧν τὸ μὲν μέγιστον καὶ ἐξωτάτω ρέον περὶ κύκλῳ, ὃ καλούμενος Ὠκεανός ἐστι. i.e. 'There are many other both great and all-various rivers, but principally four; the greatest and last of which, flowing round the earth in a circle, is called the *ocean*.'

I only add that this expression is perfectly philosophical, as will be evident from considering the ever-flowing condition of the ocean, by means of which it admirably corresponds with the nature of a *stream*. Homer indeed was so sensible of this truth, that he generally (if not always) speaks of the ocean in this manner; and there is no doubt, but he derived his conviction from the first and most profound philosophy in the world. After this the expression, a *blameless tide* of abundance is objected to. But if the epithet *blameless* may be applied to *abundance*, which it is in the original; (ἡδ' ὄλβον ἀμεμῶῃ) and if a *tide* of wealth, is an usual expression, I see no reason why abundance, when conferred with moderation, may not be said to be poured in a *blameless tide*. The objections to the translations of (θνητῶν ζήριγμα) 'basis of mankind,' and the first part of the hymn to Protogonus, are too contemptible to deserve any reply. This too would be the case with the epithet 'Bacchic King,' which is literally translated from the Greek; (Βακχεῖον ἄνακτα) but very fortunately these *sagacious critics* have employed a correspondent expression, in their Review of Wharton's Milton: for in page 1. they speak of the *Miltonic muse*, which I presume must fall under the same imputation of *impropriety*, and want of *beauty* with *Bacchic king*. I shall only adduce one instance more, and then proceed to take notice of the pride of these uncandid and *ignorant* censors. In the hymn to Boreas, that deity is requested to dissolve the all-misty station of the air:

Λῦέ τε παννέφελον ζάσιν ἡέρος——

Which I have accordingly translated,

'The *misty station* of the air dissolve.'

And I must confess, that as I cannot find the least impropriety in speaking of the air as being in a *misty station*, I must conclude that this was exactly the *station* of the Reviewers, at the time when they composed the present

criticism; the whole of which appears to have been the result of *misty* visions, *clouded* conceptions, and *uncertain* conjectures.

Let us now proceed to a *review* of their *pride*. In the first place then, they very pompously inform us of their natural gravity as follows: ‘Grave though we be, *our own* risibility has been provoked,’ &c. As if it was of any consequence to the public, whether they are *grave* or *facetious*, solemn or *ludicrous*, sanguine or *bilious*: whether they possess the qualities of the *owl*, or the *ape*; and whether they laugh like the tickled Hyæna, or like Milton’s *death* ‘grin horribly a ghastly smile.’ In the next place, after having praised my paraphrase of Plotinus on the Beautiful, they add: ‘this praise ought to convince Mr. Taylor, that we are neither insensible to the real value of his author’s work, nor blind to the merits of the translation’. As if the praise of a Reviewer could be of any importance to a man, whose writings, are not calculated for the multitude: or as if the censure of ignorant judges, was not preferable to their most unbounded approbation! I only add that from men who are critics by profession on the writings of others, the most perfect composition may be justly expected: and yet the *Monthly Reviewers* have grossly failed in this respect, as the following instances will evince: Polybius makes use of the expression, νεύειν πρὸς ἓνα καὶ τὸν αὐτὸν σκοπὸν, i.e. ‘to verge to one and the same end:’ and this our admirable critics *translate* (p. 122.) ‘to verge to one *point*, and conspire to one *end*:’ which is obviously a most ridiculous tautology. For it is impossible that any thing can verge to one *point*, and at the same time conspire to an *end*, different from that *point*. Again, in their review of Bell’s Shakespeare, (p. 156.) they make use of the following simile: ‘Shakespeare, now stands (among the French) as a *Colossus*, while the most that can be done by Voltaire, and indeed the very best of our modern writers at home, is to *creep* under his *feet*.’ But here we may very justly enquire, what similitude there is between *modern wits* endeavouring to imitate Shakespeare, considered as a *dramatic writer*, and *men* crawling under his feet, considered as a *Colossus*? If Shakespeare indeed had been a quadruped, men by creeping under his feet might be considered as his *groveling* imitators: but I cannot conceive any similitude between a *creeping*, and an *upright* figure. I only add, that the Analytical Reviewers, are not more fortunate in their review of my translation of Proclus. For after asserting that the original is not remarkable for its elegance (though the contrary is the opinion of the best ancient and modern writers) and that I have too faithfully copied my author in this respect, they inform us, among other *interesting* particulars, ‘that the employment of an ancient philosopher did not consist in relieving the *distresses* of the *wretched*, and the *wants* of the *miserable*!’ After such a specimen of tautology, we cannot wonder that Proclus is considered as an inelegant writer: for though his language is always overflowing and majestic, it never degenerates into weak and needless repetition. While on the other hand, there is such a *perfect* sameness, in the above sentence, that, ‘to relieve the *distresses* of the *wretched*, and the *wants* of the *miserable*,’ is indeed no other, than ‘to verge to one *point*, and conspire to one *end*!’

And thus much for the Reviewers, whom in any other cause than that of verbal criticism, I should consider as too mean for censure, and even too insignificant for *contempt*. For what attention can those writers deserve, who decide dogmatically on subjects they have never studied; who endeavour by malevolent aspersions to ruin the reputation of men they have never seen; and who abuse the credulity of the ignorant, by a monthly compilation of criticisms, which originate from vanity, and ultimately tend to illiberal gain?

T H E E N D .

FOOTNOTES:

[1] the two preceding books of this work our author has displayed an uncommon degree of philosophic elegance and depth; and in the present two, he no less manifests the greatest geometrical accuracy and skill. In the former he elevates us from participated truth to truth itself; and from the glimmering light of universals reflected in the catoptric bosom of the phantasy, to the bright refulgence of ideas. In the latter he combines geometry and philosophy, occasionally cloathes the rigid accuracy of demonstration with the enchanting imagery of divine imaginations, and unites the graces of diction with the precision and sanctity of truth. Yet his genius, though rapid as a torrent, never passes beyond the bounds of propriety; and though his thoughts are vehement and vast, they are at the same time orderly and majestic. For my own part I confess myself enamoured with the grandeur of his diction, astonished with the magnificence of his conceptions, and enlightened by the

irradiations of his powerful genius. And I desire nothing so much as that others may experience similar effects from this admirable work. I only add, that the study of this second part is absolutely necessary to a perfect comprehension of Euclid's method and meaning; and to the understanding geometry completely and philosophically. It is easy indeed to learn a science in a manner sufficient for mechanical purposes; for this is accomplished by the *many*: but it is arduous to learn it with a view to the perception of truth; for this is alone the province of a *few*. It is easy to be knowing in effects, for these are obvious and common; but it is difficult to investigate causes, for these are occult and rare. In short, a general and confused apprehension of a science may be readily obtained, without much labour and toil; but a particular and accurate knowledge requires liberal application, and patient endurance. For the one is like the distant prospect of a country, in which the larger parts are alone conspicuous to the observer's eye; but the other resembles a near and distinct view, in which every thing is recognized essential to the perfection of the whole.

[See](#) the second section of the Dissertation, Vol. I.

[The](#) nature of place has been a subject of much curious and deep speculation to the Peripatetic and Platonic philosophers, as may be seen in the very valuable Commentaries of Simplicius on Aristotle's Physics; so that Proclus does not affirm without reason, that place is more obscure than the natures it contains. But as the opinion of our philosopher, concerning place, is so admirably profound, subtle, and remarkable, I persuade myself the following translation from the fourth book of Simplicius on the Physics, containing his sentiments at large on this subject, will not be unacceptable to the liberal English reader. "Proclus (says Simplicius) having proved from the arguments of Aristotle, that place is neither matter nor form, concludes, that it is a certain interval:" after which, he reasons as follows. "This interval then, is either, nothing or something; and if nothing, local motion will consist in a transition from nothing to nothing; but all motion subsists according to something. But if it ought to be called something, it is either corporeal or incorporeal; and if incorporeal, an absurdity will ensue; for it is necessary that place should be equal to the thing placed. But how can body, and that which is incorporeal be equal? For equal is found in quantities, and especially in those of a similar kind, as lines are equal to lines, superficies to superficies, and bodies to bodies: place, therefore, is body, if it be a certain interval; but if body, it is either moveable or immoveable; and if moveable, in whatever manner it may be moved, it must necessarily be moved according to place, so that place again will require another place, which is impossible, as was also evident to Aristotle and Theophrastus; for Aristotle says, that a vessel is a moveable place, but place an immoveable vessel, because place is naturally immoveable. But if it be immoveable, it is either indivisible, which cannot be divided by the bodies entering its receptacle, since one body cannot penetrate another; or it is divisible, as air and water are divided by the bodies entering into their yielding natures; but if it be divisible, the whole being dissected or divided, the divided parts will be moved on each side, and place will be the first mutable, since its parts are moved; but we have demonstrated that it is immoveable. Again, the parts being separated, we ask where that which is divided betakes itself; for there must be again given or investigated another interval, intervening between the divided parts, which may receive and be placed together with that which is divided; and this will be the case, in infinitum. Place, therefore, is an indivisible body; and if an indivisible body, either material, or destitute of matter: but if material it will not be indivisible, for it is requisite that all material bodies, when permeated by other material bodies, should be divided by them, as is the case with our bodies when they fall into water. But immaterials alone resist all division and this from a necessity of nature; for every body destitute of matter is void of passion; but every thing which is divided likewise suffers. Since division is a certain affection of bodies, which extirpates and destroys their unity and connection; for that which is continuous, so far as continuous suffers no other affection or molestation than section, which destroys and takes away its continuity. That we may therefore collect together what we have separately demonstrated, place is an immoveable indivisible body, destitute of matter. And if this be admitted, it is evident that it is a body by far less material than the rest, and indeed less than the matter contained in things which are moved. Hence, if light is the most simple of these (for fire is more incorporeal than the other elements, and fire is lucid) it will be manifest, that since light is the purest among the rest, light will be place. Conceive, therefore, two spheres, of which one is composed from many bodies, and the other of light alone, and let both be of equal bulk; then, by establishing the sphere of light, together with the centre, and giving the composite sphere a revolution in the circumscribing sphere of light, you will perceive the world moved in immoveable light, and according to its whole extension, immoveable, similar to place, but moved according to its parts, because these are less than place." Now, from this demonstration of

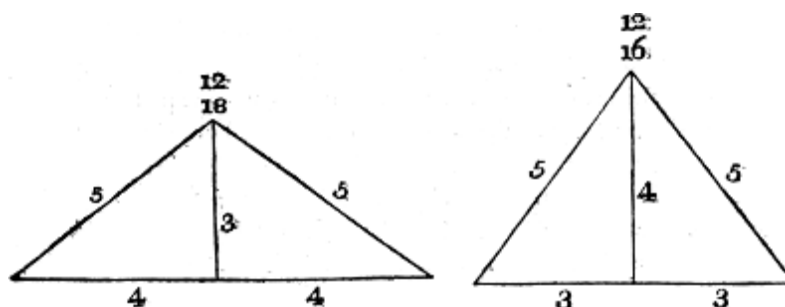
Proclus, it follows by a necessary consequence, since contraries are contained under the same genus that darkness, if it be any thing positive, is the most material of all bodies; and hence, the most material natures will participate the most of darkness, as indeed, is evident in the elements of earth and water. It likewise follows that whatever exists in perfect darkness, exists out of corporeal place, which, however paradoxical, is perhaps, no less true than wonderful to conceive.

~~See~~ Section second, of the Dissertation, in Vol. I. of this work.

~~See~~ the third figure of this problem.

~~This~~ is easily proved from the mensuration of a triangular space, which it is well known is obtained by multiplying the base into half the altitude; and this in the first triangle will be equal to 3 multiplied by 2; and in the second, to 2 multiplied by $2\frac{1}{2} = 5$.

~~The~~ quantity of this perpendicular in each triangle may be easily obtained from the 47th proposition of this book; for in the first triangle it will be three units; and in the second four. Hence, the area of each will be 12 units; but the ambit of the one will be 18, and of the other 16 units, as is evident in the following figures.



~~What~~ is angles formed from the circumferences of circles cutting or touching each other, when they are on both sides concave.

~~Most~~ probably Zeno, the Epicurean.

~~[M]~~. Simpson, in his note on the 7th proposition of this book, positively asserts, that it contains two cases, though there is but one in the Greek text; and ridicules Proclus for asserting that the second part of the present proposition was added, in order to solve objections which might be urged against the seventh. But that Euclid never added any more than one case, is, I think, evident, not only from no such case being found in the Greek copies so early as the age of Proclus; but from his not converting it in the 6th proposition. Besides, it is employed with advantage in the solution of objections against the 9th proposition, as the reader will perceive in its commentary; and the objection there started merits the appellation of a case, as much as the 7th. But Mr. Simpson seems to have been ignorant of Euclid's design in these elements;—the tradition of that only which is accommodated to an elementary institution. Hence, Euclid every where avoids a multiplicity of cases; and anticipates objections where he foresees they may be urged. Mr. Simpson adds in support of his dogmatical assertion, “that the translation from the Arabic has this case explicitly demonstrated.” As if an Arabic translation was of greater authority than the Greek text which Proclus consulted! And lastly, he concludes, with observing, that “whoever is curious, may read what Proclus says of this in his commentary on the 5th and 7th propositions; for it is not worth while to relate his trifles at full length.” If an accurate knowledge of the nature, beauty, and tendency of a science, or a collection of scientific propositions, is trifling, Proclus, indeed, deserves this accusation; as I doubt not the liberal reader, is, by this time, fully convinced. But Mr. Simpson was no philosopher; and therefore the greatest part of these Commentaries must be considered by him as trifles, from the want of a philosophic genius to comprehend their meaning, and a taste superior to that of a *mere mathematician*, to discover their beauty and elegance. It is common, indeed, to hear geometricians of the present day exclaiming, *What need of a comment on Euclid! Is he not perspicuous to every one?* I will readily admit that such gentlemen know enough of geometry for all mechanical and sensible purposes: but I fear they are totally ignorant of its end; and have never dreamt that when properly studied it is the handmaid of true philosophy, the purifier of the rational soul, and the bridge by which we may pass from the obscurity and delusion of a material nature, to the splendor and reality of intellectual vision. I add farther, that I am greatly inclined to doubt,

whether such geometricians ever considered what kind of subsistence geometrical forms possess Whether they have any certainty, or are only imaginary? Where these forms, if real, reside? And a multitude of other questions which are discussed in these Commentaries. And lastly, what is most material of all, if geometry be a science, what science itself is? This last question, indeed, they would doubtless consider so *trifling* and easy of solution, that they would readily and confidently answer with young Theætetus in Plato, “that sciences are such things as may be learned from Mathematicians, geometry, and the like; shoe-making, and other mechanical arts; and that all, and each of them are no other than sciences!” To which admirable definition we may justly reply in the words of Socrates, “Generously and magnificently O my friends, when interrogated concerning *one thing*, have you given instead of something simple, things *many* and *various*.”

[11] Angular numbers, are 1, 3, 6, 10, &c.; and sexangular numbers, 1, 6, 15, 28, &c. But concerning their formation, see note to page 95, Vol. I. of this work; by means of which, the truth of this assertion will be evident.

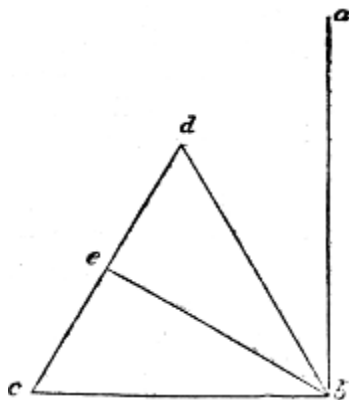
[12] Concerning the meaning of total predication, see page 45 of the Dissertation, Vol. I. of this work.

[M]. Simson in his note to this proposition observes, that he thought proper to change its enunciation, so as to preserve the same meaning; “because (says he) the literal translation from the Greek, is extremely harsh, and difficult to be understood by beginners.” Whatever difficulty learners may find in conceiving this proposition abstractedly, is easily removed by its exposition in the figure; and therefore, I conceive, that Mr. Simson acted very injudiciously in altering its enunciation. Besides, the following comment of Proclus shews, that there is great beauty in Euclid’s statement of this proposition; the greatest part of which is lost in Mr. Simson’s indiscreet alteration. It would appear strange that such liberties should be taken by one, who professes in his preface, *to remove blemishes, and restore the principal books of the Elements to their original accuracy*, if Mr. Simson had not informed us in his note, that the present Commentary of Proclus unfolding the beauty and *accuracy* of this proposition, is too *trifling* to merit a relation! See more concerning this proposition in the note to Prop. 5.

[S] See the Comment of Clavius on this proposition.

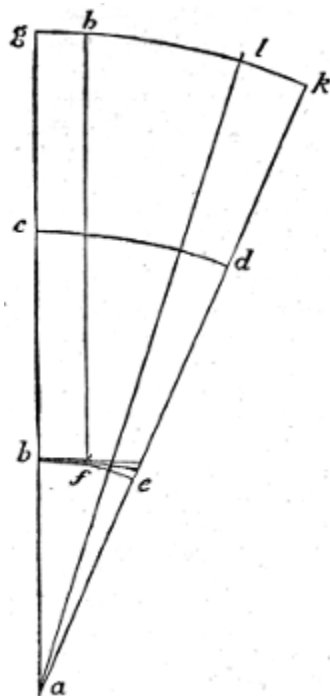
[15] And from hence, also appears the emptiness and arrogance of Mr. Simson’s note to this proposition, which we have already exploded.

[16]



This too may be easily effected by means of the first problem, and the present. Thus let abc be a right angle, which it is required to trisect; then, upon the side ab , describe an equilateral triangle cd , and bisect the angle dbc , and the angles abd , dbe , ebc , shall be equal. For the angle dbc , is one third of two right angles, or two thirds of one right, and consequently the angle dba , is one third of a right angle; and this is equal to dbe , the half of dbc . Therefore they are all equal.

[17]



The method of dividing an angle in any given ratio, by means of a right line and circle only, seems to have been entirely unknown to the ancients, as well as to the moderns. However, the author of this translation presumes he has discovered the means of solving this arduous problem; and that such as admit the truth of his demonstration respecting the quadrature of the circle in page 56 of his Dissertation, vol. I. of this work, must necessarily subscribe to the following method of dividing an angle in any required proportion. Let there be an acute angle given $g a k$, which it is required to divide in the ratio of the right line $a c$ to $c g$. Bisect $a c$ in b , and from the centre a , with the radius $a b$, describe the arch $b e$, and with a radius equal to $a c$, describe an arch touching $b e$, in the point b . Likewise with a radius double to $a c$, describe another tangent arch at the point b , and with a radius equal to $a g$, a tangent arch at the same point, according to the figure; and lastly, let the arches $c d, g k$, from the centre a be drawn. Then $\frac{1}{2}$ of the arch $b e$, shall be equal to $\frac{1}{4}$ of $c d$, and to $\frac{1}{8}$ of the arch similarly placed, described with a radius the double of $a c$, as is well known. Bisect then $b e$ in f , and make each of its two next tangent arches at b , equal to $b f$, which is easily done, from what has been already observed; and through the points of equality describe a circle, this (by the theorem in page 56, of our Dissertation), shall cut off some part of the tangent arch described with the radius $a g$, equal to $b f$, or the fourth part of $c d$. Hence, a part in the arch $g k$, may be easily taken equal to $c d$, which let be $g l$, and drawing the right line $a l$, the angle $g a l$, shall be to $l a k$, as $a c$ to $e g$, which was required to be done.

The same construction will serve for the division of a right angle in any given ratio, as is evident; and if the given angle be obtuse, the problem may be solved by a two-fold operation, that is, by bisecting the obtuse angle, and dividing either of the equal sections in the given ratio; for when this is effected, the whole angle may be easily divided in the same proportion. Hence, too, a right line may be speedily obtained equal to a given arch of a circle.

[18] the 30th Prop.

[19] See Chap. 8. Book 2d.

[20] Simson having a great objection to the word infinite, though it is adopted by Euclid, substitutes in its place the word *unlimited*; but not in my opinion with any success. For if by *unlimited*, he means infinite, the alteration is ridiculous: but if he means only *indefinite*, or a line which has boundaries, though they are not ascertained, the problem will not succeed, as the ensuing commentary most beautifully evinces. I only add, that the reader, if he be a man of taste, and possesses any spark of the philosophic genius, must be greatly delighted with the digression of Proclus in this comment, concerning the nature of infinite, as it is perfectly philosophical and truly sublime.

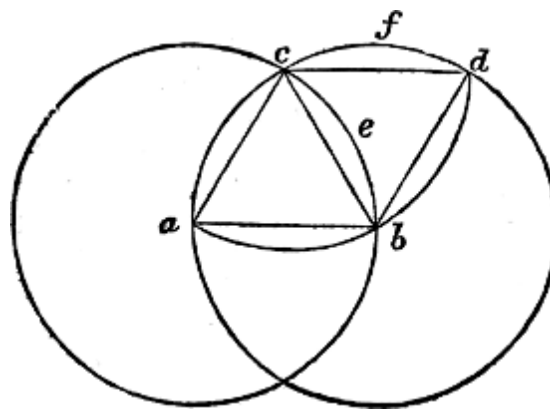
[20] That no other figure besides these can fill place, will be evident, if its angle, when found, is multiplied by any number: for, as Tacquet well observes, it will always either exceed, or be deficient from four right angles. For a more particular demonstration of this admirable theorem, see Tacquet *Elementa Geometriæ*, p. 88.

[22] Pappus in *Mathem. Collect.* shews that any two sides, whose ratio to the external sides is less than two to one, may be inscribed after this manner in a triangle.

[23] See likewise concerning these triangles, ol. I. p. 173 of these Commentaries.

[24] the second comment of this book.

[25] This will be manifest from the following figure. Let the circles $a c$, $b d$, be drawn passing through their respective centres a , b ; and from the centre c , with the radius $c b$, equal to $a b$, describe the arch $a b d$, and draw the lines $c b$, $c d$, $c a$. Then because $a c b$ is an equilateral triangle, as also $c b d$, each of the angles $a c b$, $b c d$, shall be equal to $\frac{2}{3}$ of one right angle; and because the biline $c d$, is equal to the biline $c b$, hence, the angle formed by the arch $c b$, and the arch $c d$, viz. the angle $a c f$, shall be equal to the angle formed by the right line $c b$, and the right line $c d$, i.e. to $\frac{2}{3}$ of one right angle. Q. E. D.



[26] See more concerning these universal genera in the third section of the Dissertation, Vol. I. of this work.

[27] Book II. Comment 2. of this work.

[28] the Comment on the 32d proposition.

[29] book III. chap. I. and in Com. 3.

[30] lib. i. de Cælo, tex. 35.

[31] Pappus and Simson have employed a multitude of propositions in the demonstration of this petition, but their demonstrations fall far short in my opinion of the elegance of the present.

[32] Lib. III. Com. 2.

[33] In his last analytics. See the second section of the Dissertation, Vol. I, of this work.

[34] the definitions which are with great propriety called by the Platonists hypotheses, because their evidence is admitted without proof, which at the same time they are capable of receiving from the first philosophy.

[35] his last Analytics. See page 49, of the Dissertation, Vol. I. of this work.

[36] See the last preceding figure.

[37] This is a well known property of the hyperbola, and its asymptotes; and is thus expressed by Mr. Simson, in his *Conic Sections*, Lib. 3. Prop. 16. "If from a point in the hyperbola, any two lines are drawn to the asymptotes, and if from any point, in the same or opposite hyperbolas, there are drawn to the asymptotes other right lines parallel to the former; then the rectangle contained by the lines first drawn, shall be equal to the rectangle contained by the other drawn lines."

[38] Thus let there be a square, whose side is equal to three, and a parallelogram whose longest side is equal to four, and its shortest to two; the ambit of each figure will indeed be equal to twelve, but the

area of the square will be equal to nine, and of the parallelogram to eight.

[39]is will be evident by conceiving a rectangular parallelogram equal to that which is non-rectangular, described on the same base: for the ambit of the former will be less than that of the latter, and consequently less than the parallelogram, with an ambit equal to the non-rectangular parallelogram.

[40]om hence it is evident that it was the intention of Proclus to comment on the whole of Euclid: but it does not appear that he ever carried this design into execution.

[41]he present Commentary is imperfect, both in the Greek, and the translation of Barocius; who observes that the conclusion is wanting in all the copies which he had an opportunity of perusing. Those who are curious may consult his scholium, in which he has endeavoured to complete it.

[42]See Comment 8, of the third book, with its note.

[43]arocius is of opinion, that this Commentary was originally mutilated; and that the part which follows the word *drawn*, was added by some skilful geometrician, as necessary to the perfection of the demonstration. See his Scholium to this Commentary.

[44]e Commentary of Proclus, on this proposition is wanting in the Greek, and, as we are informed by Barocius in all the MS. copies which he had an opportunity of consulting. Barocius has endeavoured to supply this deficiency, after the manner of Proclus; but he appears to have fallen into prolixity, by a too minute division of the problem.

[45]See Sims. Sect. Con. Lib. I. Prop. 13. and Lib. II. Prop. 23, and Lib. III. Prop. 48.

[46]the sixth Commentary of this book.

[47]the 31st proposition.

[48]ασα γὰρ ἡ παρ' Ἑλλήσι θεολογία τῆς Ὀρφικῆς ἐς μυσαγωγίας ἔκγονος· πρώτου μὲν Πυθαγόρου παρὰ Ἀγλαοφήμου τὰ περὶ Θεῶν ὄργια διδασχθέντος. Procl. in Plat. Theol. p. 13.

[49]Lib. ii. Florid.

[50]seb. Prepar. Evang. lib. xiv.

[51]is pestilence was in the time of the emperor Gallienus, and raged so vehemently, according to Trebellius Pollio, that five thousand men perished with an equal disease in one day. This happened in the year of Christ 262, and of Gallienus, 9, 10; and not long after Porphyry applied himself to Plotinus. Fabricius.

[52]is is the Firmus Castricius to whom Porphyry inscribes his books on Abstinence.

[53]is was probably nothing more than a small serpent resembling the form of a dragon.

[54]is strange that Fabricius should think it ought to be entitled περὶ τῆς ἀπαθείας τῶν σωμάτων: for he who reads this book must see that such a title would be ridiculous. Vide Bibl. Grec. tom. iv. p. 143.

[55]is Rogatianus is the person Porphyry alludes to in his Treatise on Abstinence, lib. i. p. 106, when he speaks as follows. "There was once an instance, where a negligence of terrene concerns and a contemplation and intuition of such as are divine, expelled an articular disease, which had infested a certain person for the space of eight years. So that at the very same time, that his soul was divested of a solicitous concern for riches, and material affairs, his body was freed from a troublesome disease."

[56]ave, however, the happiness of being intimate with a lady, who is a noble exception to this remark; and is both an excellent Greek scholar, and skilled in the Platonic philosophy.

[57]or this is alone the virtue of traffic; and is the chief support of its professors.

[58]ncerning the Ἱεροὶ Γάμοι, or holy marriages, which were first celebrated by the Orphic Theologers, see the note to the hymn to Proserpine, in my translation of the Orphic Initiations.

[59]line somewhat altered from Homer. The original is βάλλ' οὕτως αἶ κέν τι φόως Δαναοῖσι γένηται. Iliad. 8. l. 282.

[60]the original.

[61]p. 6.

[62]de Procl. in Plat. Theol. p. 89, et 90.

[63]This passage is cited by Ficinus, in the second volume of his works, p. 1186. and is, I doubt not, taken from the manuscript Commentary of Proclus on the Parmenides.

[64]τοσ γάρ (i.e. Moderatus) κατὰ τοὺς Πυθαγορείους τὸ μὲν πρῶτον ἐν ὑπὲρ τὸ ὄν καὶ πᾶσαν οὐσίαν ἀποφαίνεται. Simp. in Ar. Phys. fol. 50.

[65]the note to page 14, of this volume, we have shewn from Proclus that light is place, and is an immaterial body. Indeed that light is something superior to sensible matter may, I think, be evinced by the following consideration. As the supreme principle of the universe, (who can be compared to nothing so properly as light,) is the light of the intelligible world, and is at the same time more exalted than every thing which it contains; so in this sensible, which is the image of the intelligible world, it is necessary that corporeal light, should be more excellent than any material nature. But as Proclus prosecutes this speculation on light, in the above mentioned passage, in a most admirable and uncommon manner; let us attend him in the abstruse investigation. In the first place speaking of that light which emanates from the fontane soul of the world, and which, according to the Zoroastrian oracle,

Ἄρδην ἐμψυχοῦσα φάος, πῦρ, αἰθέρα, κόσμους,

abundantly animates, light, fire, ether, and the world; he says that this light is one fire, above the three others, viz. the empyrean, etherial, and material fire; and that it first unfolds the eternal quiet of the gods, and illustrates essential spectacles to such as are worthy of their inspection. For by the assistance of this light, things destitute of figure and impression, are impressed according to reason and intelligence. And perhaps (says Simplicius) he calls light place, because it is a certain description and type of the universe, and gives distance to things, which without the presence of light would not appear to be distant. But here Proclus very philosophically doubts against his own position. In the first place, how body can be received and penetrated by body. In the second place, whether place, i.e. light, is inanimate, or participates of soul. But (says he) it cannot be inanimate, because it is better than the animated natures which it contains, and because rational beings are said to be animated. Hence this body, will be animated the first of all others. But if it is animated, how can it be immoveable? And he dissolves the first doubt, because there are some immaterial bodies void of affection and passion. For (says he) a body void of matter neither resists nor is resisted; since that which is impelled and repelled, is eminently passive in such operations. But neither can such a body be divided, since it is void of passivity. Hence neither can the absurdity be adduced, that the universe is received by a minimum, or penetrates a minimum. For if it cannot be divided, neither can it be equally divided with the smallest; and if this is impossible, neither can it be permeated by the universe.

But he removes the second objection, by asserting, that this body, i.e. light, is animated by that soul, which is the fountain of others; and that it possesses a divine life, and an essence self-motive, but not in energy. For if we assert that soul is self-motive, in a two-fold manner, in one respect according to essence, but in the other according to energy; we must likewise affirm that place, or light is partly moveable, and partly immoveable. For what should hinder our asserting, that place participates of such a life, and that it lives according to an immutable essence, and not according to a self-motive energy? But if you are desirous (says he) of considering the motion of place, according to energy, you will plainly perceive, that it is, as it were, the mover of the bodies which are moved, and which turn their parts according to a certain distance; since these cannot be every where, and are incapable of being present to all the parts of place, according to each of their parts. And this is that intervention and affinity, which it possesses with soul moving without dimension. For, life so far as life, appears to produce motion: and since place is that which first participates of life, motion to place, will confer a true motion on every part; and will produce a desire in every part of the moved body of arriving at its whole, and since it is not able to accomplish this, on account of the natural property of interval, of subsisting divisibly in the universe. For whatever desires any nature, which it is not able to reach, through its peculiar defect, is then more enflamed in the pursuit of that, which it

cannot possess through the imbecility of its nature. For it is requisite (says he) that between an incorporeal and immutable life, such as that of soul, the fountain of the rest; and one corporeal and mutable, that one immutable and corporeal should intervene as a proper medium. And thus much from Proclus concerning place or light, whose reasonings appear to me equally elegant and philosophical, subtle and profound.

But the opinion of the Phœnicians respecting light, as preserved by the emperor Julian in his elegant oration to the sun, is no less admirable than that of Proclus; and at the same time asserts its immaterial nature. “According to the Phœnicians (says Julian) who are skilled in divine science and wisdom, the universally-diffused splendor of light, is the sincere energy of an intellect perfectly pure: i.e. of the solar intellect, which as Julian expresses it, scattering its light from the middle region of the heavens, fills all the celestial orbs with powerful vigour, and illuminates the universe with divine and incorruptible light.” With great reason, therefore, does Plotinus assert, that light is nothing else than that which is visible: for this must be the necessary property of a nature, possessing a triple dimension without sensible matter.

[66] 2. l. 9. c. i.

[67] Intellectual System, p. 562.

[68] Timæum. p. 93. 94.

[69] Plat. Theol. p. 158

[70] Plat. Theol. p. 27.

[71] Inead, 5. p. 492.

[72] Intellectual System, p. 568.

[73] Prepar. Evan. lib. iv. c. 6.

[74] b. x. cap. 29.

[75] It is necessary to inform the genuine Platonic reader, that Synesius was not a bishop, and had not embraced Christianity, when he composed the present excellent book. This is not only evident from the recondite wisdom of antiquity, which it every where displays, but from his expressly using in one place, p. 151, the epithet, *the mundane god* (θεὸς ἐγκόσμιος) in the same manner as it is always employed by heathen Platonists. There are three Latin versions of this admirable work: the first and best by Ficinus, the next both in the order of time and excellence, by Cornarius, and the last and worst by Petavius. This industrious Jesuit, indeed, though sufficiently learned on other subjects, appears by his version to be very ignorant of the Greek philosophy. This will be evident by comparing his translation with the Greek, and, I hope, I may add, with the above version, which contains the greatest, and most important part of this inestimable work. With respect to Synesius, he was a native of Cyrene, in Africa; travelled into Egypt for improvement; and perfected his studies under the celebrated Hypatia: a woman no less eminent for her uncommon abilities, than remarkable for her tragical death, effected by the infernal revenge of the Alexandrian church, and the *orthodox, patriarchal, Christian* malice of St. Cyril. Though Synesius was reluctantly consecrated bishop of Ptolemais about the year 410, yet we may collect from his tenets, that he was not a perfect convert to Christianity; and from his epistles, that he lamented his episcopal station. For with respect to the former, he denied the immediate creation of the world, its final destruction, and the resurrection of the dead: and with regard to the latter, he frequently and earnestly begs to be discharged from his office; and declares that as he was by education a heathen, and by profession a philosopher, he had met with no success since he presumed to serve at the altar. Particularly in a letter to his friend Olympius (Epist. 95.) he declares that if his duty as a bishop should be any hinderance to his philosophy, he would relinquish his diocese, abjure his orders, and remove into Greece. For farther conviction in this particular consult Epist. 11, 13, 57, 67, 80, 105: and the Clidophorus, and Hypatia of Toland, who largely and curiously descants on the life and writings of Synesius. I shall only add, that even Petavius, the editor of his works, affirms, that in some of the books composed after his profession of Christianity, Synesius appears as great a heathen as before: and this conduct may be reconciled, by what Synesius himself asserts in one of his letters: (Epist. 105.) “that it may be sometimes expedient to lye, in order to do good; since philosophical truth is not necessary for the vulgar, who may receive hurt from their knowledge. *Wherefore* (says he) *if this method*, (that is of concealing the truth), *be consistent with the rights of episcopal dignity, I may be consecrated. I shall*

freely philosophize at home, and tell fables abroad; neither teaching, nor unteaching, but suffering people to live in the prejudices they have imbibed." This must be allowed to be sound philosophy, excellent policy, and true orthodoxy!

[76] This is excellently amplified by Porphyry in Senten. 32. p. 232, "As the soul's residence on the earth, (says he) does not consist in being conversant in the earth, as bodies, but in presiding over bodies situated on the earth: so likewise the soul is said to be in Hades, when she presides over her image, which has a nature accommodated to place, but then obtains a subsistence in darkness. Hence if Hades is a subterranean dark place, the soul though not separated from being, dwells in this case in Hades, attracting to herself her image. For the spirit which she had collected from the spheres, attends her in her departure from a solid body. But from her affection towards body, reason having a partial object, according to which, she obtained a habitude towards a body of this kind, while she lived; from this propense affection, a form of the phantasy is impressed on the spirit, and so she attracts the image. But she is said to be in Hades, because she obtains a spirit of a formless and dark nature: and since a heavy and humid spirit, extends to subterranean places, on this account also the soul is said to dwell under the earth; not because her essence passes from place to place, and subsists in place, but because it receives the habits of bodies endued with a natural inclination to local transitions, and the possession of place. And bodies of this kind indeed, receive the soul, according to aptitudes, from a certain disposition towards her nature. For according to her particular disposition, she finds a body defined, correspondent to her order and properties. Hence when the soul is in a more pure condition of being, a body is natural to her, approximating very nearly to that which is immaterial; such as an ethereal body. But when she proceeds from reason to the object of imagination, she naturally obtains a solar-form body; and when effeminated and captivated with the love of forms, she is united with a lunar-form body. Lastly, when she falls into bodies composed from humid vapours, a perfect ignorance of being succeeds, and darkness and infancy. And indeed in her egress from body, when she retains a spirit disturbed by a humid evaporation, she attracts a shade, and is weighed down; a spirit of this kind, endeavouring to dwell naturally in the bottom of the earth, unless some other cause draws it into a contrary place. As therefore when surrounded with this terrene shell, it is necessary she should reside on the earth; so likewise when she attracts a humid spirit, it is necessary she should be surrounded with the image. But she attracts moisture, when she continually studies to be present with a nature, whose operation consists in moisture, and is mostly subterranean. But when she endeavours to depart from nature, she becomes a dry splendor, without a shadow, and without a cloud: for humidity constitutes a cloud in the air; but dryness produces from vapour, a dry splendor."

[77] This *sacred discourse* composed by Pythagoras was two-fold; one in verse, mentioned by Heraclides, and the other ΠΕΡΙ ΘΕΩΝ, in Doric prose, both which are unfortunately lost. It appears, however, from the present passage, that a part of one of them consisted in explaining the labours of Hercules, who, like Ulysses, is an allegorical character, representing the progress of a man from the impurity of a sensible life, till he acquires the perfection and purity of a life intellectual and divine. Hence Proclus on Plato's Republic, p. 382. "Hercules being purified by sacred initiations, and having acquired undefiled advantages, deserved a perfect establishment among the gods." We may conceive, therefore, that by the club of Hercules is meant philosophy, and by his lion's skin, prudence; through whose assistance he tamed the passions, those monsters of the soul, and destroyed vain cogitations: both which are occultly signified by the twelve labours he endured. I only add that Petavius appears to have been entirely ignorant, that any such writings as the *sacred discourses*, ever existed: for he translates them in one place *sacræ historiae*, and in another, where they are mentioned in the singular number, (λόγος) *nostra hæc oratio*. Such ignorance may be excusable in the modern priest; but as a man of learning Petavius cannot be defended by any apology in such vile translations.

[78] The Scholium of Nicephorus upon this passage is as follows: "Synesius (says he) calls the dross of matter, that which the phantasy, descending from the supernal spheres, derives from the natural elements of fire and air. And this he observes it is not lawful to leave in the precipice of matter, viz. in the world every way obscure and dark, but it is requisite to draw it upwards, by the assistance of temperance, continence, and the other virtues, and re-elevate it to the ethereal world; since there is a place allotted for nature, and the image, in the region every where lucid. But he calls the *image*, the phantastic spirit, as being connate to the rational soul, though of a subordinate nature. For as *intellect* is the resemblance and image of the divinity, but the *rational soul* of intellect: so the *phantastic spirit*, or irrational soul, is the image of the rational soul. Hence, as *nature* or the natural soul, is the

image of the phantastic spirit, and of this again *the body*, and of body, *matter*: on this account he observes that the phantasy obtains a certain familiar proportion to the natures which communicate with it on each side, by a certain union and affinity, and by a tendency to the same common end. Hence in its descent it not only acquires fire and air, from the rational soul, which has a supernal situation, but likewise from the natural elements, which are placed beneath.” Nicephori Scholia, p. 390.

[79] Synesius says that there are two extreme seats, the summits of the universe, one beneath, every way dark, the other above, wholly luminous; and that the former is miserable, but the latter blessed. But because opposites cannot be immediately copulated together, he asserts that in the middle of these two seats, there are many other regions, differing from each other, which he denominates in one part luminous, and in the other obscure, participating of the extremes, and beneath the region totally resplendent with light. Just in the same manner as *cogitation* is the medium between intellect and soul: but *opinion* is seated between the phantasy and the soul; and *sense* between the phantasy and body. And these mediums mutually penetrate, and extend through each other, from body to intellect, and participate all their surrounding natures. As air possesses on each side two extremities and seats, by means of which, it is hot in its supreme situation, through its proximity to elementary fire, but humid in its inferior extremity, through its vicinity to water.’ Nicephori Schoia, p. 392.

[80] The Greek is very defective in this place, which I have endeavoured to supply, as the learned reader may observe.

[81] Pope’s Odyssey, book xix.

[82] There are three editions of this excellent work. The first Greek and Latin by Holstenius, Cantab. 1655; the second by Barnes, prefixed to his Homer; and the third by some German editor, which I have not seen.

[83] e. about ten Italian miles.

[84] z. a mile and a half.

[85] These verses are not to be found in any of the hymns now extant, ascribed to Homer.

[86] Macrobius in the 12th chapter of his comment on Scipio’s dream, has derived some of the ancient arcana which it contains, from the present part of this admirable work. What he has farther added, I shall translate on account of its excellence, and connection with the above passage of Porphyry. “Pythagoras (says he) thought that the empire of Pluto, began downwards from the milky way, because souls falling from thence, appear already to have receded from the gods. Hence he asserts, that the nutriment of milk is first offered to infants, because their first motion commences from the galaxy, when they begin to fall into terrene bodies. On this account, since those who are about to descend, are yet in *Cancer*, and have not left the milky way, they rank in the order of gods. But when by falling they arrive at the *Lion*; in this constellation, they enter on the exordium of their future condition. And, because in the *Lion*, the rudiments of birth, and certain primary exercises of human nature commence; but *Aquarius* is opposite to the *Lion*, and presently sets, after the *Lion* rises: hence, when the sun is in *Aquarius*, funeral rites are performed to departed souls; because he is then carried in a sign, which is contrary, or adverse to human life. From the confine, therefore, in which the zodiac, and galaxy touch each other, the soul descending from a round figure, which is the only divine form, is produced into a cone by its defluxion. And as a line is generated from a point, and proceeds into length, from an indivisible; so the soul from its own point, which is a monad, passes into the duad, which is the first protraction. And this is the essence which Plato in the *Timæus*, calls indivisible, and at the same time divisible, when he speaks of the fabric of the mundane soul. For as the soul of the world, so likewise that of man will be found in one respect ignorant of division, if the simplicity of a divine nature is considered; and in another respect capacious of division, if we regard the diffusion of the former through the world, and of the latter through the members of the body.

As soon, therefore, as the soul gravitates towards body, in this first production of herself, she begins to experience a material tumult, that is, matter flowing into her essence. And this is what Plato remarks in the *Phædo*, that the soul is drawn into body, staggering with recent intoxication; signifying by this the new drink of matter’s impetuous flood, through which the soul becoming defiled and heavy, is drawn into a terrene situation. But the starry *cup*, placed between *Cancer* and the *Lion*, is a symbol of this mystic truth, signifying that descending souls first experience

intoxication in that part of the heavens, through the influx of matter. Hence, oblivion the companion of intoxication, there begins silently to creep into the recesses of the soul. For if souls retained in their descent to bodies, the memory of divine concerns of which they were conscious in the heavens, there would be no dissension among men, concerning divinity. But all, indeed, in descending drink of oblivion; though some more, and others less. On this account, though truth is not apparent to all men on the earth, yet all exercise their opinions about it: because *a defect of memory, is the origin of opinion*. But those discover most, who have drank least of oblivion: because they easily remember what they had known before in the heavens. Hence, that which is called *lectio* by the Latins, is called by the Greeks ἀνάγνωσις, or *repeated knowledge*; because when we learn any truths, we recognize what we naturally knew, before material influxion, rushing into the body, had intoxicated the soul. But it is this *hyle* or matter which composes all that body of the world, which we every where perceive adorned with the impressions of forms. Its highest and purest part, however, by which divine natures, are either sustained, or composed, is called nectar, and is believed to be the drink of the gods: but its more inferior and turbid part, is the drink of souls. And this is what the ancients have denominated the river of Lethe. But according to the Orphic writers the νοῦς ὑλικός, or *material intellect*, is Bacchus, who proceeding from that indivisible part, is divided into particulars. Hence, in the Orphic mysteries, he is reported to have been torn in pieces, by Titanic fury, and the fragments being buried, are said to have risen entire, and collected into one; because intellect by passing into a divisible from an indivisible nature, and again returning from divisible to indivisible, both accomplishes the duties of the world, and does not desert the arcana of its own nature.

The soul, therefore, falling with this first weight, from the zodiac, and milky-way into each of the subject spheres, is not only clothed with the accession of a luminous body, but produces the particular motions, which is to exercise in the respective orbs. Thus in Saturn, it energises according to a ratiocinative and intellective power, which they call λογικὸν and θεωρητικόν: in the sphere of Jove, according to the power of acting, which is called πρακτικόν: in that of Mars, according to the ardour of courage, which is denominated θυμικόν: in the orb of the sun, according to a sensitive and phantastic nature, which they call αἰσθητικὸν and φαντασικόν: but according to the motion of desire, which is denominated ἐπιθυμητικόν, in the planet Venus: of pronouncing and interpreting what it perceives, which is called ἑρμηνευτικόν, in the orb of Mercury; and according to a plantal nature, and a power of acting on body, which is denominated φυτικόν, when it enters the lunar globe. And this sphere, as it is the last among the divine orders, so it is the first in our terrene situation. For this body, as it is the dregs of divine concerns; so it is the first substance of an animal. And this is the difference between terrene and supernal bodies (under which last, I comprehend the heavens, the stars, and the other elements) that the latter are called upwards to be the seat of the soul, and merit immortality from the very nature of the region, and an imitation of sublimity; but the soul is drawn down to these terrene bodies, and is on this account reported to die, when it is inclosed in this fallen region, and the seat of mortality. Nor ought it to cause any disturbance, that we have so often named the death of the soul, which we have pronounced to be immortal. For the soul is not extinguished by its own proper death, but is only overwhelmed for a time. Nor does it lose the benefit of perpetuity, by its temporal demersion: since when it deserves to be purified from the contagion of vice, through its entire refinement from body; it will be restored to the light of perennial life, and will return to its pristine integrity and perfection.”

[87]ad. lib. xx. l. 223, &c.

[88] the original ἰζᾶσιν οὖν καὶ σύμβολον τῆς μεσημβρίας καὶ τοῦ νότου, ἐπὶ ἣ θύρα μεσημβριάζοντος τοῦ θεοῦ. Which Holstenius translates most erroneously, as follows: “Austrum igitur meridei symbolum statuunt: cum Deus meridiano tempore ostio immineat.”

[89]ad. lib. ix. l. 579.

[90]ad. lib. viii. l. 395.

[91]ad. lib. viii. l. 393.

[92]is assertion of Pythagoras that *the people of dreams*, δῆμος ὀνειρώων, are souls situated in the milky way, admirably contributes to elucidate the following passage in the 24th book of the Odyssey, respecting the descent of the suitors’ souls to the region of spirits:

Πὰρ δ' ἴσαν Ὠκεανοῦ τε ῥοὰς καὶ Λευκάδα πέτρην,
Ἡ' δὲ παρ' ἡελίοιο πύλας καὶ δῆμον ὄνειρων
Ἦσαν, αἴψα δ' ἵκοντο κατ' ἀσφοδελὸν λειμῶνα,
Ἐνθά τε ναίουσι ψυχαὶ εἰδῶλα καμόντων.

i.e. “But they passed beyond the flowing waters of the ocean, and the rock Leucas, and the gates of the sun, and *the people of dreams*: and they immediately came into meadows of asphodel, where souls the images of the dead reside.” For it is evident from hence that the souls of the suitors passed through the galaxy, or the seats of the blessed, according to the most ancient theology: and I doubt not but Homer describes in these lines the complicated progression of an impure soul till it regains its original habitation in the stars, and again begins to gravitate to this terrene abode. This, I presume, will be manifest from the following elucidation of these admirable lines.

In the first place these souls are said to pass over *the flowing waters of the ocean*, and *the Leucadian*, or *white rock*. Now by this nothing more is meant than the flight of the suitors' souls to the extremity of the earth, in order to a subterranean descent: for, according to the most ancient opinion, the earth is bounded by the ocean; and the Leucadian rock may, as Eustathius observes, be some rock on the earth's extremity, which receives the last rays of the sun. Afterwards they are said to pass through *the gates of the sun*, by which, as Porphyry informs us above, we must understand the tropics of Cancer and Capricorn: and as Capricorn is subterranean, and affords a passage to ascending immortals, we must conceive that they enter through this prior to the tropic of Cancer. But in order to comprehend the perfect propriety of this transition, we must observe that the souls of the suitors on account of their impurity, are punished in the recesses of the earth, before they enter the celestial tropics, and pass into the meadows of Pluto. This the poet evidently evinces by the *screaking noise*, which they utter, and the squalid paths, through which they descend: a noise of this kind as Proclus well observes, (in Plat. Repub.) “representing a species of life solely given to appetite and imagination.” After they have been purified therefore by subterranean punishment, they are fit to ascend to *the people of dreams*, or the souls of the blessed situated in the milky way. However as the soul, on account of her middle nature, is incapable of a perpetual sameness of situation, but is formed for infinite circulations (as will be demonstrated in the following elements); hence Homer, without mentioning her duration among the gods, though it is doubtless very extended, agreeable to the mystic brevity of his Muse, makes her immediately pass into *the meadows of Asphodel, where souls the images of the dead reside*. Now these *meadows of Asphodel*, form the supreme part of Plato's dominions: for, according to Pythagoras, as we are informed by Macrobius, in the preceding note to page 287, the empire of Pluto commences downwards from the milky way; so that these meadows are most probably situated in *the Lion*, the constellation into which souls first fall, after they leave the tropic of Cancer. But the description of the Asphodel, perfectly corresponds with, and confirms the preceding exposition. For the Asphodel is a plant, bearing a naked *starry flower*, which comprehends an ovarium, or *orbicular figure*, after the manner of a cup. And what can more aptly symbolize with the stars, than such a flower? It was usual too with the ancients to scatter this plant in the tombs of the deceased; occultly intimating most probably, the similitude of this flower to the pristine and proper habitations of the soul. And hence it derived its appellation from σποδός, *ashes*, διὰ τὴν τῶν καιομένων νεκρῶν σποδόν, i.e. *from the ashes of burnt dead bodies*.

In these meadows, then, *the images of the dead* are said to reside. Now these εἰδῶλα, or *images*, are no other than those vehicles of the soul, which have been so copiously, and admirably discussed by Synesius, in the preceding quotation from his book on dreams, and which, from their residence in the starry regions, must be luciform, ethereal, and pure. It is this phantastic spirit, or primary vehicle of the soul, which Virgil alludes to, in these beautiful lines:

—exinde per amplum
Mittimur Elysium, et pauci læta arva tenemus:
Donec longa dies perfecto temporis orbe
Concretum exemit labem, purumque reliquit
Ætherium sensum, atque auræ simplicis iguem.

i.e. “We are afterwards sent through ample Elysium, and a few of us possess the joyful plains: till a long period, when the revolving orb of time has perfected its circulation, frees the soul from its concrete stains, and leaves the ethereal sense pure, together with *the fire (or splendour) of simple æther*.” For here he evidently conjoins the rational soul, or *the ethereal sense*, with its splendid vehicle, or *the fire of simple ether*; since it is well known that this vehicle according to Plato, is

rendered by proper purgation αὐγοειδέες, or *luciform*, and divine. It must here however be observed that souls in these *meadows of asphodel*, or *summit of Pluto's empire*, are in a falling state; or in other words through the secret influx of matter, begin to desire a terrene situation. And this explains the reason why Hercules in the infernal regions, is represented by Homer boasting of his terrene exploits and glorying in his pristine valour: why Achilles laments his situation in these abodes; and souls in general are engaged in pursuits, similar to their employments on the earth: for all this is the natural consequence of a propensity to a mortal nature, and a desertion of the regions every way lucid and divine. Let the reader too observe, that, according to the arcana of the Platonic doctrine, the first and truest seat of the soul, is in the intelligible world, where she lives entirely divested of body, and enjoys the ultimate felicity of her nature. And this is what Homer divinely insinuates when he says:

Τὸν δὲ μέτ', εἰσενόησα βίην Ἡρακλεΐην,
Εἶδωλον· αὐτὸς δὲ μετ' ἀθανάτοισι θεοῖσι
Τέρπεται ἐν θαλίῃς, καὶ ἔχει καλλίσφυρον Ἥβην.—κ. τ. λ.

i.e. “after this I saw the Herculean power, or *image*: but Hercules himself is with the immortal gods, delighting in celestial banquets, and enjoying the beautiful-footed Hebe.” Since for the soul to dwell with the gods entirely separated from its vehicle, is to abide in the intelligible world, and to exercise, as Plotinus expresses it, the more sacred contests of wisdom.

Should it be enquired why departed souls, though in a state of felicity are compared by Homer to *dreams* and shadows, I answer with Porphyry (apud Stob. p. 132.) that they are shadows with respect to human concerns, both because they are destitute of body, and are void of memory: for after they have passed the Stygian river, they are entirely ignorant of their pristine life on the earth, though they recognize, and converse with each, other, as is evident from the discourses between Patroclus, Ajax, and Antilochus. Indeed together with memory, they lose all knowledge of corporeal resemblances, which are rendered apparent through the ministry of the phantasy. For since the phantasy consists from memory, as Plato asserts in the *Phelebus*; whatever we imagine perishes with the memory; and when this is taken away all the perturbations of the soul are removed, as she then becomes wholly intellectual and passes into a state divinely prudent and wise. However, by means of the blood, which, as we have before observed, is, according to Homer, the instrument of the phantastic soul, departed spirits recognize material forms, and recollect their pristine condition on the earth. And to the phantasy reasoning pertains; since it is nothing more than an aggregation of memory, collected through imaginations, into the judgment of universals. But this is very different from the *intellective energy*, acquired by the soul beyond Acheron, which Cocytus and Pyriphlegethon fill, from the whirling streams of the dreadful Styx. Let the reader, however, remember that the phantasy is two-fold, communicating in its supreme part with the rational soul, and in its inferior part with sense; and that it is this inferior part which the soul deserts, when it acquires an intellectual condition of being.

[91] ad xxiv. l. 527.

[92] This was no doubt fully proved by Porphyry, in some of his unfortunately lost writings; such as his book *on the philosophy of Homer*; or that, *on the Allegories of the Grecian and Egyptian Theology*, of which we have already made mention. However, it does not seem impossible, from the hints afforded us in this excellent treatise, for a person conversant in the Platonic philosophy, to evince the truth of this assertion. Presuming, therefore, that an attempt of this kind will be acceptable to the liberal reader, though my abilities are far inferior to those of Porphyry; I shall request his attention to the contents, and his pardon for the length of the ensuing discourse. I only promise, that I shall make use of a small treatise in Greek, on the wanderings of Ulysses, by an anonymous author, where he appears to have penetrated the sense of the allegory; and freely reject his interpretation, when foreign from the leading character of Ulysses, above mentioned, according to Numenius and Porphyry.

The first apparently fabulous adventure then of Ulysses, is that of the Lotophagi; a coast upon which he was driven by the violence of the winds; and which produced a fruit called Lotos, whose enchanting taste caused oblivion, and a vehement desire of perpetually abiding with the Lotophagi. Or as it is beautifully described by Homer, in the elegant version of Mr. Pope:

The trees around them all their fruit produce,
Lotos the name, divine, nectarious juice!
(Thence call'd Lotophagi) which whoso tastes,
Insatiate riots in the sweet repasts,
Nor other home, nor other care intends,
But quits his house, his country, and his friends:
The three we sent from off th' enchanting ground
We dragg'd reluctant, and by force we bound:
The rest in haste forsook the pleasing shore,
Or, the charm tasted, had return'd no more.

Lib. ix. l. 94. &c.

Here we view Ulysses subject to different desires, and blown about by contrary blasts of fortune, inconsiderately permitting the various affections of his soul to be engaged in improper employments; who, after they have tasted of sensual pleasures, and fraudulent manners, refuse to return to ruling intellect, and the natural good habits of the soul. Hence, fascinated by delight, they consign their country and kindred to oblivion; and desire to live forever lost in the intoxication of pleasure. Reason, however, no longer sustaining that her dignity should be obscured by depraved enjoyment, immediately recalls the affections from fraudulent deeds; which is represented by Ulysses withdrawing his men from the coasts of Lotophagi. After this she binds and restrains them, by the most weighty arguments, so as to prevent them from afterwards returning to evil. And indeed, she treats the affected parts of the soul, in a manner accommodated to their disease; and strengthens and fortifies such as are not affected, and removes them from the possession of noxious delight.

The next adventure of Ulysses is that of the Cyclops, whom he deprived of sight, and irritated with reproaches. Now, according to Porphyry in the above excellent treatise, this is no other than a malignant natal dæmon, who destroys by his stratagems the good affections of the soul; which is occultly signified by Ulysses loosing his companions, through the brutal appetite of the destructive dæmon. Ulysses, however, lamenting the dreadful effects of his voluntary confinement, meditates, and at length accomplishes his deliverance, together with that of the remaining good habits of his soul; and glories in resuming the liberty he has lost. But this fable indeed contains one of the greatest arcana of philosophy: for this is what Synesius occultly intimates, in the preceding quotation from his book on dreams; when he speaks of matter marking her votaries in her secret volume, and severely punishing such as desert from her service. Thus Ulysses after a voluntary submission to his natal dæmon, by indulging the irrational appetites and desires of his soul, flies from his base servitude; and adds irritations to his flight. He is, however, pursued by the anger of the marine and material dæmons, and punished for his escape. For he who blinds the eye of sense, and extinguishes its light, after his will has profoundly assented to its use, must expect punishment for the attempt; as necessary to his own private good, and the general order of the universe. Indeed, troubles and misfortunes resulting from such undertakings, not only contribute to appease the anger of their malevolent authors, but likewise purify and benefit the subjects of their revenge. He, therefore, who in the present life, perceives himself like Ulysses, attended by uncommon misfortunes, while he is in the road of virtue, and eagerly searching for wisdom, may safely conclude, that either here, or in a prior state of existence, he has voluntarily submitted to the power of his natal dæmon, and has now deprived him of sight; and that he has been profoundly delighted with the nature of matter, and is now abrogating the confessions which he made. This too is insinuated in the beautiful story of Psyche, by Apuleius, when the terrestrial Venus, sends Mercury, with a book in which her name is inscribed, to apprehend Psyche as a fugitive from her mistress. For this whole story relates to the descent of the soul into this terrene body, and its wanderings and punishments till it returns to its true country and pristine felicity. And this I may probably demonstrate in some future period, by publishing a translation of this admirable fable, and a comment on the divine mysteries it contains. We must here, however, observe, that as the advancements of Ulysses in virtue are but moderate, till he leaves Calypso; so the greatest of his troubles commence after that period, as our discourse will shortly evince.

In the next fable, which is that of Æolus, a being, as the name implies, various and versatile, though hospitable and benignant, we see Ulysses sorrowful and wandering, through the anger of his natal dæmon, enquiring after a refuge from misfortune; though not as becomes one studious of piety, but committing himself to enchanters and magicians, and relying on their incantations for his deliverance from danger. He cannot, however, accomplish his end, by such undertakings, but remains frustrated of his hope, and filled with shame for his disappointment. The Poet too, by

attributing the ill success of Ulysses to his sleep, egregiously insinuates that the rational soul was in a dormant state, when he confided in practices so incapable of producing the desired end, and so inconsistent with the goods which intellect confers. Such methods, indeed, as they increase the desire of success, so they present strongly to our view, the distant object of our pursuit; but this is immediately succeeded by the sleep of reason, and the destruction of hope. And this is what Homer, appears to signify, by the following verses:

Nine prosp'rous days, we ply'd the lab'ring oar;
The tenth presents our welcome native shore:
The hills display the beacon's friendly light,
And rising mountains gain upon our sight.
Then first my eyes by watchful toils opprest,
Comply'd to take the balmy gifts of rest;
Then first my hands did from the rudder part,
(So much the love of home possess'd my heart.)
Lib. x. l. 28, &c.

And hence;

—— ——— the thongs unbound,
The gushing tempest sweeps the ocean round;
Snatch'd in the whirl, the hurried navy flew,
The ocean widen'd, and the shores withdrew.

After this succeeds the adventure of the Lestrigons which seems to indicate the yet imperfect condition of Ulysses' nature, unable to distinguish the coast of virtue, from the infamous regions of vice. Hence he becomes an involuntary prey to the depredations of depraved manners; and is for some time incapable of exerting the power of reason. However, at length perceiving the magnitude of the evils with which he is surrounded, he cuts the detaining cables of vice, and flies from his dangerous situation; deploring, indeed the ruined state of his better manners, but rejoicing that his principal part has escaped, and that he is not totally destroyed. And this the poet appears to me to insinuate by the following verses:

Whilst thus their fury rages at the bay,
My sword our cables cut, I call'd to weigh;
And charg'd my men, as they from fate would fly
Each nerve to strain, each bending oar to ply.
The sailors catch the word, their oars they seize,
And sweep with equal strokes the smoky seas;
Clear of the rocks th' impatient vessel flies;
Whilst in the port each wretch encumber'd dies.
With earnest haste my frightened sailors press,
While kindling transports glow'd at our success;
But the sad fate that did our friends destroy
Cool'd every breast, and damp'd the rising joy.
Lib. x. l. 125, &c.

In the next fable, which is the beautiful allegory of Circe, we shall find some deep arcana of philosophy contained, exclusive of its connection with Ulysses. By the Æean island then, in which the palace of Circe was situated, we must conceive the region of sorrow and lamentation; for this word is evidently derived from the interjection αἶ, *alas!* and the adjective αἰνὴς, *lamentable*. And by Circe we must understand the goddess of sense: for thus Porphyry in Stobæus p. 141. Ὅμηρος δὲ τὸν ἐν κύκλῳ περίοδον καὶ περιφορὰν παλιγγενεσίας, Κίρκην προσηγόρευκεν, ἡλίου παῖδα, τοῦ πᾶσαν φθορὰν γενέσει, καὶ γένεσιν αὖ πάλιν φθορᾷ συνάπτοντες αἰεὶ καὶ συνείροντος. i.e. "Homer calls Circe, the daughter of the sun, the period and revolution of regeneration in a circle, who ever connects and combines all corruption with generation, and generation again with corruption." Hence, we may observe that the Æean isle, or this region of sense, is with great propriety called the abode of trouble and lamentation. In this region then, the companions of Ulysses, that is, the thoughts and natural powers of his soul, are changed by the incantations of the goddess; and his opinions and natural motions, rashly wandering from the authority of ruling intellect, are converted through the allurements of delight, into an unworthy and irrational habit. Ulysses, however, or the rational soul, is by the assistance of Mercury, or reason, prevented from destruction. Hence, intellect roused by its impassive power, and recollecting the ills which its natural faculties endure; at the same time, being armed with prudent anger, and the plant moly, or virtue, which is able to repel the allurements of pleasure, wars on the goddess of sense, and prevents the effects of her fascinating charms. Nor is

reason alone free from the dire incantations of delight, but it likewise restores to their proper form the powers of nature, which had been previously corrupted; and thus departs a gainer by its loss. For he who returns to himself from the dominion of vice, derives at least this advantage in his return, that he becomes afterwards more prudent in resisting its incursions, and employs his first defeat as an incitement to the acquisition of virtue. It must here, however, be observed that Ulysses is an involuntary offender, in all his adventures posterior to that of the Cyclops. His passions, indeed, hurry him into various vices and misfortunes, but his will by no means concurs with their endurance. But his connection with his natal dæmon was voluntary; and after his departure from hence, he must be considered as in a gradual course of purification, though his progress in virtue is but small, till the latter part of his abode with Calypso.

But Homer's account of Circe, exclusive of its relation to Ulysses, contains, according to Porphyry (in Stob. p. 141), an admirable explanation of the soul. "For thus, (says he) Homer speaks:

No more was seen the human form divine;
Head, face, and member, bristle into swine:
Still curst with sense, there *minds* remain alone,
And their own voice, affrights them when they groan.
Lib. x. l. 239, &c.

This fable, therefore, is the enigma of the opinions of Pythagoras and Plato respecting the soul; signifying that it is of an incorruptible and eternal nature, but not void of passion and mutability; since by dissolution and death, it is capable of being transmuted and changed, into other corporeal forms; and by its desire of pleasure it pursues a form adapted and allied to the condition of its life. And in this the assistance of learning and philosophy is perceived, if the soul, mindful of what is honest, and disdaining base and unlawful pleasures, can govern and defend herself from being changed into a beast; and from embracing a brutal and impure body, which increases and nourishes a nature dull and irrational, together with desire and anger, rather than reason. Indeed, the order and nature of this transmutation is predicated by the dæmon of Empedocles, when he says:

Σαρκῶν ἀλλόγνωντι περιέλλουσα χιτῶνι.
Καὶ μετ' ἀμπέχουσα τὰς ψυχάς.

i.e. *folding round the external garment of flesh; and afterwards investing souls with its covering.* But the Æean isle, which receives the dead body is that part of the continent, into which souls first descending wander and lament, and are ignorant

—— ——— what coast before them lies
Or where the sun shall set, or where shall rise.

Indeed, since through the love of pleasure, they desire an association, and nourishment in the flesh, and in conjunction with its nature, they again fall into the confused mixture of generation, truly mingling things eternal and mortal, prudence and passion, celestials, and terrestrials; ensnared and fascinated by pleasures, again leading to the fluctuating realms of generation. And in this case, souls particularly require the greatest felicity and prudence; lest pursuing the most base concerns, and becoming bound to their parts and passions, they obtain an unhappy and brutal life. For that which is called the τριῶδος, or triple path of Hades, is perceived in the *rational*, *irascible*, and *desiderative* parts of the soul; each of which contains the principle of a life convenient to its nature. And these assertions are not to be reckoned the figments of fables and the inventions of poets, but are to be esteemed as true and natural discourses. For those whose *desires*, in their mutation and generation, obtain the principality, will be changed into assinine bodies, and an impure life, through the dominion of gluttony and lust. But when the soul raging with weighty contentions, and odious cruelties, seeks a second generation, it betakes itself full of recent severity, into the nature of a wolf, or lion; acquiring a body of this kind as a defensive organ, adapted to its ruling affection. Hence, it is requisite that every one should be pure with respect to death, as in the sacred mysteries of initiation, by banishing every depraved affection, mitigating every desire, and expelling envy and anger from all connection with the body. And this is the true Mercury with his rod of gold, the clear indicator of honest conduct, who entirely prohibits and restrains the soul from the mixture of generation; or if she should drink the envenomed potion, preserves her in a human life, as long as can possibly be effected."

After this follows the allegory, respecting the descent of Ulysses into the infernal regions, which, exclusive of its connection with Ulysses, contains likewise some of the greatest arcana of the Grecian theology. As it respects Ulysses, it appears to me to insinuate, his flying to the assistance of necromancy, in order to know the result of the ills with which he is surrounded, through the anger of his natal dæmon. Hence Tiresias is nothing more than a departed spirit evoked by magical art, for the purpose of disclosing the secrets of futurity, and informing Ulysses how he may return to the true empire of his mind. The success, however, was not answerable to the certainty of the information: and perhaps Homer meant to intimate by this allegory, that the end of such illicit practises is never correspondent to the desires by which they are undertaken. Hence he plainly indicates the madness of such a conduct, by the consequences which may possibly attend its execution; and by the horror which forced Ulysses to hasten its conclusion: for thus Ulysses speaks:

Curious to view the kings of ancient days,
The mighty dead that live in endless praise,
Resolv'd I stand; and haply had survey'd
The god-like Theseus, and Perithous' shade;
But swarms of spectres rose from deepest hell,
With bloodless visage, and with hideous yell,
They scream, they shriek; sad groans and dismal sounds
Stun my fear'd ears, and pierce hell's utmost bounds.
No more my heart the dismal din sustains,
And my cold blood hangs shiv'ring in my veins;
Lest Gorgon rising from th' infernal lakes,
With horrors arm'd, and curls of hissing snakes,
Should fix me stiffen'd at the monstrous sight,
A stony image, in eternal night!

Lib. ii. l. 627. &c.

Indeed by such a conduct, he becomes impious, profane, and execrable; till he returns to that condition of mind, in which the judgement of reason, and the light of intellect emerges through the gloom of impiety and folly; and prudent cogitations dance round the liberated soul. For in this case it may be truly said:

Here the gay morn resides in radiant bow'rs,
Here keeps her revels with the dancing hours.
Lib. xii. l. 3. & 4.

With respect to the recondite wisdom contained in the description of the infernal regions, I shall only observe from Porphyry (ap. Stob. p. 131.) that the reason why departed spirits, are represented as possessing no knowledge of human concerns, till they inhale the vapour of blood, is because according to Homer and many of his successors, human intelligence or prudence consists in blood. And this says Porphyry is confirmed by the testimony of most writers posterior to Homer, who inform us that when the blood is inflamed by a fever or the bile, imprudence and foolishness is produced. But Empedocles considered the blood as an instrument of prudence, when he says:

Αἷματος ἐν πελάγεσι εθραμμένη ἀντιθορόντος,
Τῇ τε νόημα μάλιχα κυκλήσεται ἀνθρώποισιν.
Αἷμα γὰρ ἄνθρωποις περικάρδιόν ἐστι νόημα.

The sense of which is, “that the blood surrounding the heart is the seat of intelligence in men.”

But we must now view Ulysses passing from sense to imagination; in the course of which voyage, he is assailed by various temptations, of surprising power, and destructive effect. We shall perceive him victorious in some of these, and sinking under others; but struggling against the incursions of all. Among the first of these is the enchanting melody of the Sirens,

Whose song is death, and makes destruction please.

By which the poet evidently signifies alluring and fraudulent pleasures, which charm the soul in its passage from a sensible life, with flattering and mellifluous incantations. These delights however will be vanquished by him, who, imitating the example of Ulysses, closes, with divine reasons and energies as with wax, the powers of the soul, and the organs of sense; so that every passage being barred from access, they may in vain warble the song of extacy, and expect to ruin the soul by the enchanting strain. It will however be requisite that besides this, the corporeal assaults, should be restrained by the bands of philosophy, and rendered irresistible by external machines: for thus like

Ulysses, we shall employ the senses, without yielding to their impetuous invasions; and experience delight without resigning the empire of reason to its fascinating controul.

Ulysses having escaped the danger of the Syrens, passes on to the rocks of Scylla and Charybdis, of terrific appearance, and irresistible force. By these two rocks, the poet seems to signify the affections compressing human life on both sides; and which every one must experience; who proceeds like Ulysses in a regular manner to an intellectual state of existence. Some of these, which are conversant with the soul, are like Scylla, of a lofty malignity; fraudulent yet latent, and obscure, as concealed in the penetralia of the mind. And such is pride, and other depraved affections of the soul. In these recesses, a dæmon, the prince of such affections resides, a dire and enraged dog, who partly exposes his own malice, and partly hides it in impenetrable obscurity. Hence he is capable of producing mischief in a two-fold respect: for he privately hurts by malignant stratagems, openly ravishes the soul, on the lofty rock of haughtiness; and rends it with the triple evil of deadly teeth, I mean revolt, hatred of humanity, and haughty arrogance. Indeed a dæmon of this kind will be perpetually vigilant, in endeavouring to destroy, at one time the whole, and at another time a part of the soul, struggling like Ulysses against passion, and yielding reluctantly to its invasions.

But the other affections which belong to the body, are indeed lofty, and evidently destructive, but far inferior to the others; since their baseness is conspicuous, and not concealed by ostentation. A wild fig-tree, that is the will, is produced on the top of this rock; wild indeed, on account of its free nature, but sweet in fruition; and under which, often through the day, the impetuosities of the boiling body, are accustomed to absorb; and disturb the man, agitating upwards and downwards inflamed desire, so that mighty destruction both to soul and body, is produced by their mutual consent. But it is highly proper that a rock of this last kind, should be anxiously avoided by one, who like Ulysses is labouring to return to his true country and friends. Hence if necessity requires he will rather expose himself to the other: for there the energy of cogitation, and of the souls simple motions, is alone necessary to be exerted; and it is easy to recover the pristine habit of the soul. In short the poet, seems to represent by this allegory of the two rocks, as well the dangers spontaneously arising from the soul, as those which happen from the external mass of matter; both of which must be sustained, or one at least by a necessary consequence. For it is impossible that neither of them should be experienced by any one, who is passing over the stormy ocean of a sensible life.

After this succeeds the allegory of the Trinacrian isle, containing the herds sacred to the god of day, which were violated by the companions of Ulysses; but not without destruction to the authors of this impiety, and the most dreadful danger to Ulysses. By the result of this fable, the poet evidently shews that punishment attends the sacrilegious, and the perjured; and teaches us that we should perpetually reverence divinity with the greatest sanctity of mind, and be cautious how we commit any thing in divine concerns contrary to piety of manners, and purity of thought. But Homer by attributing sense to the flesh and hides of the stain herds, manifestly evinces, that every base deed, universally proclaims the iniquity of its author: but that perjury and sacrilege are attended with the most glaring indications of guilt, and the most horrid signatures of approaching vengeance, and inevitable ruin. We may here too observe that the will of Ulysses, was far from consenting to this impious deed; and that though his passions prevailed at length over his reason, it was not till after frequent admonitions had been employed, and great diligence exerted to prevent its execution. This indeed is so eminently true, that his guilt was the consequence of surprize, and not of premeditated design; which Homer appears to insinuate, by relating that Ulysses was asleep, when his associates committed the offence.

In the next fable, we find Ulysses impelled, by the southern wind, towards the rocks of Scylla and Charybdis; in the latter of which he found safety, by clinging on the fig-tree which grew on its summit, till he refunded the mast, on which he rode after the tempest. But the secret meaning of the allegory appears to me as follows: Ulysses who has not yet taken his leave of a sensible life, is driven by the warmth of passion, represented by the southern gales, into the dire charybdis of insane desires, which frequently boiling over, and tossing on high the storms of deprived affections, plunges into ruin, the soul obnoxious to its waves. However he is far from shameful idleness, and careless security; but perceiving the danger to which he is exposed, when the base storms begin to swell, and the whirlpool of depravity roar, he seizes the helm of temperance, and binds himself fast to the solid texture of his remaining virtue. The waves of desire are indeed tempestuous in the extreme; but before he is forcibly merged by the rage of the affections, into the depths of depravity, he tenaciously adheres to his unconsenting will, seated as it were on the lofty summit of terrene desire. For this like

the wild fig-tree affords the best refuge to the soul struggling with the billows of base perturbations. Hence he by this means recovers the integrity which he had lost, and afterwards swims without danger over the wares of temptation; ever watchful and assiduous while he sails through this impetuous river of the flesh, and is exposed to the stormy blasts of heated passion and destructive vice. Hence too while he is thus affected, and anxious lest the loss from unworthy affections should return upon himself, he will escape being lacerated by the teeth of arrogance, though she should terribly and fiercely bark in the neighbourhood of desire, and endeavour like Scylla, to snatch him on her lofty rock. For these who are involuntarily disturbed like Ulysses by the billows of desire, suffer no inconvenience from the depraved rock of pride: but considering the danger of their present situation, they relinquish confident conceit, for modest diffidence, and anxious hope.

Hitherto we have followed Ulysses in his voyage over the turbulent and dangerous ocean of sense; in which we have seen him struggling against the storms of temptation, and in danger of perishing through the tempestuous billows of vice. We must now attend him in the region of imagination, and mark his progress from the enchanted island, till he regains the long lost empire of his soul. That the poet they by Calypso occultly signifies the phantasy, is I think evident from his description of her abode, (for the anonymous Greek author, affords us no farther assistance.) For she is represented as dwelling in a cavern, illuminated by a great fire; and this cave, is surrounded with a thick wood, is watered by four fountains, and is situated in an island, far remote from any habitable parts, and invironed by the mighty ocean. All which particulars correspond with the phantasy, as I presume the following observations will evince. In the first place then, as the phantasy is situated between sense and cogitation, it communicates with each in such a manner that its beginning is the end of the cogitative power, and its end is the commencement of the senses. Hence on account of its two-fold nature it partakes of a two-fold light; receiving in its supreme part the splendor of cogitation, and in its inferior part, a light corresponding to that of sense. Now it is this inferior part or the common phantasy, which is represented by the cave of Calypso, for its light is artificial and external like that of fire: and this correspondence is evident from the etymology of the phantasy, which is derived from φάος, or *light*. In the next place the island is said to be surrounded with a thick wood, which evidently corresponds to a material nature, or this humid body, with which the phantasy is invested: for ὕλη, or *a wood*, implies matter according to its primary signification. But the four fountains by which the cave is watered occultly intimate those four gnostic powers of the soul discovered by the Pythagoreans, and embraced by Plato; *intelligence, cogitation, opinion, and imagination*. And these fountains are said, with great propriety and correspondence to communicate with each other. In the last place the island is said to be invironed with the ocean, which admirably agrees with a corporeal nature, forever flowing without admitting any periods of repose. And thus much for the secret agreement of the cavern and island with the regions of imagination.

But the poet by denominating the goddess, Calypso, and the island, Ogygia, appears to me, very evidently to confirm the preceding exposition: for Calypso is derived from καλύπτω, which means *to cover as with a veil*; and Ogygia, is from ὀγύγιος, *ancient*. Now, as we have been previously informed by Synesius, the phantastic spirit, is the primary vehicle of the rational soul, which it derived from the planetary spheres, and in which it descended to the corporeal world. It may therefore with great propriety be said to cover the soul, as with a fine garment, or veil; and it is no less properly denominated *ancient*, when considered as the first vehicle of the soul.

In this region of the phantasy then, Ulysses is represented as an involuntary captive; continually employed in bewailing his absence from his true country, and ardently longing to depart from the fascinating embraces of the goddess. For thus his situation is beautifully described by the poet:

But sad Ulysses by himself apart,
Pour'd the big sorrows of his swelling heart;
All on the lonely shore he sat to weep,
And roll'd his eyes around the restless deep;
Tow'rd his lov'd coast, he roll'd his eyes in vain,
'Till dim'd with rising grief they stream'd again.
Lib. v. l. 82. &c.

His return however, is at length effected by means of Mercury, or reason, who prevails on the goddess to yield to his dismissal. Hence after her consent, Ulysses is said with great propriety, to have placed himself on the throne, where Mercury had sate: for reason now resumes her proper seat, and begins to exercise her authority with undisturbed controul. But Homer appears to me to

insinuate something egregious, when he represents Ulysses on his departure from Calypso, sailing by night, and contemplating the order and light of the stars, in the following beautiful lines:

And now rejoycing in the prosp'rous gales,
With beating heart Ulysses spread his sails;
Plac'd at the helm he sate, and mark'd the skies,
Nor clos'd in sleep his ever watchful eyes.
There view'd the Pleiads, and the northern team,
And great Orion's more refulgent beam,
To which around the axle of the sky
The bear revolving, points his golden eye;
Who shines exalted on th' ætherial plain,
Nor bathes his blazing forehead in the main,
Far on the left those radiant fires to keep
The nymph directed, as he sail'd the deep.
Full seventeen nights he cut the foamy way;
The distant land appear'd the following day:
Then swell'd to sight Phæacia's dusky coast,
And woody mountains half in vapour lost;
That lay before him, indistinct and vast,
Like a broad shield amid the wat'ry waste.
Lib. v. l. 269, &c.

Indeed as Ulysses is the image of a man passing in a regular manner from a sensible life, and advancing from darkness to light, he is very properly represented as sailing by the splendor of the stars, and directing his course by the most conspicuous of these illustrious orbs. For star-light corresponds to the light of the mathematical sciences, which are the proper employment of one who is departing from the sensible phantasy, and her detaining charms. And the stars themselves correspond to ideas, from which the light of science is derived. Ulysses therefore who is hastening to an intellectual life, contemplates these lucid objects with vigilant eyes, rejoycing in the illuminations and assistance they afford him, while sailing over the dark ocean of a material nature.

But as he is now earnestly engaged in departing from sense, he must unavoidably be pursued by the anger of Neptune, whose service he has forsaken; and whose offspring he has blinded by stratagem, and irritated by reproach. Hence in the midst of these delightful contemplations, he is almost overwhelmed by the waves of misfortune, roused by the wrath of his implacable foe. He is however through divine assistance or Leucothea, enabled to sustain the dreadful storm: for receiving from divinity, the immortal fillet of true fortitude, and binding it under his breast, (the proper seat of courage) he encounters the billows of adversity, and bravely shoots along the boisterous ocean of life.

Ulysses therefore having with much difficulty escaped the dangers arising from the wrath of Neptune, lands at length on the island of Phæacia, where he is hospitably received, and honourably dismissed. Now as it is proper that he who like Ulysses departs from the delusions of imagination, should immediately betake himself to the more intellectual light of thought, the land of Phæacia, ought to correspond to the realms of cogitation: and that this is the case the following discourse will I persuade myself abundantly evince. In the first place then this island is represented by the poet, as enjoying a perpetual spring; which plainly indicates, that it is not any terrene situation. Indeed the critical commentators have been so fully convinced of this, that they acknowledge Homer describes Phæacia, as one of the Fortunate Islands; but they have not attempted to penetrate his design, by such a description. Now if we consider the perfect liberty, unfading variety, and endless delight, which the regions of cogitation afford, we shall find that it is truly the fortunate island of the soul. In the next place the poet by the description of the palace of Alcinous, the king of this island, egregiously insinuates the pure and splendid light of cogitation: for thus he speaks:

The front appear'd with radiant splendors gay,
Bright as the lamp of night, or orb of day.
The walls were massy brass: the cornice high
Blue metals crown'd, in colours of the sky:
Rich plates of gold the folding doors incase;
The pillars silver, on a brazen base;
Silver the lintels deep projecting o'er,
And gold the ringlets that command the door.
Two rows of stately dogs, on either hand,
In sculptur'd gold, and labour'd silver stand.
These Vulcan form'd intelligent to wait
Immortal guardians at Alcinous' gate.

Lib. vii. l. 84, &c.

And he represents it, as no less luminous internally, by night.

Refulgent pedestals the walls surround,
Which boys of gold with flaming torches crown'd;
The polish'd ore reflecting ev'ry ray;
Blaz'd on the banquets with a double day.

For this palace is not like the cavern of Calypso naturally obscure, but remarkably bright and refulgent. Indeed Homer by his description of the outside of this palace, sufficiently indicates its agreement with the planet Mercury, who is the god of speech; and cogitation as Plato beautifully observes in the Theætetus, is nothing more than inward discourse. For, according to astronomers, the planet Mercury is resplendent with the colours of all the other planets. Thus Baptista Porta in Cœlest. Physiog. p. 88. Videbis in eo Saturni luridum, Martis ignem, Jovis candidum, Veneris flavum, nec-non utriusque nitor, hilaritasque, et ob id non peculiaris formæ, sed eorum formam capit, cum quibus associatur, ob id in describendo ejus colore astrologi differunt. That is, "you may perceive in this planet the pale colour of Saturn, the fire of Mars, the whiteness of Jupiter, and the yellow of Venus: likewise the brilliancy and hilarity of each; and on this account it is not of a peculiar form, but receives the form of its associates, and by this means causes astrologers to differ in describing its colour."

But that the island of Phæacia, is the region of cogitation, is indisputably confirmed by Homer's account of the ships fabricated by its inhabitants: for thus he beautifully describes them:

Ὅφρα σε τῇ πέμπωσι τιτυσκόμεναι φρεσὶ νῆες.
Οὐ γὰρ Φαιήκεσσι κυβερνητῆρες ἔασιν.
Οὐδέ τι πηδάλι' ἐστὶ τὰτ' ἄλλαι νῆες ἔχουσιν.
Ἀλλ' αὐταὶ ἴσασι νοήματα καὶ φρένας ἀνδρῶν.
Καὶ πάντων ἴσασι πόλιας καὶ πῖονας ἀγροῦς
Ἀνθρώπων. καὶ λαῖτμα τάχισθ' ἄλως ἐκπερόωσιν,
Ἥερι καὶ νεφέλῃ κεκαλυμμέναι· οὐδέ ποτέ σφιν,
Οὔτε τι πημανθῆναι ἔπι δέος οὔτ' ἀπολέσθαι.

Lib. viii. l. 556, &c.

i.e. "That ships intently directed by intellect, may send you to your country. For the Phæacians have no pilots, nor have the ships helms like others: but they know the thoughts and minds of men. They likewise know the cities and fertile fields of all men; and swiftly swim over the waters of the sea, covered with darkness and clouds: for they never are afraid of sustaining any damage, or of being utterly lost." Or in verse:

So shalt thou instant reach the realm assign'd,
Is wond'rous ships self-mov'd, instinct with mind;
No helm secures their course, no pilot guides,
Like man intelligent they plow the tides,
Conscious of ev'ry coast, and ev'ry bay,
That lies beneath the fan's all seeing ray;
And veil'd in clouds impervious to the eye,
Fearless and rapid thro' the deep they fly.

Now it is absurd to suppose that Homer would ever employ such an hyperbole, in merely describing the excellency of the Phæacian ships: for it so eminently surpasses the bounds of probability, and is so contrary to the admirable prudence, which Homer continually displays, that it can only be admitted as an allegory, pregnant with latent meaning, and the recondite wisdom of antiquity.

It must here however be observed, that as the energies of cogitation are two-fold, according to the objects on which they are employed (for they are either sensible or intellectual); so the manners of Alcinous and his nobles, are perfectly opposite to those of the other inhabitants. For these latter are thus described by the poet:

A race of rugged mariners are these;
Unpolish'd men, and boistrous as their seas;
The native islanders alone their care,
And hateful he that breathes a foreign air.
These did the ruler of the deep ordain
To build proud navies, and command the main;
On canvas wings to cut the wat'ry way;
No bird more light, *no thought more swift than they.*
Lib. vii. l. 33, &c.

The last of which lines, so remarkably agrees with the preceding account, that I presume no stronger confirmation can be desired. Nor is the original less satisfactory:

Τῶν νέες ὠκεῖται ὥσει πτερόν ἢ ἐ νόημα.

i.e. "The ships of these are swift as a wing, or as *a conception of the mind*." But the inhabitants of the palace, are represented as spending their days in continual festivity, and unceasing mirth: in listening to the harmony of the lyre; or in forming the tuneful mazes of the joyful dance. And this distinction of manners, admirably agrees with the difference between vulgar, and intellectual cogitations: for the former of these are boisterous, and rough, selfish and proud; skilled indeed in rapidity, but groveling and unpolished. But the latter are constantly employed in intellectual festivity and mirth; in tuning the melodious lyre of divine recollection, or forming the responsive dance of refined imaginations. It was with the greatest reason therefore, that Ulysses exclaimed on this occasion:

How sweet the products of a peaceful reign?
The heav'n taught poet, and enchanting strain:
The well fill'd palace, the perpetual feast,
A land rejoycing, and a people blest.
How goodly seems it ever to employ
Man's social days in union, and in joy?
The plenteous board high-heap'd with cates divine,
And o'er the foaming bowl the laughing wine.
Lib. ix. l. 3, &c.

And here we may observe how much the behaviour of Ulysses at the palace of Alcinous, confirms the preceding exposition, and agrees with his character as a man passing in a regular manner from the delusions of sense, to the realities of intellectual enjoyment. For as he is now seated in the palace of cogitation, it is highly proper that he should call to mind his past conduct, and be afflicted with the survey; and that he should be wakened to sorrow by the lyre of reminiscence, and weep over the follies of his active life. Hence when the divine bard Demodocus, inspired by the fury of the muses, sings the wrath of Ulysses and Achilles, on his golden lyre; Ulysses is vehemently affected with the relation. For:

Touch'd at the song, Ulysses strait resign'd
To soft affliction all his manly mind:
Before his eyes the purple vest he drew,
Industrious to conceal the falling dew:
But when the music paus'd, he ceas'd to shed
The flowing tear, and rais'd his drooping head.
Lib. viii. l. 83, &c.

And when the inhabitants of the palace, or refined cogitations, transported with the song, demanded its repetition:

Again Ulysses veil'd his pensive head,
Again unmann'd a shower of sorrow shed.

For reminiscence is delightful to the former, because purified from guilt: but afflictive to the latter, because he has not yet arrived at the summit of virtue, and acquired the perfection of contemplative good.

But while Ulysses is at the palace of Alcinous, Homer takes occasion of introducing, some admirable mythological arcana, in the story of Mars and Venus; the explication of which from Proclus, on Plato's Republic, page 388. will I persuade myself be acceptable to the Platonic reader, and vindicate Homer from acting improperly by introducing this excellent fable. "Both Vulcan and Mars then (says Proclus) operate about the universal world. And Mars indeed separates, perpetually nourishes, and constantly excites the contrarieties of the universe, that the world may exist perfect and entire from all its parts. But Vulcan adorns the whole sensible machine by his artifice, and fills it with certain reasons, proportions, and powers of nature. Hence he is feigned by Homer to construct twenty tripods about the heavens, that he may adorn them with the most perfect of many-sided figures: and he is likewise said to fashion a multitude of sublunary forms, variously figured, and artificially adorned; as when he says:

Chains, bracelets, pendants, all their toys I wrought.
Iliad. xviii.

But each of these deities requires the assistance of Venus; Mars that he may insert order and harmony in contrary natures; but Vulcan that he may induce beauty, in sensible operations, and thus render the world beautiful and divine. However since Venus is every where, Vulcan indeed always enjoys her, according to the superior orders of beings. As for instance, if Vulcan is super-mundane, Mars is mundane; and if the former is in the heavens, the latter enjoys Venus, in the regions subject to the moon. Hence Vulcan is said to have taken Venus for his wife according to the will of Jove; but Mars is reported to have committed adultery with Venus. For the cause of beauty, and conciliation, is naturally conjoined with the artificer of sensible concerns: but is after a manner foreign from the presiding deity of division and contrariety: for the collective, are opposed to the separating genera of gods. And hence fables have given the appellation of adultery, to this consent of dissimilar causes. But a consent of this kind was necessary to the world, that contraries might join in mutual conciliation, and the war of the universe terminate in peace. Indeed as in the celestial regions, beauty, elegance of form, and the fabrications of Vulcan, are particularly conspicuous: but in the realms of generation, elementary war, opposition, contrariety of natural powers, and the gifts of Mars, reside: on this account I say, the sun beholding from the heavens the congress of Mars and Venus, betrays it to Vulcan, with all whose actions he unites in amicable co-operation.

After this, Vulcan entangles them with all-various bonds, invisible to other gods, that he may connect the mundane genera, with artificial reasons; and may produce one constitution from the contrarieties of Mars, and the conciliating benefits of Venus. For both are required in generation. But according to Timæus, there are some bonds of celestial, and others of sublunary concerns: and the latter are dissoluble, but the former of a perpetual nature. Hence Vulcan at last dissolves the bonds, with which he had confined Venus and Mars; and this he effects through the persuasion of Neptune.

Who sues importunate to loose the god.

For Neptune desirous that the perpetual continuity of generation, may be preserved, and that the circle of mutation may revolve into itself, causes begotten natures to be corrupted, and such as are corrupted to be renewed. Is there any reason then, why we should wonder at Homer's feigning Mars and Venus to be fettered with the bonds of Vulcan, since Timæus also gives the appellation of bonds, to the demiurgic reasons, by which the celestial gods constitute generated natures? Is it not likewise congruous to the nature of things, that these bonds should be again dissolved, since they are the bonds of generation? Indeed the first, and most perfect artificer of things seems to have composed the world from contrary elements, and to have given them friendship in a certain proportion, by uniting the effects of Vulcan, Venus, and Mars, in association with each other. For producing the contrarieties of the elements, he operates within himself, according to Mars: but when he fabricates friendship, he energizes according to the power of Venus; and by connecting the natures of Venus and Mars, he appears to contain in himself the primitive exemplar of Vulcan's art. *Hence he is all, and operates with all the gods.* Besides the junior artificers imitating their father, produce mortal animals, and again receive them when they perish, fabricating at the same time with Vulcan, mundane bonds, and primarily containing in themselves the causes of their dissolution. For every where, he who possesses a bond, knows also the necessity of its solution."

We may here too observe that Ulysses, with the greatest propriety relates his past adventures in the palace of Alcinous: for as he now betakes himself to the intellectual light of thought, it is highly

necessary that he should review his past conduct, faithfully enumerate the errors of his life, and anxiously solicit a return to true manners, and perfect rectitude of mind. But the description of his departure from Phæacia is no less pregnant with philosophical mystery, than poetical beauty. For as he is now passing by the pure energy of thought, to his true country, the rational soul; he is represented as departing by night, and falling into so profound a sleep in his voyage, as to be insensible for some time of its happy consummation. For thus according to the poet:

He climb'd the lofty stern; then gently prest
The swelling couch, and lay compos'd to rest.
Lib. xiii. l. 75.

And the vehemence of his thoughts, is finely represented by the rapidity of the vessel:

Now plac'd in order the Phæacian train
Their cables loose, and launch into the main:
At once they bend, and strike their equal oars,
And leave the sinking hills, and less'ning shores.
While on the deck the chief in silence lies,
And pleasing slumbers steal upon his eyes.
As fiery coursers in the rapid race
Urg'd by fierce drivers thro' the dusty space,
Toss their high heads, and scour along the plain;
So mounts the bounding vessel o'er the main.
Back to the stern the parted billows flow,
And the black ocean foams and roars below.
Thus with spread sails the winged galley flies;
Less swift an eagle cuts the liquid skies;
Divine Ulysses was her sacred load,
A man in wisdom equal to a god!
Much danger long and mighty toils he bore,
In storms by sea, and combats on the shore;
All which soft sleep now banish'd from his breast,
Wrapt in a pleasing, deep, and death-like rest.

By the night, therefore, Homer intimates the stillness and tranquillity which attends intellectual contemplation: and by the sweet and death-like sleep of Ulysses, his being abstracted from all sensible concerns, while merged in the profound and delightful energies of thought. For he has now bid adieu to the storms of passion, and the conflicts of desire; and is hastening to expel these dangerous foes, from the secret recesses of his soul.

Nor is it without reason that the poet represents Ithaca as presenting itself to the mariners' view, when the bright morning star emerges from the darkness of night. For thus he speaks:

But when the morning star, with early ray,
Flam'd in the front of heav'n, and promis'd day;
Like distant clouds the mariner descries
Fair Ithaca's emerging hills arise.
Lib. xiii. l. 93, &c.

Since it is only by the dawning beams of intellect, that cogitation can gain a glimpse of the native country, and proper empire of the soul.

But when Ulysses awakes from the delightful sleep of his corporeal energies, and, through the assistance of Minerva, recognizes his native land, he immediately enters into a consultation with the goddess, how he may effectually banish the various perturbations and inordinate desires which yet lurk in the penetralia of his soul. For this purpose it is necessary that he should relinquish all external possessions, mortify every sense, and employ every stratagem, which may finally destroy these malevolent foes. On this occasion the garb of poverty, the wrinkles of age, and the wants of life, are symbols of mortified habits, desertion of sensible pursuits, and an intimate conversion to intellectual good. For the sensitive eye must now give place to the purer sight of the rational soul; and the strength and energies of a corporeal nature must yield to superior vigor of intellectual exertion, and the severe labour of moral investigation. And this Homer appears most evidently to insinuate by the following beautiful lines:

Now seated in the olive's sacred shade,
 Confer the hero and the martial maid,
 The goddess of the azure eyes began:
 Son of Laertes! much-experienc'd man!
 The suitor train thy earliest care demand,
 Of that luxurious race to rid the land:
 Three years thy house their lawless rule has seen,
 And proud addresses to the matchless queen.
 But she thy absence mourns from day to day,
 And inly bleeds, and silent wastes away:
 Elusive of the bridal hour she gives
 Fond hopes to all, and all with hopes deceives.
 Lib. xiii. l. 372.

Hence:

It fits thee now to wear a dark disguise,
 And secret walk unknown to mortal eyes,
 For this my hand shall wither ev'ry grace,
 And ev'ry elegance of form and face,
 O'er thy smooth skin a bark of wrinkles spread,
 Turn hoar the auburn honours of thy head,
 Disfigure every limb with coarse attire,
 And in thy eyes extinguish all the fire;
 Add all the wants and the decays of life,
 Estrange thee from thy own; thy son, thy wife;
 From the loath'd object ev'ry fight shall turn,
 And the blind suitors their destruction scorn.
 Lib. xiii l. 397, &c.

After this follows the discovery of Ulysses to Telemachus, which is no less philosophically sublime than poetically beautiful. For by Telemachus we must understand intellectual virtue, the true progeny of Ulysses, or the rational soul. Hence Ulysses, while employed in the great work of mortification, recognizes his legitimate offspring, and secretly plans with him the destruction of his insidious foes. The assistance however of Minerva, or wisdom, is requisite to this discovery, who beautifies and adorns the rational soul, and restores it on this occasion, to its pristine dignity and excellence of form. But it is necessary that this should be nothing more than a temporary change, till the enemies of reason are destroyed, and the dominion of intellect regained. With great propriety, therefore, is Telemachus represented as exploring his absent father, and impatient for his return: for the rational soul then alone associates with true virtue, when it withdraws itself from sensible delights, and earnestly meditates a restoration of its fallen dignity, and original sway.

And now Ulysses presents himself to our view in the habits of mortification, hastening to his long deserted palace, or the occult recesses of his soul, that he may mark the conduct, and plan the destruction of the malevolent passions, who are secretly attempting to subvert the empire of his mind. Hence the poet very properly and pathetically exclaims:

And now his city strikes the monarch's eyes,
 Alas! how chang'd! a man of miseries;
 Propt on a staff, a beggar old and bare,
 In tatter'd garments, flutt'ring with the air!
 Lib. xvii. l. 201, &c.

However as this disguise was solely assumed for the purpose of procuring ancient purity and lawful rule, he divests himself of the torn garments of mortification, as soon as he begins the destruction of occult desires; and resumes the proper dignity and strength of his genuine form. But it is not without reason that Penelope, who is the image of intellectual purity, furnishes the instrument by which the hostile rout of passions are destroyed: for what besides the arrows of purity can be sufficient to extirpate the leading bands of impurity and vice? Hence as soon as he is furnished with this irresistible weapon, he no longer defers the ruin of his insidious foes, but:

Then fierce the hero o'er the threshold strode;
 Stript of his rags, he blaz'd out like a god.
 Full in their face the lifted bow he bore,
 And quiver'd deaths a formidable store;
 Before his feet the rattling show'r he threw,
 And thus terrific to the suitor crew.
 Lib. xxii. l. 1. &c.

But Homer represents Penelope as remaining ignorant of Ulysses, even after the suitors are destroyed, and he is seated on the throne of majesty, anxious to be known, and impatient to return her chaste and affectionate embrace. For thus he describes her:

Then gliding through the marble valves in state,
Oppos'd before the shining fire she sate.
The monarch by a column high enthron'd,
His eye withdrew, and fix'd it on the ground
Anxious to hear his queen the silence break:
Amaz'd she sate, and impotent to speak;
O'er all the man her eyes she rolls in vain,
Now hopes, now fears, now knows, then doubts again.
Lib. xxiii. l. 88, &c.

Nor ought this to appear strange, for purity has been so long absent from his soul, that it is difficult to obtain a recollection of their pristine union, and legitimate association with each other. However in order to facilitate this discovery, he renders all harmonious and pure, within the recesses of his soul; and by the assistance of Minnerva, or wisdom, resumes the garb and dignity which he had formerly displayed.

Then instant to the bath, (the monarch cries)
Bid the gay youth and sprightly virgins rise,
Thence all descend in pomp, and proud array,
And bid the dome resound the mirthful lay;
While the sweet lyrist airs of raptures sings,
And forms the dance responsive to the strings.
Lib. xxiii. l. 131, &c.

And afterwards Ulysses is described, as appearing through the interposition of Minerva, *like one of the immortals*:

So Pallas his heroic form improves
With bloom divine, and like a god he moves.
Lib. xxiii. l. 163.

For indeed he who, like Ulysses, has destroyed the power of his passions, and purified himself from their defiling nature, no longer ranks in the order of mortals, but assimilates with divinity itself. And now that he may become entirely known to intellectual purity, that chaste Penelope of the soul, he need only relate the secrets of their mystic union, and recognize the bower of celestial love. For then perfect recollection will ensue; and the anxiety of diffidence will be changed into transports of assurance, and tears of rapturous delight.

And thus we have attended Ulysses through his various wanderings and woes, till he recovers the ruined empire of his soul. We must however remember, according to the beautiful observation of Porphyry, in the above treatise, that he is not freed from molestation till he has passed over the raging sea of a material nature, has entirely appeased the anger of divinity, and is become so insensible to the concerns of a corporeal life, as to be ignorant of their nature and use. For:

Then heav'n decrees in peace to end his days,
And steal himself from life by slow decays;
Unknown to pain, in age resign his breath,
When late stern Neptune points the shaft of death;
To the dark grave retiring as to rest;
His people blessing, by his people blest.
Lib. iii, l. 281, &c.

[95] Ὀρkuνος δέ τις ἐξὶ λιμὴν ἁλίοιο γέροντος.

[96] is Sopater, succeeded Plotinus in his philosophical school. Vide Sozom. Hist. Eccles. 1, 5.

[97] The exact time of Iamblichus' death is unknown; it is however certain, that it was during the reign of Constantine; and according to the accurate Fabricius, prior to the year of Christ 333. Biblioth. Græc. Tom. 4. p. 283.

[98] e vol. I. of this work.

[99] l. I. Sect. 4.

[100]I have already mentioned this admirable work, with the praise it so justly deserves, in my account of the life and writings of Proclus. (Vol. I. of this work, p. 38.) I now add, with great pleasure that I have been no less fortunate with respect to the translation of these Elements than in translating the commentaries of Proclus. For the Greek text, is very frequently defective in parts essential to the meaning; and consequently necessary to the perfection of the whole. This defect I have been able to supply from a Latin version of Patricius (Ferrar, 1583) who appears to have had a perfect manuscript in his possession. But notwithstanding this assistance, I must freely own, that I never translated any thing which required so much intense thought, and severe labour in its execution. This indeed must necessarily be the case, if the abstruseness of the subject, the difficulty of finding proper terms, and the defects of the original, are properly considered. But the learned reader may be experimentally convinced of the truth of this assertion, if he only compares the Greek text with the Latin version of Æmilius Portus; in which I am sorry to say, he will scarcely find one proposition, in which Portus has not mistaken the sense of his author. Indeed were I disposed to entertain the critical reader, I might swell the volume with copious notes from the blunders of Portus; and display the superiority of my own version by contrasting the English with the Latin. But I consider verbose criticisms of this kind as both useless and pedantic; as remote from the philosophical genius; and as alone calculated to fill up the leisure hours of men, who have ruined their understandings in the study of words. The genuine Platonist who may be ignorant of Greek, will I persuade myself, rejoice to see this invaluable treasure in his native tongue: and those who have been led to consider the theology of the heathens as delusion, and absurdity, will doubtless be surprized to find, that it is replete with the sublimest knowledge, and the most important truths. Yet I must admonish the reader, that these Elements cannot be understood by any one, who is not a thorough adept in the preceding Commentaries on Euclid: for the propositions relate to the most abstract subjects that can be conceived; and the demonstrations are uncommonly subtle and profound. Indeed if opportunity permitted, I should attempt a commentary on every proposition: but this if ever I should be able to accomplish it, must be reserved for some more auspicious period. In the mean time I hope that my occasional elucidations, will be acceptable to the Platonic reader, and assist him in the study of this inestimable work. I only add, that these Elements form an admirable introduction to the six books of Proclus on Plato's theology.

[101]The absurdity of this consequence consists in this, that from the hypothesis of unity being excluded, infinite infinities would subsist separate from each other, without any bond of conciliating unity; which is evidently impossible. For though every line contains infinite infinities in *capacity*, because the smallest part of every line is capable of infinite division; yet these infinities in capacity are connected by the power of unity, and form one finite line.

[102]Because, since the whole is motive, the part which is moved will also be motive, and so will be at the same time both moving and moved. And if a part moves, and the whole is moved; then because of the motion of whole, the part will be both moving and moved.

[103]The truth of this may be exemplified in light. Thus for instance we see many species of light; one kind emanating from the sun, another from fire and the stars, another from the moon, and another from the eyes of many animals. But this light though various, is every where familiar, and discovers in its operations a unity of nature. On account of its uniformity therefore it requires one principle, and not different principles. But the sun is the only principle of all mundane light. And though there are many participants of light posterior to the solar orb, yet they scatter their uniform light, through one solar nature, property and power. But if we again seek for the principle of light in the sun, we cannot say that the solar orb is the principle of light: for the various parts of this orb diffuse many illuminations. There will therefore be many principles. But we now require one first principle of light. And if we say that the soul of the sun generates light, we must observe that this is not effected by her animal multiplicity, or she would diffuse different lights: and hence we must assert that she generates visible by intellectual light. But this production again, does not subsist through intellectual variety, but rather by a unity of intellect: and this unity is a symbol of that simple unity which is the principle of the universe. And to this principle the solar intellect is united by its unity; and through this it becomes a god. This divine unity of the sun, therefore, is the principle of the uniform light of the world; in the same manner as simple unity and goodness is the author of intelligible light to all intelligible natures.

[104]This will be evident by considering that *the one*, or the first principle of all, must produce that which first proceeds from himself, by *union* (καθ' ἑνωσιν.) And as his first production must be the

most similar of all things to himself, and must be at the same time multitude (or in what respect would it differ from the one); hence it is necessary, that this progression must be no other than self-perfect unities. In consequence therefore of this sublime doctrine, as Proclus beautifully observes (Theol. Plat. p. 123.) there is one god, and many gods; one unity, and many unities prior to beings: and one goodness, and many goodnesses after this first good. It likewise follows that the first principle of all is a super-essential one; and that after this one, there are many super-essential unities. And we may consider every unity of beings, as the flower of some certain being; and as the summit and centre about which every being subsists. For a farther account and confirmation of this sublime doctrine, study the third book of Proclus on Plato's Theology.

[103] *imparticipable*, in these Elements, we must understand that which participates nothing belonging to the characteristic of its nature, and which is likewise participated through proper mediums by all subordinate natures. Thus for instance *imparticipable being*, is that which participates nothing of *being*, but is the source of *being*, to others; and at the same time it is not participated without a medium by subordinate essences; since the participation of being, in subordinate natures, is accomplished through the immediate progeny of *being itself*. And so of the rest. Nor ought it to seem wonderful that an imparticipable should preside over every series of causes: for this is a necessary consequence, if we consider that every cause must be that to its following progeny, which the first cause of all, is to the universality of things. But the first cause of all is perfectly imparticipable; since he would not be purely *the one*, if he was mingled with *many*, and received the condition of a subordinate nature. As the divine unities therefore are participated by all the following orders of things, and are the mediums, by which every being is conjoined with the first cause of all; so the immediate progeny of every primary cause conjoins all the subsequent progressions with that cause, which from its similarity to *the first* is properly denominated an imparticipable.

[104] Cause if the one generates through motion, an infinite motion must take place, through the want of an immoveable nature.

[105] order to understand this proposition the reader must observe that the hypothesis requires that both the progression and regression subsist together. And this hypothesis is no less proper than true; for unless effects were continually converted to their causes, they could not exist; since they depend on these for their subsistence, and this can only be procured by conversion.

[106] is absurd, because every partible nature must be converted to something different from itself, on account of its parts. So likewise since a self-subsistent nature resides in itself, if such a nature was partible, one divisible whole would be in another, not different from itself.

[107] Cause every composite consists of matter and form; the former of which is base, and the latter more excellent.

[108] The truth of this reasoning may be evinced by the following considerations: Every thing which is measured by time, and such is every corporeal nature, depends on time for the perfection of its being. But time is composed of the past, present, and future. And if we conceive that any one of these periods is taken away from the nature with which it is connected, that nature must immediately perish. Time, therefore, is so essentially and intimately united with the natures which it measures, that their being, such as it is, depends on the existence of time. But time, as is evident, is perpetually flowing, and this in the most rapid manner imagination can conceive. It is evident, therefore, that the natures to which it is so essential, must subsist in a manner equally transitory and flowing. As we cannot therefore affirm with propriety of any part of time, that it *is*; since even before we can form the assertion, the present time is no more; so with respect to all corporeal natures (from their subsistence in time) before we can say that they exist, they lose all identity of being. But as it appears to me this flowing, and evanescent nature of things existing in time, may be aptly illustrated by the following similitude. Conceive a line, all whose parts are in a continual flux, like the waters of the most rapid river: and let it be distinguished by three different colours, so profoundly mingled with each other, that every part of the line may possess these colours, in the same manner as the whole line. Now a line of this kind, will very properly represent time, distinguished by past, present, and future, to which the three colours correspond. If then we conceive another line moving on this coloured line, with an equable uniform motion, corresponding to the motion of the parts of the coloured line, and so that in every instant of its motion it may be tinged with one of these colours in regular succession; we shall see the condition of a nature whose being is measured by the

progressions of time. For as the perpendicular line, in its uniform motion, no sooner assumes one colour, than it deserts it for another, and cannot on this account be said to possess any colour; so every corporeal nature, from its being profoundly mingled with the fleeting and unreal essence of time, cannot be said to possess any true and substantial being.

Such then is the unreal condition of every thing subsisting in time, or of every thing corporeal, and entangled with matter. By this shadowy essence of body is finely demonstrated by Plotinus in the sixth book of his third Ennead, as follows: "Being (says he) properly so called is neither body, nor is subject to corporeal affections: but body, and its properties, belong to the region of non-entity. But you will ask, how is it possible, that visible matter, should possess no real being: that matter, in which stones and mountains reside, the solid earth, and bodies which mutually resist; since bodies which impel each other, confess by their collision, the reality of their existence? You will likewise ask after what manner things which neither strike against, nor resist each other; which neither externally act, nor internally suffer, nor are in any respect the objects of sight, viz. soul and intellect, are to be reckoned true and real beings? We reply, that, on the contrary, things more corpulent are more sluggish and inert, as is evident in bulky masses of earth: but whatever is less ponderous is more movable and alert; and the more elevated the more movable. Hence fire, the most movable of all the elements, flies in a manner from a corporeal nature. Besides, as it appears to me, whatever is more sufficient to itself, disturbs others less, and brings less inconvenience: but such things as are more ponderous and terrene, unable, from their defect of being, to raise themselves aloft, and becoming debile and languid, strike and oppress surrounding bodies, by their falling ruin and sluggish weight. Since it is evident, that bodies destitute of life, fall with molestation on any proximate substance, and more vehemently impel and pain whatever is endued with sense. On the contrary, animated beings, as participating more of entity, by how much the more of being they possess, by so much the more harmless they impinge on their neighbouring bodies. Hence motion, which is a kind of life, or soul, or an imitation of life in bodies, is more present with whatever is less corpulent; as if more of body was necessarily produced where a defect of being happens in a greater degree. Again, it will more manifestly appear from passivity, that whatever is more corpulent is more passive; earth in greater degree than the other elements; and the rest in a similar proportion. For some things, when divided, suddenly return to their former union, when no obstacle prevents their conjunction; but from the section of a terrene body, the divided portions, always remain separate; as if destitute of natural vigour, and without any inherent desire of union and consent. Hence they are ready, by every trifling impulse, to remain as they are impelled; to rush from the embraces of *bound*, and hasten into multitude, and non-entity. So that whatever becomes corporeal in an eminent degree, as falling fast into non-entity, has but little power of recalling itself into one. And on this account ponderous and vehement concussions are attended with ruin, when by mutual rushing one thing impels another. But when debility runs against debility, the one is valid against the other, in the same manner as non-entity rushing on non-entity. And this we think a sufficient confutation of their opinion, who only place being in the genus of body, persuaded by the testimony of impulses and concussions, and the phantasms perceived through the senses, which testify that sense is alone the standard of truth. Such as these are affected in a manner similar to those in a dream, who imagine that the perceptions of sleep are true. For sense is alone the employment of the dormant soul: since as much of the soul as is merged in body, so much of it sleeps. But true elevation, and true vigilance, is a resurrection from and not with the dull mass of body. For indeed a resurrection with body, is only a transmigration from sleep to sleep, and from dream to dream, like a man passing in the dark from bed to bed. But that elevation is perfectly true, which entirely rises from the dead weight of bodies: for these possessing a nature repugnant to soul, possess something opposite to essence. And this is farther evident, from their generation, their continual flowing and decay; properties entirely foreign from the nature of being, substantial and real."

[111]The account of eternity given by Plotinus, in which its *total* nature is copiously demonstrated, is so inimitably sublime and profound, that I cannot refrain from presenting the reader with the following paraphrased translation, of the most considerable part of the seventh book of his third Ennead, relative to the nature of eternity. Proclus (in Theol. Plat. p. 149.) observes that the definition of eternity given by Plotinus in this discourse, viz. "Infinite life, which at once openly exhibits itself, and manifestly declares its own being," is a definition framed *in a most divinely inspired manner* (ἐνθεακικώτατα.)

“Since we reckon eternity and time to be different from each other, and that the one respects a nature which always is, and that the other is conversant with things in generation, unstable and flowing, from which this sensible universe is composed; we are apt to believe that from our natural ingenuity, and a sudden view of intelligence, an instinctive knowledge concerning time and eternity, is profoundly inherent in our souls; since we always affirm the same properties of these, and never vary in the appellations we assign them. But when we endeavour to approach nearer to these, and to explain their natures in a more perfect manner, we are immediately perplexed with doubts; and receiving the different sentiments of the ancients on this subject, according to the difference of our opinions, and perhaps conceiving the same sentiment, in a manner different from other men, we advance no farther in our enquiries; esteeming it sufficient, if when interrogated, we are able to relate the opinions of antiquity on this important subject. Indeed it is proper to believe that some of the ancient, and blessed philosophers, have discovered the truth: but it is highly necessary to enquire what method they pursued in their investigation, and how we may arrive at the knowledge of the same exalted truths. And first, it will be proper to enquire what they think of eternity, who judge it to be different from time: since from the knowledge of that which is, as the exemplar, we may perhaps more plainly understand, the nature of its image, which is denominated time. But if any one previous to his contemplation of eternity, should conceive the nature of time in his imagination, he may perhaps happen, by advancing into the intelligible world, and by a certain reminiscence, to comprehend that to which time is assimilated: since time possesses a similitude to eternity.

What then shall we say eternity is? Is it intelligible essence itself, in the same manner, as some are reported to affirm, that time is the whole heavens, and the world? For since we imagine and understand eternity to be something especially venerable, which we likewise affirm of an intelligible nature, we are not able to discern which of these is most venerable. And as that which is superior to these, ought not to be described by appellations of this kind, perhaps eternity, and an intelligible nature, may with propriety be received as the same. Again, both eternity, and the intelligible world, comprehend in themselves beings entirely the same. But when we say that the one abides in the other, placing intelligibles in eternity, and when we pronounce every thing intelligible eternal, as in the *Timæus* of Plato, where it is written, “If the nature of the exemplar is eternal, &c.” in these cases, I say, we affirm eternity to be something different from an intelligible nature; at the same time confessing that eternity dwells about, or is resident in, or is present with an intelligible nature. For indeed it is not necessary that because both are venerable, they should be perfectly the same: since perhaps the venerableness of the one, is derived from the other. And what is farther added to establish their identity, that the same things are contained by each, is to be thus explained: that an intelligible nature contains every thing belonging to itself, as parts; but that eternity comprehends at the same time the whole, not as a part, but because all things of this kind are called eternal, from its continual presence. But ought we to judge of eternity according to permanent station in the intelligible world, as in this sensible world we are accustomed to consider time according to motion? But it may be properly enquired, in the first place, whether eternity, is the same with station belonging to being. For if it is the same with station, we cannot with mere propriety, speak of a station which is eternal, than we can of an eternal eternity: since that is eternal which participates of eternity. Besides, if station is the same with eternity, how can motion be eternal? For in this case, it must lose its nature, and become permanent and stable. Besides, how is the notion of station contained in the *ever itself*? I do not mean the *ever*, or *always* which respects time, but that which we understand when we speak of eternity. But if the *ever itself*, is contained in the station of being, we must consequently segregate the other genera of beings from eternity. Besides, it is requisite to conceive of eternity, not in station alone, but as abiding in *one*; and to preserve its indistant property, lest it should become entirely the same with time. But station, considered simply as station, neither contains in itself the intelligence of one, nor of that which is indistant. Hence we predicate of eternity, that it abides in one: and consequently it participates of station, without being station itself.

What then is the nature of that, by which we call the whole of the divine world eternal and perpetual? And what is perpetuity itself? Is it the same with eternity, or does eternity subsist according to the perpetual? Perhaps we ought to conceive of eternity, as a certain one collected from many; viz. either as one intelligence, or one nature; whether consequent to things in the intelligible world, or existing together with it, or beheld as situated in the depths of its essence. All these, I say, reduced into eternity a one, which is also many, and is endued with a various capacity. Indeed he who beholds a various capacity, when he considers it as a subject, denominates it essence; but so far as he perceives life, he denominates it motion, and afterwards station, considered as abiding in a

manner entirely the same. He will likewise behold difference and sameness, so far as they are many, bound in one. So that he who contracts the difference, subsisting in things which are many, into one life alone, and contemplates an unceasing sameness of energy, never passing its intelligence, or life, from one thing into another, but ever abiding in the same manner in itself, far remote from all distance; he, I say, who beholds all these, contemplates eternity, viewing life ever possessing a present whole, where all things abide together in sameness, without the order of first and last, and are comprehended in an indivisible bound. Where all things are collected into one, as into a point, not yet proceeding into a linear flux, but abiding in sameness, that is, in itself, in an ever present *now*; because nothing of its nature is past, nothing in it is future; but what it is, it always is. Hence eternity is not a subject, but that which beams as it were from its subject, according to the possession of an ever present identity; promising itself, that its ever abiding nature, will never be changed. For what should happen to this in future, which it is not at present? Since it is a perfect and present plenitude of all things. Nor can the term *was*, the appellation of time past, belong to eternity. For what can that be, which *was* present with its nature, and *is past*? It is in like manner independent of all connection with futurity. And hence eternity is that which neither *was*, nor *will be*, but alone *is*, which it possesses in a stable manner; because it is neither changed into a future, nor altered from a past duration. So that the eternity, which we are now investigating, is life total and full, abiding in its essence about being; and is every where indistant and one.

Nor indeed is it to be thought, that eternity happens extrinsically to the first essence; but we should consider it, as dwelling in this essence, as emanating from, and abiding together with it. For it is beheld as profoundly resident in its essence; because from perceiving every thing else which we affirm to abide there, as perfectly intimate, we say that all things emanate from essence, and abide together with essence. For it is requisite that first natures should abide with, and reside in first beings, and in these have the whole of their essence, because the beautiful dwells in these, and emanates from them: and this also may be asserted of truth. And partly indeed the whole itself, is as it were in a part, but partly, as parts in the whole: as if this true nature was a whole, not congregated from parts, but itself generating parts, that on this account it may be truly all. Besides, truth which flourishes there, is not a certain consonance towards another, but is peculiar to every thing of which it is the truth. Indeed it is requisite that this total truth, if total, should not only be total, so far as it is all things, but should likewise so possess the whole, that nothing may be wanting to itself. And if this is the case, no affection of futurity can possibly belong to this being: for if the circulation of time can add any thing to its essence, it must be defective till such an accession takes place. Besides, nothing contrary to nature can happen to this true being; for it suffers nothing. So that since nothing is generated in it, its being is independent of all temporal connections; and hence it is neither *about to be*, nor *will be*, nor *was made*, nor *is made*. If indeed the affections of futurity are taken away from generated natures, because they are conversant with perpetual acquisition, they cease to exist. But to things of a nature essentially different, the addition of any future circumstance of being causes them to fall from the stable seat of essence. So that it is manifest from hence, that being itself could not reside in these natures, if they were composed from the past, and future processions of time. Since essence in generated natures appears to be nothing more than a certain procession from *real being*, from the beginning of generation, till it arrives at the extremity of time, when it ceases any longer to exist. And that which is called the being of these, subsists in such a manner that by diminishing any part of this extension, life is likewise diminished: for in this manner it is necessary that every thing which participates of futurity should exist. Hence it continually hastens, by a natural desire, to what is future, without desiring to repose; since it continually draws fresh supplies of being to itself, while it performs one thing after another, and is moved in a certain orb of progression, by its desire of essence. And thus we have discovered the existence of temporal beings, and the cause of their motion to eternal duration, through the assistance of futurity. But to such things as are first, and to beings truly blessed, the desire of futurity is unknown; for they are already total; and whatever of life they ought to possess, they now totally possess: so that they seek after nothing, because nothing which is requisite to the full energy and perfection of their essence is future, and consequently nothing happens to them connected with futurity. The absolute and total essence, therefore, of a being which is not separated into parts, but is total in all its parts, and in no one circumstance of being deficient, and to which nothing of non-entity can possibly happen (for it is not only requisite that all beings should be present with that which is universal; and total, but that everything appertaining to non-entity, should be always excluded) is eternity: for that is called eternal which always is."

And shortly after this, he adds: “Hence eternity is something especially venerable, and is the same with a god, as inherent intelligence affirms. But intelligence likewise dictates that eternity is the same with that god, whom we denominate being and life. And it may with the greatest propriety be said, that eternity is a deity shining and unfolding himself in intelligible light, such as he is in his essence: in an essence, I say, perfectly unchangeable, and the same, and thus firmly abiding in an unceasing energy of life. Nor ought any one to wonder that we speak of eternity, as composed from many. For every thing which abides in the intelligible world, is called many, on account of its infinite power; since infinite there receives its denomination, because it never falls off from the consummate intellectual plenitude of its nature. And indeed it is particularly called after this manner, because it loses nothing of its own: and if any one should describe eternity after this manner, as life already infinite because universal, and because it never deserts the integrity of its nature; (since it cannot be diminished by the past, nor increased, by the future, because it is a perfect whole) he who thus describes eternity, will approach very near to its true definition. For what is afterwards added, that it is a perfect whole, and loses nothing of its integrity, is only a certain exposition of the definition which affirms it to be infinite life. But because a nature of this kind, thus all-beautiful and eternal, abides about the *one itself*, emanating and in no respect departing from it, but ever abiding about and in it, and living with it, in indissoluble union; hence it is said by Plato, not rashly, but in a manner truly beautiful, and profound, *that eternity abides in one*: so that he not only reduces that which eternity contains into one; but the life of being in like manner reduces itself, about *the one itself*. This then is what we investigate, and that is eternity, which thus abides. For that which is the energy of life abiding from itself, and residing in the depths of unity, without any deception, either in essence or life, is without all controversy eternity. Since truly *to be*, is never *not to be*, and to possess no diversity of being. But when in discoursing on eternity, we use the term *ever*; also when we say, it is not sometimes *being*, and sometimes *non-being*, we must consider these appellations as adopted only for the purpose of explanation; because the term *ever* is not perhaps principally assumed, but is employed, in order to shew, an incorruptible, and never-failing nature.

But here it is proper to enquire, whether we affirm all this concerning eternity, by means of a power foreign from its nature, and as of a foreign concern? But how is this possible? For how can intelligence take place in the soul, unless by an intimate union of intellect, with the object of its intellection? And after what manner can this profound union subsist, by an intelligence of such things as are foreign from our intellectual nature? It is therefore requisite, that we should be partakers of eternity; or we could not thus define its nature, and exhibit the properties which it contains.”

[112] The nature of *time itself*, which every thing in generation participates, has been a subject of much subtle and profound speculation to the Pythagoric, Platonic, and Peripatetic philosophers. According to Archytas the Pythagorean (as we are informed by Simplicius, on Aristotle’s Categories) “time is a generative number, proceeding from the motion of the mundane soul, as from unity.” But others of the ancients considered it as a certain extension of soul about intellect. Others confessed, that time was the period of soul and intellect. According to the determination of others, it is a natural extension about intellect. And others described time, by circulations in a circular nature; all which the Pythagoric sect comprehends. Time, therefore, according to Archytas and the most ancient philosophers, is a self-moving motion, and the number of temporal unity: for as eternity abides in *one*, so time is moved in *number*. According to Plato, “time is a certain image flowing in number, of eternity abiding in one.” But according to Aristotle, “it is the number, or measure of motion according to prior, and posterior.” But the opinion of Plotinus respecting time, is perhaps the most profound of all: and though the preceding definitions when attentively considered contain an abundance of intellectual truth, yet the doctrine of Plotinus on this subject *rekindles* a greater light in our intellectual conceptions. In his book then on eternity and time, which we have cited in the preceding proposition, after many admirable doubts concerning the nature of time, he enters on its definition, as follows:

“But it is requisite to recal ourselves again to that nature which we have said resides in eternity: a nature immutable and total, and a life infinite and indeclinable, and abiding in, and about the one. But time was not as yet, or at least was not among these true beings, but was about to follow, by a certain reason and nature of a posterior energy. Supernal natures therefore acting quietly among themselves, if any one desires to know how time first fell, he will not perhaps improperly call upon the Muses, who then were not, for the important explanation. Perhaps also he will opportunely

invoke them, since the Muses then existed, according to true conceptions of their nature. And perhaps such a one will find time already generated, such as it now is, and manifests itself to our intellectual view. But he will speak concerning it after the following manner:

“Before priority originated, and was indigent of that which is posterior, time not yet existing according to procession, reposed together with priority in being itself, with a tranquil and intellectual energy. But at length time desiring a nature engaged in a multiplicity of affairs, and anxious to govern itself, and become independent, and always chusing to possess more of that which is present, began to be agitated, and at the same time commenced a flowing duration, subsisting in continual succession, and a transition into different periods of existence. And by conceiving that from this motion, a certain longitude of procession is effected, we establish time, as an image of eternity. For since it is a certain restless power of the mundane soul, desirous of transferring what it there beholds, into another, it cannot sustain that a collected whole should be at once present with its nature: but as the spermatic reason, unfolding itself from a certain quiet seed, acts according to an abundant discursive energy; dispersing in the mean time that which is called abundant, by a certain division, and instead of the one which abides in itself, unfolding and consuming a non-abiding one, and thus proceeding into a more debile extension: so likewise the nature of the mundane soul fabricating the sensible world in imitation of that which is superior, and agitated with a motion, not such as the intelligible world contains, but rather similar to the motion which is there; with a motion I say affecting to represent its image, it renders itself in the first place temporal, as a substitute for eternity. But afterwards it endeavours to become subservient to time, because it compels the world to reside in time, comprehending all its transitions in its forming nature. For the world is moved in this nature; since there is no other place for the universe than the mundane soul; and in the time of this soul, it is agitated without end. For this soul applying one energy after another in succession, generates that which is consequent, together with its energy, and at the same time proceeds with a cogitation posterior to this energy, and which before this had no existence, because cogitation was not yet in energy. Its present life too, is now not entirely similar to the preceding life: and hence together with this energy, another life succeeds; and in consequence of this another time is produced. Time, therefore, is allotted a distance and protraction of life; and a perpetual prorogation of life, possesses a perpetual time. If any one, therefore, should say that time is an energy of the mundane soul, perpetually proceeding in a certain transitory motion, from one life to another, he will perhaps appear to assert something correspondent to the truth. For if eternity is life abiding in station, and in a habit similar and the same, and already infinite; and if it is requisite that time should be the image of eternity, in the same manner as this universe is the image of the intelligible world, certainly instead of the life which flourishes there, another life must reside in a certain power of the mundane soul, equivocal with respect to intelligible life; and instead of intellectual motion, it is requisite that there should be a motion of some part of the soul. Likewise instead of identity, and a similar and abiding habit, that which does not abide in the same, but always pursues a different energy must succeed. And again instead of a property indistant, and one, a resemblance of one possessing unity in a continued succession. Besides instead of that which is infinite and total, that which perpetually propagates itself in infinite succession. Lastly, instead of that which is a collected whole, that which sustains parts, and is always about to be total and perfect. For thus it will imitate that which is total, collected, and infinite, if it wishes its essence to consist in a continual pursuit of being; and thus resembles the being of eternity itself. But it is requisite not to receive time external to soul, in the same manner as eternity is not external to being itself. Nor again, must we conceive that time exists as an appendage, or any thing posterior, in the same manner as conceptions of this kind, are not proper to eternity: but we must contemplate it as beheld seated in the recesses of soul, with which it exists in conjunction, just as eternity subsists in being itself.”

[11B] the last of beings, he means matter, which on account of its formless nature may be considered as nothing more than the shadow of being; or something if possible still more privative and simple.

[11B] ὑπαρξίς, *hyparxis*, in these Elements is meant that *characteristic* or *summit* of any nature, through which it subsists: and in the gods is the same with the *unity* and *deity* of their natures. And it is likewise necessary to inform the reader, that by ὑπόστασις, *hypostasis*, or subsistence, is meant any individual nature whether essential, or super-essential, considered as something distinct and different from *accident*.

[115] matter proceeding from the cause of all, nothing more is meant than that it depends entirely on the first cause for its shadowy and unreal subsistence: for as the emanations of causes are extended in proportion to their eminence; hence the precessions of the one extend beyond those of every other cause, and even leave faint traces of their illuminations in the dark receptacle of matter.

[116] It follows from Prop. 69. For every whole prior to parts, may be said to be a whole of parts, because every whole formed from parts, participates of that totality which is prior to parts.

[117] Thus matter possesses a certain obscure image of being, but does not preserve the most debile impression of form. For as the gradations of being are more extended than those of form, and as matter is the last of things; thence matter may be said to retain the footstep of *being* in its dark receptacle, whilst the processions of form are reflected like echoes from its rebounding seat.

[118] The proposition ends here in the Greek, though very erroneously; and its conclusion forms the beginning of the next proposition. It is strange that Portus would not have detected, this egregious mistake, which the alteration of a single letter, would have enabled him to rectify. Thus if after ἀλλὰ μὲν' αἰεὶ ὑπόκειται γινόμενον. ἀπειρον ἄρα ἔχει την' του γίνεσθαι δυναμιν, which is the conclusion of the proposition; πᾶν' τὸ ὄντως ὄν, &c. is retained, instead of ἕαν τὸ ὄντως ὄν, and is made the beginning of the next proposition, the whole will be correct and plain. Such mistakes, are dreadful instances of the danger attending the understanding from the study of words alone.

[119] If a reader not skilled in the Platonic philosophy, it will doubtless appear strange, that *being* should be prior to *eternity*, and yet each participate of one another. This apparent paradox may be easily solved, by considering that the mode of participation is different in each. For *being* participates of eternity, as establishing, illuminating, replenishing, and deifying eternity: but *eternity* participates of being, as depending upon, established and deified by being. So that when a superior, participates of an inferior nature, the participation consists in the energy of the former on the latter: but when an inferior participates of a superior nature, the participation consists in its receiving the communications of the superior nature.

[120] The reader must not be surprised to find that among infinities, some are more infinite than others. For as among beings some are truer than others, and possess more of real being in proportion as they approach nearer to being itself; at the same time that they are all in a certain respect beings: so infinities possess more of infinity, as they approach nearer to the infinite itself. Thus for instance eternity possesses infinity more truly than time, though time also is infinite; because the infinity of eternity, is a stable indivisible life, but the infinity of time consists in an unceasing progression, or as it were an unwearied pursuit of infinity, which it can alone obtain in an extended and partible manner. And this difference among infinities extends even to matter itself, which is the most degraded and abject of all infinities, because it is infinite only in the most dormant capacity.

[121] Thus for instance the first being, or being *itself*, does not flow from any being as its cause, because it is without any participation of being; and though it proceeds from a superior cause, i.e. the *one itself*, yet it does not proceed from this cause, on account of its being an imparticipable, but on account of its being subordinate to *the one*, and consequently a participant of the one. And this is likewise true in every other imparticipable.

[122] ὄντως in the original, which means indeed literally, *truly*. But the philosophical reader will please to observe that as it is an adverb derived from the participle ὄν, or *being*; its full meaning is *truly according to being*. So that by *true life*, we must understand *life according to the truth of being*; and so of the rest.

[123] If that which is primarily eternal, is eternal according to essence, but not according to energy, it will be the same with the world, which is the first participant of time. For the world is essentially eternal, because though its parts are subject to change, yet considered as a whole, it is perpetually the same. But then it is not eternal in *energy*, but in *capacity* alone. For it subsists in a perpetual capacity of existence; and its flowing essence is perpetually composed from the past, present, and future circulations of time.

[124] In the original it is κατ' οὐσίαν, but it should doubtless be read, according to our translation κατ' ἐνέργειαν.

[125] Thus for instance the qualities which subsist about bodies are incorporeal, and consequently eternal and incorruptible; but yet they are not immortal, because they are destitute of life.

[126] the Greek θανάτου is erroneously printed, instead of αθανάτου, which the reader will evidently perceive must be the true reading. But though this is sufficiently obvious to those who understand the proposition; yet Portus, who seldom attends to the meaning, has, from not rectifying this mistake, given the most ridiculous translation of this concluding sentence, that can possibly be imagined. The original is: τὸ δὲ ὄν τῆς ζωῆς περιληπτικώτερον, καὶ τοῦ θανάτου, (lege αθανάτου) ἄρα τὸ ἄϊδιον. The translation of Portus: “Illud vero, quod est, vitam, et mortem magis complectitur, vel, ipsum vero ens, plura complectitur, et latius patet, quam vita et mors. Ergo ipsum eternum est illud, quod non recipit ipsum non esse, et quod semper est;” i.e. “But that which is, comprehends in a greater degree, life and death; or being itself comprehends more, and is of greater extent than life and death. Therefore, eternity itself, is that which does not receive non-being, and which always is.”

[127] that is neither according to time alone, nor eternity alone: but from the participation of both.

[128] instead of ὡς δι’ ὁμοίων ἀνόμοιον, read ὡς δι’ ὁμοίων εἰς ὅμοιον. And read with astonishment the version of Portus; who not perceiving the error of the text, has made nonsense of the passage, as usual.

[129] such as understand these Elements, this argument for the existence of a multitude of gods, is perfectly demonstrative and clear. Indeed as every production of nature possesses the power of generating its similar, it is much more necessary that the first cause of all should generate a multitude the most similar to himself, that can possibly be conceived. For every being produces that which is similar prior to the dissimilar; as indeed a contrary mode of proceeding would be absurd and impossible. The immediate or first productions therefore of the first god, must be a multitude of gods: or otherwise his first progeny would not be perfectly similar to himself. Nor does this doctrine, in any respect derogate from the dignity of the supreme god, as the ignorant suppose; but on the contrary tends to exalt his majesty, and evince the ineffable beneficence and perfection of his nature. For though it establishes a multitude of gods, yet it teaches that they are dependant on the first, who is perfectly incomprehensible, and without participation. So that it leads us to consider the subordinate deities, as so many lesser luminaries shining before the presence of the sun of good, and encircling with awful grandeur his ineffable radiance, and occult retreats. And that this doctrine fully displays his superlative goodness, is sufficiently manifest; since by a contrary assertion we must ascribe imperfection to the fountain of excellence, and leave deity impotent and barren. Indeed this opinion is so natural and reasonable, that, excepting the Jews, it was embraced by every nation of the ancient world. Nor ought we to wonder that the Jews were an exception to the universal consent of mankind, in this important particular. For with respect to the origin of this despicable, *though chosen people*, it was scarcely known to the Heathens; “the greater part of whom,” as Dr. Burnest justly observes (De Origin. Rer. cap. 7.) “supposed them to be natives of Egypt, sprung from the same root, or considered them as a vile, and inconsiderable people.” And as to their learning we may remark with the same author, “that they never excelled in philosophical, or mathematical knowledge; and never gave the world a famous example of the strength of human wit; from whence arose that bitter reproach of Apollonius, *That the Jews were the most trifling of all the barbarians, and that they were the only people who had never found out any thing useful for life.*” The example of the Jews, therefore, ought to be so far from deciding against Polytheism, that when their character is impartially estimated, it will strengthen the evidence of its reasonableness and truth.

[130] though the first cause of *the one itself*, confers on every thing a proper symbol of his ineffable nature; yet this occult unity, or impression is not divine in things subject to generation and decay, but in true essences alone, in the number of which rational souls must be ranked. Such of these, however, as are of a partial nature, and on this account are not the immediate progeny of *the first one*, do not contain a unity which can be called a god; because they are connected with motion, and are in a certain respect composite essences. But where there is a most true essence, as in separate intellects, and celestial souls, the unity of each is a god. And indeed on account of these unities, which are as it were expressive characters of the first unity, the essences of the gods contain all things, and extend their providential care to every part of the universe, with unbounded beneficence, and immaculate power. But these divine unities are perpetually united to the first one, like rays to light, and lines to a centre. They likewise subsist in the most perfect union with each other. For since union in other natures is effected through the power of unity, these divine unities must be much more closely united, through their subsisting much nearer to the first and most perfect one. Every divine unity, therefore, though it is neither essence, nor obnoxious to essential multitude, yet abides in essence; or is rather the summit, and as it were blossom of essence. And as every thing is established

in its proper species through form, and as we derive our being through soul; so every god is a deity, from the secret unity which he contains. Hence these divine unities subsist in the intelligible world, and in the essences of the gods, like so many splendid lamps in diaphanous spheres, mingling their rays with an ineffable union, energy, and consent. And situated in most admirable order, in the vestibule of the good, they occultly signify divine silence, and solitary beauty; and perspicuously announce to posterior natures the awful sanctuary of their incomprehensible cause.

[134] may farther infer the necessity of providence in the gods, from considering that as they are the producing causes of all things, so all things abide, and are radically established in their natures. For where can any thing subsist, which is not contained in their unknown and all-pervading comprehensions? But if this be the case, since all things are in reality the offspring of the gods, they must continually be the objects of their providential exertions. For as goodness is the characteristic of these divine natures, it is impossible that they should abandon their progeny, or cease to impart their beneficent, unenvying, and all-powerful communications. Nor must we think that these providential exertions are laborious to the gods; since, as Proclus well observes, (Theol. Plat. p. 41.) that which is according to nature, is not laborious to any thing. "For neither (says he) is it laborious for fire to give warmth, nor for snow to refrigerate, nor for bodies themselves to energize according to their peculiar powers. Nor prior to bodies, is it laborious to natures themselves to nourish, or generate, or increase: for these are the operations of natures. Nor again, prior to these, is it laborious to souls to exert their peculiar energies: for many of their energies are attended with delight, many are the result of their essence; and many motions are produced by their presence alone." Hence if the communication of good naturally belongs to the gods, providence also is natural to these divinities, which they exert in a tranquil, unpolled, and incorporeal manner.

Should it be enquired in what manner providence operates, the following beautiful passage from Proclus on the Parmenides, as cited by Ficinus, in his commentary on that dialogue, will give us abundant satisfaction: "Let us conceive (says he) a ship agitated by the winds and waves, and let us suppose, that the imagination of some one is so powerful, that while he imagines the sea, the sea immediately flows; that while he imagines the ship, the ship is constructed; and that the winds and waves, arise agreeable to his imagination, and as the consequences of its vehement energy. Now it is evident, that such a one would not be compelled, in surveying these particulars, to employ a confused and distracted vision; but both his knowledge and operation, would equally subsist in a uniform manner. And such is the simplicity of divine intelligence, with respect to the intuition and fabrication of inferior concerns."

[135] The reader must remember that the gods are alone super-essential, through their *unities*, which are the characteristics of their natures: for as irrationality is the essential signature of a brute, and rationality of a man; so a divine unity, is the invariable characteristic of a god.

[136] By this ἀπὸ ψυχῆς, or *resounding echo of soul*, we must understand that vital quality, by which the soul is united to the body; and which is nothing more than the extreme image and shadow of the soul. The necessity of such a connecting quality, will easily appear, from considering that an *incorporeal* nature, like that of soul, cannot be connected with *body*, without a vital medium. In consequence of this we may consider with Plotinus, (Ennead. iv. lib. 4.) the animated body as resembling illuminated and heated air; and the pains and pleasures of the body, will be conversant with this shadow of the soul.

[137] This must be understood of immediate participation; and on this hypothesis, it is certainly true, that neither two or more unities are participated by one being. For since there is an order among the unities, and some are more universal than others, two cannot be immediately participated by one being, without an alteration taking place in the subsistence of each. This is evident from the cxxxii. Proposition, in which it is proved *that all the orders of the gods are bound in union by a medium*: and consequently since all the unions are connected by proper mediums, it is impossible that any two can be immediately participated.

[138] For as being itself, is no other than the highest order of the gods and the most uniform multitude, and as the characteristic of every god is a divine unity; hence the characteristic of being itself, will be the, unity proceeding from bound. But as all the divine unities are super-essential; hence *being itself* according to its characteristic will be super-essential.

[139] For as matter is deservedly called *non-being*, because it is *worse* than all things; in like manner this appellation is proper to the first cause, as he is *better* than all things.

[137] who understands this divine sentence, *that all things sympathise with all*, will see that the *magic* cultivated by the ancient philosophers, is founded in a theory no less sublime than rational and true. Such a one will contemplate the universe as one great animal, all whose parts are in union and consent with each other; so that nothing is foreign and detached; nothing strictly speaking void of sympathy and life. For through various parts of the world, when considered as separated from the whole, are destitute of *peculiar* life; yet they possess some degree of animation however inconsiderable, when considered with relation to the universe. Life indeed may be compared to a perpetual and universal sound; and the soul of the world resembles a lyre, or some other musical instrument, from which we may suppose this sound to be emitted. But from the unbounded diffusion as it were of the mundane soul, every thing participates of this harmonical sound, in greater or less perfection, according to the dignity of its nature. So that while life every where resounds, the most abject of beings may be said to retain a faint echo of the melody produced from the mundane lyre. It was doubtless from profoundly considering this sympathy between the mundane soul, and the parts of the world, that the ancient philosophers were enabled to procure the presence of divinity, and perform effects, beyond the comprehension of the *vulgar*. And that this was the opinion of Plotinus, the following passage evinces: “It appears to me that the ancient wise men, who wished to procure the presence of the gods, by fabricating statues and performing sacred rites, directed their intellectual eye to the nature of the universe, and perceived that the nature of soul was every where easy to be attracted, when a proper subject was at hand, easily passive to its influence. But every thing adapted to imitation, is readily passive; and is like a mirror able to seize a certain form, and reflect it to the view.” Ennead iv. lib. 3.

[138] as this divine light operates according to the debilitated nature of the subject into which it is received (and there is no other way in which it can operate), it necessarily appears to receive a dominion foreign from its own. Hence it appears both obscure and impotent, though in reality neither: for these are the imperfections of the subject which it actuates and illumines.

[139] According to the Chaldaic theology, which is the same with the Platonic, the intelligible order, subsisting immediately after *the one*, consists of three triads; the summit of each being called *father*, the middle, *power*, and the third *intellect*. This is profoundly and beautifully explained by Proclus in Theol. Plat. lib. 3. to which I refer the reader desirous of copious information, on this sublime and interesting subject. I only add that Plato in the *Timæus*, ascribes these appellations to the demiurgus of the universe: and this sufficiently proves that words of this kind are not the invention of the latter Platonists, according to the ignorant assertion of Hyde, in the preface to his book, “On the Religion of the Persians;” a work of an alluring title, but of insignificant execution.

[140] consequence of this the cause of protection must be superior to the cause of purgation, or purity. For since protection preserves things in their proper order, but purity separates things excellent from such as are base, and the latter is preparatory to the former; hence protection must be superior in the order of causes to purgation.

[141] That *being* and *form* differ from each other is evident from the 73d and 74th proposition of these Elements, in which it is demonstrated that *being* is superior to *form*; because being is above that which is total; and that which is total is above form.

[142] who understands this will see the propriety of the appellation *fabricator*, and *father*, given by Plato in the *Timæus* to the artificer of the world; and why *fabricator* is placed before *father*; concerning which consult Proclus on Plato’s Theology, lib. v. cap. 16.

[143] is must be understood, not as if the first intellect understood nothing but itself; but that it understands no other intellect besides itself. For the divine unities, and the first one, are the objects of its continual speculation: and in this exalted employment its life and felicity invariably and eternally consists.

[144] is last sentence of the proposition, is in the original: καὶ νοητὸν ἐστὶ. τοῦτο, τὸ μὲν, ὃ ἐστὶ, τὸ δέ, ἄφ’ οὗ ἐστίν. And this Portus renders with his usual unskilfulness: *et est mente-perceptum. Hoc autem partim quidem, est illud, quod est, partim vero, est illud, a quo est.* That is, “and it is intelligible. But this is partly that which it is, but partly is that from which it is.” The wrong pointing after ἐστὶ, which instead of period should be a comma, and τοῦτο being erroneously printed for τοῦτῳ were the causes of Portus’ mistake: and this he would have rectified, had he understood the proposition.

[145]us for instance intellect in *being itself*, which comprehends the highest order of intelligibles, is νοῦς νοητός, or an *intelligible intellect*; because it is the object of intelligence to all subordinate natures, and because its vision is transcendently simple and occult. But every intellect is indeed the same with the intelligible in its own nature, but is subordinate to *intelligible itself*.

[146]Intellect in energy, or in the act of understanding is the same with the object of its intellection. For the object of its perception, must be resident in its essence, or it would perceive externally like sense; and thus would not behold the thing itself, but only its image. But if that which is intelligible is seated in the essence of intellect, it will in no respect differ from intellect: for it will be essential to its nature, and will consequently be intellectual, as well as intelligible.

[147]By an intellection *according to one* (καθέν) we must understand a various intelligence subsisting indivisibly, and without mutation. Just as when by one and the same energy of vision, we survey the various parts of the same countenance though distant from each other. And an intellection of this kind belongs to all intellects subordinate to the first: for the intelligence of this is perfectly simple, and comprehends all things in one.

[148]That part of his proposition, within the crotchets is in one place very defective in the Greek, which may be corrected as follows: Instead of reading οἱ δὲ μέτ' ἐκείνους, δια πλειόνων, ελασσώ. κατὰ τὴν δυ[ν]αμιν οὖν ελλαμψιν ἐκείνοι δι' ἐλασσόνων πλείονα παραγούσιν, ὀλικοίτερα ἐν αὐτοῖς εἶδη: read after πλειωνων:—ελασσώ, κατὰ τὴν δυναμιν ἄλλειψιν. εἰ οὖν ἐκεῖνοι δε and the rest as above.

[149]The reader must observe that this is to be understood of immediate participation.

[150]Instead of ἡ γὰρ θεοτης συνεξαπτεῖ τῷ νῷ. κ. τ. λ. as in the original, the sense requires that we should read: ὁ γὰρ εὖς συνεξαπτεῖ τῷ θεοτητεῖ. κ. τ. λ.

[151]For inferiors are comprehended in superiors, and particulars in universals; so that he who knows universals, knows particulars also; though the reverse of this is not true. The soul, therefore, by possessing a natural capacity of knowing herself, and things superior to her own nature, will from the illuminations attending her knowledge of the latter, know herself in a much more eminent and perspicuous manner.

[152]us for instance the body of the world, or of any particular animal, is vital from its receiving the echo of soul; but as it does not at the same time, essentially possess *life*, (or else it would be soul) it is composed from life, and non-life. For it is nothing more than body, or non-life, united with the last image of soul, or a debile life.

[153]His truly divine sentence, is derived from the most profound theory; and can alone be understood by those who have deeply studied the six books of Proclus on Plato's Theology. I shall, therefore, only observe for the sake of the *intellectual reader*, that as *essence*, *life*, and *intellect* subsist in occult union, in the first being, which comprehends the highest order of the gods; and as *intelligence* is the medium between that which is *intelligible*, and *intellect*; it is evident that life itself is intelligence.

[154]Because eternity subsists in life, and is no other than infinite life, as is copiously proved from Plotinus, in the note to Proposition LII.

[155]For that which is properly eternal, is perfectly stable, and is never subject to mutation.

[156]Proposition CLXXXIV. And from hence it appears, that by particular souls in this Proposition, are meant such as are capable of being changed from the possession of intellect, into its privation.

[157]The reader must observe that these vehicles or divine bodies, the first participants of their correspondent souls, are no other than those vehicles, so beautifully described by Synesius, and inserted in the preceding history of Theology.

[158]For a demonstration of the truth of this sentence, see the note to page 59, vol. I. of this work.

[159]Instead of ὥστε καὶ ἡ μερική ψυχὴ, καὶ τὸ αἴτιον. κ. τ. λ.—I read, ὥστε τὸ αἴτιον τῆς μερικῆς ψυχῆς, καὶ ἄρα. κ. τ. λ.

[160]ρόνων is erroneously printed in the Greek, instead of χειτᾶνων.

[161]That part of this proposition, within the crotchets, is neither in the Greek which is here defective, nor in the translation of Patricius, but is added from my own conjecture; whether or not with

propriety, the reader must determine.

Transcriber's Notes:

1. Obvious printers' punctuation and spelling errors have been corrected silently.
2. Where appropriate, the original spelling has been retained.
3. Some hyphenated and non-hyphenated versions of the same words have been retained as in the original.

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